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Research Article

Study Of Some Plane Gravitational Waves In Bimetric Relativity

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ABSTRACT

Z = $\left(\sqrt{x^2+y^2+z^2}-t\right)$ Z = $\left\{\frac{t}{\sqrt{x^2+y^2+z^2}}\right\}$, Z = (z-t),

Z = $\left(\frac{\sqrt{3}\ t}{x+y+z}\right)$, Z = $\left[\frac{x^1+x^2+x^3+\cdots\dots\dots+x^{n-1}}{\sqrt{n-1}}\right]$

Z = $\left(\frac{\sqrt{n-1}\ t}{x^1+x^2+x^3+\cdots\dots\dots+x^{n-1}}\right)$, Z = $[(x^1)^n+(x^2)^n+\cdots\dots+(x^{n-1})^n]^{\frac{1}{n}}-t$

and Z = $\left\{\frac{t}{[(x^1)^n+(x^2)^n+\cdots\dots\dots+(x^{n-1})^n]^{\frac{1}{n}}}\right\}$

Plane gravitational waves in Rosen‟s Bimetric Relativity. The remarkable work of the background and inspiration for the present research work. In "Mathematical theory of plane gravitational waves in general relativity", introduced the concept of plane gravitational waves from the cosmological point of view, gave a quite clear mathematical formulation and developed its consequences. His remarkable work, forms the background and motivation of our investigation in this research.

Introduction

General Theory of Relativity

As we all know that, in the year 1915 Einstein formulated the theory of relativity as a theory of gravitation nearly ten years after the special theory of relativity. He very well knew the conflicts between Newtonian Gravitation and Special Relativity. His theory deals with the uniform motion of the bodies in free space, where there is negligible gravitational effect. Einstein had achieved a great success in geometrization of gravity in his theory of relativity; which successfully demonstrates the fusion of mechanics with the theory of gravitation with geometry. Einstein further combined the intrinsic property of a space-time region with its geometry i.e.

$$s^2 = ct^2 - x^2 - y^2 - z^2 \qquad (1.1.1)$$

The geometry of space-time is pseudo-Euclidean [In a Euclidean geometry, the generalized Pythagoras theorem applies. In equation (1.1.1) the right hand side has positive as well as negative squares. Hence, the word "Pseudo" is used. Since this difference is not significant in many geometrical properties of space-time. Thus the word non Euclidean will imply, „Other than Euclidean as well as Pseudo Euclidean‟ in special relativity.

The general theory of relativity has been formulated by considering two main points i.e. (1) Physical and (2) Mathematical.

While formulating this theory; Einstein considered three main general principles: Firstly, the Covariance Principle, which helps us to write the physical laws in covariant form so that, their form remains invariant in all system of co-ordinates. Secondly, the Principle of equivalence is an axiom of distinguishability between gravity and inertia, which automatically includes the effect of gravity into the development of the theory. Lastly, he also used Mach‟s Principle to determine the geometry of the space-time and thereby inertial properties of a test particle, from the information of density and

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mass energy distribution in its neighborhood. Hence, we can say that the theory of relativity of space-time is described by Pseudo-Riemannian metric

$ds^2 = g_{ij}\, dx^i dx^j$, $i = j = 1,2,3,4$ (1.1.2)

where the components of metric tensor play the role of gravitational potentials. The gravitational field manifes ts through the curvature of the space-time and the general field equation governing the gravitational field are given by under mentioned equation;

$R_{ij} - \frac{1}{2} R g_{ij} + \lambda g_{ij} = -8\pi T_{ij}$ (1.1.3.)

Where R_{ij} is the Ricci tensor, R is the curvature scalar, g_{ij} is the metric tensor of space-time. T_{ij} is the energy momentum tensor due to matter and λ is the cosmological constant.

Since, the Einstein tensor $R_{ij}\ \frac{1}{2} R g_{ij}$ is divergence free; the equation (1.1.3) yields, as under

$T_{ij;j} = 0$ (1.1.4)

which can be considered as energy – momentum conservation equation.

The theory of relativity put further by Einstein also explains how material bodies will have the movement in presence of gravitational field and which can be shown by mathematical equation by giving the precise value of curvature caused by bodies of different masses. General relativity also shows that gravity can bend light, which has been proved already by experiment during complete Solar Eclipse. This bending of light by gravity leads to formation of multiple images of equidistant astronomical objects in the sky.

