

## Common Fixed Point Theorem For Six Mappings In Dislocated Metric Spaces

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### Abstract

In this paper, by employing the notions of compatibility, weak compatibility, and commutativity, we establish several common fixed-point theorems for six mappings in complete dislocated metric spaces. Our work enhances and extends several earlier results in the literature.

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### 1. Introduction

The notion of a metric space, introduced by Maurice Fréchet [3] in 1906, is a foundational concept not only in mathematics but also in various quantitative sciences. Due to its significance and potential applications, this notion has been extended, refined, and generalized in numerous ways. Some of these extensions include symmetric spaces, b-metric spaces, partial metric spaces, quasi-metric spaces, fuzzy metric spaces, dislocated metric spaces, dislocated quasi-metric spaces, as well as right and left dislocated metric spaces. In 1922, Stefan Banach [1] proved a fixed point theorem for contraction mappings in complete metric spaces. The Banach contraction theorem is a cornerstone of functional analysis and has widespread applications across various mathematical fields, including differential equations and integral equations. Following the Banach contraction theorem, numerous fixed point theorems have been established by various authors, leading to different generalizations of this theorem.

The concept of a dislocated metric (d-metric) was introduced by Hitzler and Seda [5], particularly valuable in logic programming semantics. Over time, numerous papers have been published on fixed point and common fixed point theorems that satisfy certain contractive conditions in dislocated metric spaces.

Sessa [21], initiated the tradition of improving commutativity conditions in metrical common fixed point theorems. While doing so Sessa [21] introduced the notion of weak commutativity. Motivated by Sessa [21], Jungck [8] defined the concept of compatibility of two mappings, which includes weakly commuting mappings as a proper subclass. Jungck and Rhoades [10] introduced the notion of weakly compatible (coincidentally commuting) mappings and showed that compatible mappings are weakly compatible but not conversely. Many interesting fixed point theorems for weakly compatible maps satisfying contractive type conditions have been obtained by various authors. One may note that this definition never needs to involve underlying metric. The following one way implication is obviously true but not conversely.

Commuting maps  $\Rightarrow$  Weakly commuting maps  $\Rightarrow$  Compatible maps  $\Rightarrow$   
 $\Rightarrow$  Coincidentally commuting maps.

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The class of coincidentally commuting maps is wider than other classes of maps mentioned herein. Several examples supporting this fact can be found in Dhage [2] and Jungck and Rhoades [10]. In this paper, we establish some common fixed point results, using the concepts of compatibility, weak compatibility, and commutativity in complete dislocated metric ( $d$ -metric) spaces. Our results generalize several well-known findings in the existing literature.

## 2. Preliminary Notes

We begin by recalling some basic concepts of the theory of dislocated metric ( $d$ -metric) spaces. Throughout this work  $R^+$  represent the set of non-negative real numbers. Now, we collect some known definitions and results from the literature which are helpful in the proof of our results.

**Definition 2.1** Let  $X$  be a non-empty set and let  $d: X \times X \rightarrow [0, \infty)$  be a function satisfying the following conditions:

- (i)  $d(x, y) = d(y, x)$
- (ii)  $d(x, y) = d(y, x) = 0$  implies  $x = y$
- (iii)  $d(x, y) \leq d(x, z) + d(z, y)$  for all  $x, y, z \in X$

Then  $d$  is called dislocated metric (or simply  $d$ -metric) on  $X$ .

**Definition 2.2** A sequence  $\{x_n\}$  in a  $d$ -metric space  $(X, d)$  is called a Cauchy sequence if for given  $\epsilon > 0$ , there exists  $n_0 \in N$  such that for all  $m, n \geq n_0$ , we have  $d(x_m, x_n) < \epsilon$ .

**Definition 2.3** A sequence in  $d$ -metric space converges if there exists  $x \in X$  such that  $d(x_n, x) \rightarrow 0$  as  $n \rightarrow \infty$ .

**Definition 2.4** A  $d$ -metric space  $(X, d)$  is called complete if every Cauchy sequence is convergent.

**Definition 2.5** Let  $(X, d)$  be a  $d$ -metric space. A map  $T: X \rightarrow X$  is called contraction if there exists a number  $\lambda$  with  $0 \leq \lambda < 1$  such that  $d(Tx, Ty) \leq \lambda d(x, y)$ .

