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Research Article

Reverse Zagreb And Reverse Hyper-Zagreb Indices Of Triangular Snake Graph And Alternate Triangular Snake Graph

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Abstract

In the fields of chemical graph theory, topological index is a type of a molecular descriptor that is calculated based on the graph of a chemical compound. In this paper, we investigate first and second reserve Zagreb indices, first and second reverse hyper-Zagreb indices, Sum connectivity reverse index, a product connectivity reverse index and atomic bond connectivity reverse index, Reverse geometric arithemetic index for the snake and alternate snake graph are obtained.

Keywords: Triangular snake graph, different topological indices.

1. Introduction

Topological indices enable us to collect information about algebraic structures and give us a mathematical approach to understand the properties of algebraic structures. With the help of topological indices, we can guess the properties of chemical compounds without performing experiments in wet lab. There are more than 148 topological indices in the literature, but none of them completely give all properties of under study compounds. Together, they do it to some extent; hence, there is always room to introduce new indices. Here, we will discuss some newly introduced first and second reverse Zagreb indices, hyper-Zagreb indices, [1–9] for *snake graph and alternating snake graph*

A graph having no loop or multiple edges is known as *simple graph*. A *molecular graph* is a simple graph in which atoms and bonds are represented by vertex and edge sets, respectively. The *vertex degree* $d_u(G)$ is the number of edges attached to that vertex [10–16]. The *maximum degree* of vertex among the vertices of a graph is denoted by $\Delta(G)$. Kulli et al. [17] introduce the concept of reverse vertex degree $c_u(G)$, defined as $c_u(G) = \Delta(G) - d_u(G) + 1$ where $d_u(G)$ is the degree of he vertex *u* in the graph *G*. For convenience sake we represent $c_u(G)$ by c_u

In discrete mathematics, graph theory in general is not only the study of different properties of objects but it also tells us about objects having same properties as investigating object. These properties of different objects are of main interest. Actually, topological indices are numeric quantities that tell us about the whole structure of graph. There are many topological indices [18, 19] that help us to study physical, chemical reactivities, and biological properties. Wiener, in 1947 [20], firstly introduce the concept of topological index while working on boiling point. In particular, Hosoya polynomial [21] plays an important in the area of distance-based topological indices; we can find out Wiener index, hyper-Wiener index, and Tratch–Stankevich–Zefirov index by Hosoya polynomial [22, 23]. Other well-established polynomials are Zagreb and hyper-Zagreb polynomials introduced by Gao.

The first and second reverse Zagreb indices are as follows:

(i).$CM_1(G) = \sum_{uv\in E(G)} c_u + c_v$ (ii) $CM_2(G) = \sum_{uv\in E(G)} c_u c_v$

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Now, the first and second reverse hyper-Zagreb indices are given as

(iii).$HCM_1(G) = \sum_{uv\in E(G)} \left(c_u + c_v\right)^2$ (iv) $HCM_2(G) = \sum_{uv\in E(G)} \left(c_u c_v\right)^2$

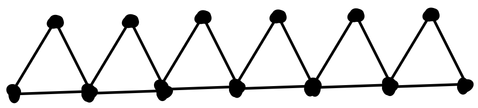
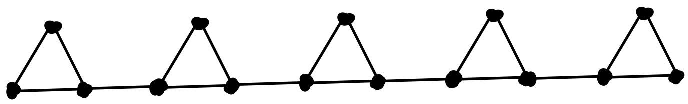
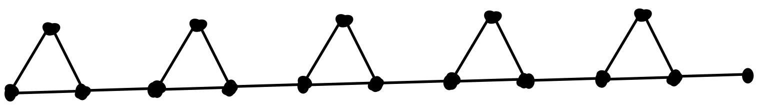
2) Atom-bond connectivity reverse index and geometric arithmetic reverse index are abbreviated as *ABCC* index and *GAC*(*G*) and defined as follows[32]

(v) $ABCC(G) = \sum_{uv\in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}$ (vi) $GAC(G) = \sum_{uv\in E(G)} \sqrt{\frac{2c_u c_v}{c_u + c_v}}$

3) Sum connectivity reverse index and product connectivity reverse index of a graph are represented as
SC(*G*) and *PC*(*G*) and are given by

(vii) $SC(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{c_u + c_v}}$ (viii) $PC(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{c_u c_v}}$

Definition 2.1: *Snake Graph and Alternate Triangular Snake graph*
The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3. An alternate triangular snake $A(T_n)$ is obtained from a path u_1, u_2,----,u_n by joining $u_i u_{i+1}$(alternately) to a new vertex v_i.