The general theory also unifies electromagnetism and Newton's Law of Universal Gravitation. It assumes gravity as a geometric property of space- time. In this theory, matter dynamics is prescribed by Einstein field equation; is a system of partial differential equations. Einstein got very little success for unification. Later, Weyl H. tried to introduce electromagnetic potential as geometric quantities into general relativity.

General relativity is a metric theory of gravitation. At its core are; Einstein's equations, which describes the relation between the geometry of four dimensional semi Riemannian manifold, representing space-time on one hand and energy momentum contained in that space-time on the other. Thus, the energy momentum tensor for matter must be free of any divergence. The curvature of space-time is directly related to the four momentum of whatever matter and radiation are present.

Einstein's equations are based on nonlinear partial differential equations which are difficult to solve exactly. From the exact solution, very few have direct physical applications. The Schwarzschild solution, the Reissnar - Norstrom solution and Kerr metric; each corresponds to a certain extent of black hole; otherwise in the empty universe are the best known exact solutions. The Friedman-Lemaitre-Robertson-Walker and de Sitter universe each of them describe an expanding cosmos. Perturbation theories lead to some approximate solutions like linearized gravity and its generalization.

Methods

Plane Gravitational Waves Z $= \left(\sqrt{x^2+y^2+z^2} - t\right)$ AND $Z = \frac{t}{\left(\sqrt{x^2+y^2+z^2}\right)}$ In Four and Higher Dimensional Space Time in Bimetric Relativity.

In this method, we have studied Z $= \left(\sqrt{x^2+y^2+z^2} - t\right)$ and Z $= \frac{t}{\left(\sqrt{x^2+y^2+z^2}\right)}$ Type plane gravitational waves in four- dimensional, five- dimensional as well as six-dimensional space-times in Rosen's Bimetric Relativity.

Takeno H. (1958a) has propounded a rigorous discussion of plane gravitational waves by defining various terms and obtained various results.

A fairly general case of plane gravitational waves is represented by the metric,

$ds^2 = -Adx^2 - 2Ddxdy - Bdy^2 - dz^2 + dt^2$ (3.1.1)

He has found out the solutions of plane gravitational waves of the field equations in general relativity proposed by Einstein (1915) and those of the various field equations in non-symmetric unified theories for the space-time.

Takeno H. (1961) has solved the field equations and obtained plane wave solutions in the form of Z=(z-t)-type and Z = $\frac{t}{z}$- type plane waves.

On the line of Takeno H. (1961), Thengane and Karade (2000) have obtained the plane wave solutions in Bimetric relativity proposed by Rosen (1973, 1974) and established the existence of these two types of plane wave's solution even in Bimetric Relativity.

Zade V.T. (2002) has modified Takeno's (1961) definition of plane waves and obtained the plane wave solutions with respect to cosmological terms.

His thesis is mainly devoted to some of the investigations regarding the plane wave solutions of the field equations; concerning higher dimensional space-time in general relativity and established the existence of Z = $\left(\sqrt{x^2+y^2+z^2}-t\right)$ and Z $=\frac{t}{\left(\sqrt{x^2+y^2+z^2}\right)}$ type plane gravitational waves in this theory by investigating the line elements as:-

$$ds^2=\frac{-3A}{(x^2+y^2+z^2)}(x^2dx^2+y^2dy^2z^2dz^2)+Adt^2 \qquad (3.1.2)$$

Where $A= A(Z)$ and $Z=\left(\sqrt{x^2+y^2+z^2}-t\right)t$

and

$$ds^2=\frac{-3At^2}{(x^2+y^2+z^2)}(x^2dx^2+y^2dy^2z^2dz^2)+Adt^2 \qquad (3.1.3)$$

Where

$A= A(Z)$ and $Z=\left\{\frac{t}{\sqrt{x^2+y^2+z^2}}\right\}$

In this chapter, we have discussed $Z=\left(\sqrt{x^2+y^2+z^2}-t\right)$ and

$Z=\left\{\frac{t}{\sqrt{x^2+y^2+z^2}}\right\}$ -type plane waves defined in equation (2.1.2) and (2.1.3) with the various matters like cosmic cloud string, perfect fluid, massive scalar field, massive scalar field coupled with perfect fluid respectively in bimetric theory of relativity proposed by Rosen N.

$Z=\left(\sqrt{x^2+y^2+z^2}-t\right)$ type plane gravitational wave with cosmic cloud string solutions

With the proper choice of co-ordinate systems, for $Z=\left(\sqrt{x^2+y^2+z^2}-t\right)$ type plane gravitational waves, we have investigated the line element in V_4 in general relativity as

$$ds^2=\frac{-3A}{(x^2+y^2+z^2)}(x^2dx^2+y^2dy^2+z^2dz^2)+Adt^2 \qquad (3.2.1)$$

Where $A= A(Z)$ and $Z=\left(\sqrt{x^2+y^2+z^2}-t\right)$

Corresponding to the equation (2.2.1), we consider the line element for background metric γ_{ij} as

$$d\sigma^2=-(dx^2+dy^2+dz^2)+dt^2 \qquad (3.2.2)$$

Since, γ_{ij} is the Lorentz metric i.e. (-1,-1,-1, 1), γ- covariant derivative becomes the ordinary partial derivative.

And, $T_i{}^j$ the energy momentum tensor for cosmic cloud string is given by equation (1.15.3) and (1.15.4) where ρ the rest energy density for a cloud is with particle attached along the extension. And ρ_p is the particle energy density, λ is the tension density of the strings.

As pointed out by Letelier (1983), λ may be positive or negative and v_i is the flow vector of matter.

The flow of the matter is taken orthogonal to the hyper-surface of homogeneity so that $V_4V^4=-1$ and x^i representing the direction vector of anisotropy i.e. $x_3x^3=1$ and $V_ix^i=0$.

Using the equation (1.10.7) to (1.10.9) with (2.2.1) and (2.2.2), we get,

$$N_1^1 = \frac{1}{2}\left[\begin{array}{c} (-1)\left\{\left[\frac{-3x^3A'}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right]\left[\frac{(x^2+y^2+z^2)^{\frac{1}{2}}A'}{3xA^2}\right]+\left[\frac{-3Ax^2}{x^2+y^2+z^2}\right]^{-1}\left[\frac{-3x^4A"}{(x^2+y^2+z^2)^2}\right]\right\} \\ +(-1)\left\{\left[\frac{-3x^2yA'}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right]\left[\frac{(x^2+y^2+z^2)^{\frac{1}{2}}yA'}{3x^2A^2}\right]+\left[\frac{-3Ax^2}{x^2+y^2+z^2}\right]^{-1}\left[\frac{-3x^2y^2A"}{(x^2+y^2+z^2)^2}\right]\right\} \\ +(-1)\left\{\left[\frac{-3x^2zA'}{(x^2+y^2+z^2)^{\frac{3}{2}}}\right]\left[\frac{(x^2+y^2+z^2)^{\frac{1}{2}}zA'}{3x^2A^2}\right]+\left[\frac{-3Ax^2}{x^2+y^2+z^2}\right]^{-1}\left[\frac{-3x^2z^2A"}{(x^2+y^2+z^2)^2}\right]\right\} \\ +(1)\left\{\left[\frac{3x^3A'}{(x^2+y^2+z^2)}\right]\left[\frac{-(x^2+y^2+z^2)A'}{3x^2A^2}\right]+\left[\frac{-3Ax^2}{x^2+y^2+z^2}\right]^{-1}\left[\frac{-3x^4A"}{(x^2+y^2+z^2)}\right]\right\} \end{array}\right]$$

$$N_1^1 = \frac{1}{2}\left[\frac{A'}{A^2}-\frac{A"}{A}-\frac{A'}{A^2}-\frac{A"}{A}\right]$$

$$N_1^1 = 0 \qquad (3.2.3)$$

Where $A' = \frac{\partial A}{\partial A}, A" = \frac{\partial^2 A}{\partial Z^2}$