Name of the Graph	Graph
Snake Graph T_7	
Alternate Snake Graph A(T_7)	
Alternate Snake Graph A(T_8)	

3.Main Results

In this paper we compute Reverse Zagreb topological Indices of Triangular snake graph and alternate triangular snake graph

Theorem 3.1: If $\boldsymbol{G \cong T_n}$ **is the Triangular snake graph, then**
$(i) CM_1(G) = 6\text{n} + 6$ $(ii) CM_2(G) = 3\text{n} + 17$ $(iii) HCM_1(G) = 12n + 76$

$(iv) HCM_2(G) = 3n + 173$ $(v) ABC(G) = \frac{2\sqrt{2}}{\sqrt{3}} + \frac{4}{3}$

$(vi) GAC(G) = \sqrt{6} + n - 3 + 2\sqrt{3} + \sqrt{2}(n-2)$ $(vii) SC(G) = 1 + \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{2}}(3n-7)$

$(viii) PC(G) = 3n + \frac{19}{3} + \frac{2}{\sqrt{3}}$

Proof: Let $v_{1,}v_{2,} \dots . v_n$ be the vertices of the path P_n of T_n and $v_1', v_2', \dots . v_n'$ be the tip vertices of triangle with base vertices as $v_{i,}v_{i+1}$. There are two pairs of edges with degrees 2,4 ; (*n*-3) pairs of edges with degrees 4,4 and two pairs of edges with degrees 2,2 and 2(*n*-2) pairs of edges with degrees 2,4. Further the reverse vertex degree is $c_u(G) = \Delta(G) - d_u(G) + 1$. Thus the number of edge pairs with their reverse vertex degrees are as given in the following table.

No. of edge pairs	Reverse vertex degrees
2	3,1
n-3	1,1
2	3,3
2(n-2)	3,1

With these reverse vertex degree sequences, we have

(i).$CM_1(G) = \sum_{uv\in E(G)} c_u + c_v$

$= (2(3+1) + (n-3)(1+1)) + (2(3+3) + 2(n-2)(1+1)) = 6n + 6$

(ii) $CM_2(G) = \sum_{uv\in E(G)} c_u c_v$

$= (2(3.1) + (n-3)(1.1)) + (2(3.3) + 2(n-2)(1.1)) = 3n + 17$

iii).$HCM_1(G) = \sum_{uv\in E(G)} (c_u + c_v)^2$

$= (2(3+1)^2 + (n-3)(1+1)^2) + (2(3+3)^2 + 2(n-2)(1+1)^2) = 12n + 76$

(iv) $HCM_2(G) = \sum_{uv\in E(G)} (c_u c_v)^2$

$= (2(3.1)^2 + (n-3)(1.1)^2) + (2(3.3)^2 + 2(n-2)(1.1)^2) = 3n + 173$

(v) $ABCC(G) = \sum_{uv\in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}$

$= \sqrt{\frac{3+1-2}{3.1}} + (n-3)\sqrt{\frac{1+1-2}{1.1}} + 2\sqrt{\frac{3+3-2}{3.3}} + (n-2)\sqrt{\frac{1+1-2}{1.1}}$

$= 2\,2\sqrt{\frac{2}{3}}\left(1 + \sqrt{\frac{2}{3}}\right)$

(vi) $GAC(G) = \sum_{uv\in E(G)} \sqrt{\frac{2c_u c_v}{c_u + c_v}}$

$= \sqrt{\frac{2.3.1}{3+1}} + (n-3)\sqrt{\frac{2.1.1}{1+1}} + 2\sqrt{\frac{2.3.3}{3+3}} + 2(n-2)\sqrt{\frac{1.1}{1+1}}$

$= \sqrt{6} + n - 3 + 2\sqrt{3} + \sqrt{2}(n-2)$

(vii) $SC(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{c_u + c_v}}$

$= 2\sqrt{\frac{1}{3+1}} + (n-3)\sqrt{\frac{1}{1+1}} + 2\sqrt{\frac{1}{3+3}} + 2(n-2)\sqrt{\frac{1}{1+1}}$

$= 1 + \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{2}}(3n-7)$

(viii) $PC(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{c_u c_v}}$

$= \quad 2\sqrt{\frac{1}{3.1}} + (n-3)\sqrt{\frac{1}{1.1}} + 2\sqrt{\frac{1}{3.3}} + 2(n-2)\sqrt{\frac{1}{1.1}}$

$= 3n + \frac{19}{3} + \frac{2}{\sqrt{3}}$

Theorem 3.2: If $G \cong A(T_n)$ is the Alternate Triangular snake graph, then
$(i) CM_1(G) = 5\text{n} - 1 \quad (ii) CM_2(G) = 3\text{n} + 2 \quad (iii) HCM_1(G) = 13n + 11$
$(iv) HCM_2(G) = 5n + 18 \quad (v) ABC(G) = \frac{n}{\sqrt{2}}$
$(vi) GAC(G) = \frac{2}{\sqrt{3}}(n-1) + n - 3 + \sqrt{2}\ (vii) SC(G) = \frac{1}{\sqrt{3}} + \frac{1}{2} + \frac{n-2}{\sqrt{3}} + \frac{n-3}{\sqrt{2}}$
$(viii) PC(G) = \frac{n-1}{\sqrt{2}} + n - 2$

Proof:
Case(i)*: *For odd n
Let $v_{1,} v_{2,} \dots . v_n$ be the vertices of the path P_n of $A(T_n)$ and $v_1', v_2', \dots . v_{\frac{n-1}{2}}'$ be the tip vertices of triangle with base vertices as $v_{2i-1,} v_{2i}, 1 \le i \le \frac{n-1}{2}$. Thus there are $\frac{n-1}{2}\ c_3 s$ in $A(T_n)$. Further there is one edge pair each with reverse edge degrees as 2,1 ; 1,3 and 2,2 $\frac{n+1}{2}$ pairs of edges with reverse edge degrees as 1,1, and (n-1) edge pairs of edges with reverse vertex degrees 2,1 .

(i).$CM_1(G) = \sum_{uv\in E(G)} c_u + c_v$

$= (1(2+1) + (n-3)(1+1)) + 1(1+3) + 1(2+2) + (n-2)(2+1) = 5n - 1$

(ii) $CM_2(G) = \sum_{uv\in E(G)} c_u c_v$

$= (1(2.1) + (n-3)(1.1)) + 1(1.3) + 1(2.2) + (n-2)(2.1) = 3n + 2$

iii).$HCM_1(G) = \sum_{uv\in E(G)} (c_u + c_v)^2$

$= ((2+1)^2 + (n-3)(1+1)^2) + (1(1+3)^2 + 1(2+2)^2 + (n-2)(2+1)^2) = 13n + 11$

(iv) $HCM_2(G) = \sum_{uv\in E(G)} (c_u c_v)^2$

$= ((2.1)^2 + (n-3)(1.1)^2) + (1(1.3)^2 + 1(2.2)^2 + (n-2)(2.1)^2) = 5n + 18$

(v) $ABCC(G) = \sum_{uv\in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}$

$= \sqrt{\frac{2+1-2}{2.1}} + 1\sqrt{\frac{2+2-2}{2.2}} + (n-2)\sqrt{\frac{2+1-2}{2.1}}$

$= n\sqrt{\frac{1}{2}}$

(vi) $GAC(G) = \sum_{uv\in E(G)} \sqrt{\frac{2c_u c_v}{c_u + c_v}}$

$= \sqrt{\frac{2.2.1}{2+1}} + (n-3)\sqrt{\frac{2.1.1}{1+1}} + 1\sqrt{\frac{2.2.2}{2+2}} + (n-2)\sqrt{\frac{2.2.1}{2+1}}$

$= \frac{2}{\sqrt{3}}(n-1)+n-3$

(vii) $SC(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{c_u + c_v}}$

$= \sqrt{\frac{1}{2+1}}+(n-3)\sqrt{\frac{1}{1+1}}+\sqrt{\frac{1}{2+2}}+2(n-2)\sqrt{\frac{1}{2+1}}$