Similarly, we get

$$N_2^2 = N_3^3 = N_4^4 = 0 \qquad (3.2.4)$$

And

$$T_1^1 = T_2^2 = 0, T_3^3 = -\lambda, T_4^4 = -\rho \qquad (3.2.5)$$

Using the equation (1.10.7) to (1.10.9) with (3.2.1) - (3.2.5) and (1.15.3), we get

$$K_1^1 = 0 = K_2^2 \qquad (3.2.6)$$

$$K_3^3 = 0 = 8\pi\kappa\lambda \qquad (3.2.7)$$

$$K_4^4 = 0 = 8\pi\kappa\rho \qquad (3.2.8)$$

Using equation (3.2.6) to (3.2.8), we get

$$\lambda = 0 = \rho \qquad (3.2.9)$$

It gives us nil contribution of cosmic cloud string in $Z = \left(\sqrt{x^2+y^2+z^2} - t\right)$ type plane gravitational wave in the Bimetric theory of relativity.

$\boldsymbol{Z = \left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)}$ Type plane gravitational wave with cosmic cloud strings.

With the proper choice of co-ordinate systems, for $Z = \left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ type plane gravitational waves, we have investigated the line element in V_4 as

$$ds^2 = \frac{-3A}{(x^2+y^2+z^2)}(x^2dx^2 + y^2dy^2 + z^2dz^2) + Adt^2 \qquad (3.3.1)$$

Where A= A(Z) and $Z = \left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$

Corresponding to the equation (2.3.1), we consider the line element for background metric γ_{ij} as express in equation (2.2.2).

Using equations (1.10.7) to (1.10.9) with (2.3.1) and (2.2.2), we get

$$N_1^1 = \frac{1}{2}\left[\left(\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right)\left(\frac{A'^2}{A^2}-\frac{A"}{A}\right)\right] = N_2^2 = N_3^3 = N_4^4 \qquad (3.3.2)$$

Where $A' = \frac{\partial A}{\partial A}, A" = \frac{\partial^2 A}{\partial Z^2}$

i.e.

$$N_1^1 = N_2^2 = N_3^3 = N_4^4 = \frac{1}{2}D\left(\frac{A'^2}{A^2}-\frac{A"}{A}\right) \qquad (3.3.3)$$

Where

$D = \left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right]$ (3.3.4) Where

$D = \left[\frac{t^2-(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}\right]$ (3.3.4)

Using equations (1.10.7) to (1.10.9) with (3.3.1) to (3.3.4) and (1.15.3), we get the field equations as

$$\boldsymbol{K_1^1 = K_2^2 = \left(\frac{A\prime^2}{A^2} - \frac{A"}{A}\right) D = 0} \quad (3.3.5)$$

$$\mathrm{K}_3^3 = \left(\frac{\mathrm{A}\prime^2}{A^2} - \frac{A"}{A}\right) D = -16\pi\kappa\lambda \quad (3.3.6)$$

$$\mathrm{K}_4^4 = \left(\frac{\mathrm{A}\prime^2}{A^2} - \frac{A"}{A}\right) D = -16\pi\kappa\lambda \quad (3.3.7)$$

Using equation (3.3.5) to (3.3.7), we get

$\lambda = 0 = \rho$ (3.3.8)

Thus, cosmic cloud string does not exist in $Z = \left(\frac{t}{\sqrt{x^2+y^2+z^2}}\right)$ type plane gravitational waves in Rosen's Bimetric relativity. Hence, for vacuum case $= 0 = \rho$, the field equation reduces to

$$\boldsymbol{\left(\frac{A'^2}{A^2} - \frac{A"}{A}\right) D = 0}$$

i.e.

$\left(\frac{A\prime^2}{A^2} - \frac{A"}{A}\right) = 00$ (3.3.9)

Solving equations (3.3.9), we have

$\mathrm{A} = Re^{SZ}$ (3.3.10)

where R, S are the constants of integration.