$= \frac{1}{\sqrt{3}}(n-1)+\frac{1}{\sqrt{2}}(n-3)+\frac{1}{2}$

(viii) $PC(G) = \sum_{uv\in E(G)} \sqrt{\frac{1}{c_u c_v}}$

$= \sqrt{\frac{1}{2.1}}+(n-3)\sqrt{\frac{1}{1.1}}+\sqrt{\frac{1}{1.3}}+\sqrt{\frac{1}{2.2}}+(n-2)\sqrt{\frac{1}{2.1}}$

$= n-2+\frac{n-1}{\sqrt{2}}$

Case(ii): *For even n*

Let $v_{1,}v_{2,} \dots . v_n$ be the vertices of the path P_n of $A(T_n)$ and $v_1', v_2', \dots . v_{\frac{n}{2}}'$ be the tip vertices of triangle with base vertices as $v_{2i-1,}v_{2i}, 1 \le i \le \frac{n-1}{2}$. Thus there are $\frac{n}{2}\, c_3 s$ in $A(T_n)$. Further there are two edge pairs with reverse edge degrees as 2,1; 2,2 and (n-2) pairs of edges with reverse edge degrees as 1,1; 2,1

(i).$CM_1(G) = \sum_{uv\in E(G)} c_u + c_v$

$= 2(2+1)+(n-2)(1+1)+2(2+2)+(n-2)(2+1) = 5n-4$

(ii) $CM_2(G) = \sum_{uv\in E(G)} c_u c_v$

$= 2(2.1)+(n-2)(1.1)+2(2.2)+(n-2)(2.1) = 3n+6$

iii).$HCM_1(G) = \sum_{uv\in E(G)} (c_u + c_v)^2$

$= 2(2+1)^2+(n-2)(1+1)^2+2(2+2)^2+(n-2)(2+1)^2 = 13n+24$

(iv) $HCM_2(G) = \sum_{uv\in E(G)} (c_u c_v)^2$

$= 2(2.1)^2+(n-2)(1.1)^2+2(2.2)^2+(n-2)(2.1)^2 = 5n+32$

(v) $ABCC(G) = \sum_{uv\in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u c_v}}$

$= 2\sqrt{\frac{2+1-2}{2.1}}+2\sqrt{\frac{2+2-2}{2.2}}+(n-2)\sqrt{\frac{2+1-2}{2.1}}$

$= (n+2)\sqrt{\frac{1}{2}}$

(vi) $GAC(G) = \sum_{uv\in E(G)} \sqrt{\frac{2c_u c_v}{c_u + c_v}}$

$= 2\sqrt{\frac{2.2.1}{2+1}}+(n-2)\sqrt{\frac{2.1.1}{1+1}}+2\sqrt{\frac{2.2.2}{2+2}}+(n-2)\sqrt{\frac{2.1.2}{2+1}}$

$= \frac{2n}{\sqrt{3}} + n - 2 + 2\sqrt{2}$

(vii) $SC(G) = \sum_{uv \in E(G)} \sqrt{\frac{1}{c_u + c_v}}$

$= 2\sqrt{\frac{1}{2+1}} + (n-2)\sqrt{\frac{1}{1+1}} + 2\sqrt{\frac{1}{2+2}} + (n-2)\sqrt{\frac{1}{2+1}}$

$= \frac{n}{\sqrt{3}} + \frac{n-2}{\sqrt{2}} + 1$

(viii) $PC(G) = \sum_{uv \in E(G)} \sqrt{\frac{1}{c_u c_v}}$

$= 2\sqrt{\frac{1}{2.1}} + (n-2)\sqrt{\frac{1}{1.1}} + 2\sqrt{\frac{1}{2.2}} + (n-2)\sqrt{\frac{1}{2.1}}$

$= n - 1 + \frac{n}{\sqrt{2}}$

3. Conclusion:

In this paper, we computed first and second reverse Zagreb indices, first and second reverse hyper-Zagreb indices, reverse GA index, reverse atomic bond connectivity index, first and second reverse Zagreb polynomials, and first and second reverse hyper-Zagreb polynomials for the crystallographic structure of molecules [24, 25]. Our results are important to guess the properties [26–28] and study the topology of the crystallographic structure of molecules and can be used in drug delivery [29–31].

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