Thus, substituting the value of (3.3.10) in (3.3.1), we get the vacuum line element as

$ds^2 = \frac{-3R\, e^{SZ} t^2}{(x^2+y^2+z^2)}(x^2dx^2 + y^2dy^2 + z^2dz^2) + Adt^2$ (3.3.11)

This study can be extended further with the introduction of cosmological term λ in the field equation is defined as $N_i^j = \lambda g_i^j$ (3.3.12)

Using equation (3.3.12) in (3.3.3), we get

$\left[\frac{A'^2}{A^2} - \frac{A"}{A}\right] D = \lambda$ (3.3.13)

On solving equation (3.3.13), we have

A= exp $\left[\frac{D'\lambda Z^2}{2} + EZ + F\right]$ (3.3.14)

Where E and F are constants of integration and

$$D' = \frac{1}{D} = \left[\frac{(x^2+y^2+z^2)^2}{t^2-(x^2+y^2+z^2)}\right]$$

Thus, the line element (3.3.1) takes the form

$ds^2 = -3exp\left[\frac{D'\lambda Z^2}{2} + EZ + F\right]\left(\frac{t^2}{(x^2+y^2+z^2)^2}\right)(x^2dx^2 + y^2dy^2 + z^2dz^2) + 3\exp\left[\frac{D'\lambda Z^2}{2} + EZ + F\right] dt^2$ (3.3.15)

Thus, $Z = \left(\frac{t^2}{\sqrt{(x^2+y^2+z^2)}}\right)$ plane gravitational wave exists in Bimetric relativity with or without cosmological constant λ respectively.

Conclusion

In the study of $\boldsymbol{Z} = \left(\frac{\boldsymbol{t}}{\sqrt{\boldsymbol{x^2+y^2+z^2}}}\right)$ type plane gravitational waves, there is of cosmic cloud strings and for $\boldsymbol{Z} = \left(\frac{\boldsymbol{t}}{\sqrt{\boldsymbol{x^2+y^2+z^2}}}\right)$

1. Plane Gravitational Waves: The text discusses the existence of plane gravitational waves in the framework of bimetric relativity. A plane gravitational wave is a solution to the Einstein field equations

that represents gravitational waves propagating in a specific direction through spacetime. The fact that plane gravitational waves exist in this theory, with or without the cosmological constant (denoted by), indicates that this theory can model wave-like disturbances in spacetime in a manner analogous to general relativity, but with additional complexities due to the presence of two metrics.

2. Cosmic Cloud Strings, Mesonic Perfect Fluid: The statement that there is no contribution from cosmic cloud strings or mesonic perfect fluids suggests that, in this specific analysis, such matter fields (often used to model certain exotic forms of matter or energy) do not contribute to the gravitational field within the bimetric theory. This could imply that the focus is on vacuum solutions or scenarios where matter is negligible or not present.

3. Rosen's Bimetric Theory of Relativity: This is a variant of general relativity where two distinct metrics are used to describe spacetime, as opposed to just one in the usual general relativity. In Rosen's Bimetric theory, the gravitational interactions between two metrics might lead to different behaviors for gravitational waves and other solutions compared to the standard Einsteinian framework.

4. Invariance of the Conclusions: The statements, , , and are invariant in all coordinate systems, even though they were derived in co-moving coordinates. This means that these results hold true in any observer's frame of reference, which is a crucial property for the physical validity of the theory. In co-moving coordinates, the velocity and other quantities like the scale factor (which may relate to the expansion of the universe or spacetime), pressure, and energy density are set to zero, indicating a static or vacuum scenario.

References

1. Einstein, A. (1915). Diefeldgleichungen der gravitation, Sitzungsberichte der Preussischen Akade mie der Wissenschaften Zu Berlin, 844-847.
2. Hawking, S. W., Ellis, G.F.R. (1973). The large scale structure of space- time, Cambridge University Press.
3. Letelier, P.S. (1983). String cosmologies, Physical Review D, 28 (10), 2414-2419.
4. Rosen, N. (1973). A bimetric theory of gravitation I, Gen. Rel.
5. Rosen, N. (1974). A theory of gravitation, Ann. Phys., 84(1-2), 455- 473.
6. Takeno, H. (1961). The Mathematical Theory of Plane Gravitational Waves in General Relativity, a scientific report of the Research Institute for the Theoretical Physics, Hiroshima University, Japan.
7. Takeno, H. (1958a). A Comparison of plane wave solutions in General relativity with those in non-symmetric theory. Prog. Theory. phys.20, 267-276.
8. Thengane, K.D. and Karade, T.M. (2000). Plane gravitational waves in general relativity (Ph. D Thesis), Nagpur University, Nagpur.