**Definition 2.6** Let  $A$  and  $S$  be two self mappings on a set  $X$ . Mappings  $A$  and  $S$  are said to be commuting if  $ASx = SAx \quad \forall x \in X$ .

In recent years several definitions of conditions weaker than commutativity have appeared which facilitated significantly to extend the Jungck's theorem and several others. Foremost among them is perhaps the weak commutativity condition introduced by Sessa [21] which can be described as follows:

**Definition 2.7.** Let  $S$  and  $T$  be mappings of a metric space  $(X, d)$  into itself. Then  $(S, T)$  is said to be **weakly commuting** pair if

$$d(STx, TSx) \leq d(Tx, Sx) \text{ for all } x \in X.$$

Obviously a commuting pair is weakly commuting but its converse need not be true as is evident from the following example.

**Example 2.1.** Consider the set  $X = [0, 1]$  with the usual metric. Let  $Sx = \frac{x}{2}$  and  $Tx = \frac{x}{2+x}$  for every  $x \in X$ . Then for all  $x \in X$

$$STx = \frac{x}{4+2x}, TSx = \frac{x}{4+x}.$$

Hence  $ST \neq TS$ . Thus  $S$  and  $T$  do not commute.

Again

$$d(STx, TSx) = \left| \frac{x}{4+2x} - \frac{x}{4+x} \right| = \frac{x^2}{(4+x)(4+2x)}$$

$$\leq \frac{x^2}{(4+2x)} = \frac{x}{2} - \frac{x}{2+x} = d(Sx, Tx),$$

and so,  $S$  and  $T$  commute weakly.

Obviously, the class of weakly commuting is wider and includes commuting mappings as subclass. Jungck [8] has observed that for  $X = R$  if  $Sx = x^3$  and  $Tx = 2x^3$  then  $S$  and  $T$  are not weakly commuting. Thus it is desirable to a less restrictive concept which he termed as 'compatibility' the class of

compatible mappings is still wider and includes weakly commuting mappings as subclass as is evident from the following definition of Jungck [8].

**Definition 2.8.** Two self mappings  $S$  and  $T$  from a  $d$ -metric space  $(X, d)$  into itself are called compatible if and only if

$$\lim_{n \rightarrow \infty} d(STx_n, TSx_n) = 0, \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that}$$

$$\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t \text{ for some } t \in X.$$

More recently, Jungck et. al. [9] introduced the concept of compatible mapping of type (A) which is stated as follows.

**Definition 2.9.** Let  $S$  and  $T$  be mappings from a metric space  $(X, d)$  into itself. The pair  $(S, T)$  is said to be **compatible of type (A)** on  $X$  if whenever  $\{x_n\}$  is a sequence in  $X$  such that

$$\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z \text{ in } X, \text{ then}$$

$$d(STx_n, TTx_n) \rightarrow 0 \text{ and } d(TSx_n, SSx_n) \rightarrow 0 \text{ as } n \rightarrow \infty.$$

It is shown in [9] that under certain conditions the compatible and compatible of type (A) mappings are equivalent for instance.

**Proposition 2.1.** Let  $S$  and  $T$  be continuous self mapping on  $X$ . Then the pair  $(S, T)$  is compatible on  $X$ , where as in (Jungck [8], Gajic [4]) demonstrated by suitable examples that if  $S$  and  $T$  are discontinuous then the two concepts are independent of each other.

The following examples also support this observation.

**Example 2.2.**

Let  $X = \mathbb{R}$  with the usual metric we defines,  $T: X \rightarrow X$  as follows.

$$Sx = \begin{cases} 1/x^2x \neq 0 \\ 0 \quad x = 0 \end{cases} \quad \text{and} \quad Tx = \begin{cases} 1/x^3x \neq 0 \\ 0 \quad x = 0. \end{cases}$$

Both  $S$  and  $T$  are discontinuous at  $x = 0$  and for any sequence  $\{x_n\}$  in  $X$ , we have  $d(STx_n, TSx_n) = 0$ . Hence the pair  $(S, T)$  is compatible.

Now consider the sequence  $x_n = n \in \mathbb{N}$ . Then,  $Sx_n \rightarrow 0$  and  $Tx_n \rightarrow 0$  as  $n \rightarrow \infty$  and

$$d(STx_n, TTx_n) = |n^6 - n^9| \leq |n^6| + |n^9| \rightarrow \infty \text{ as } n \rightarrow \infty.$$

Hence the pair  $(S, T)$  is not compatible of type (A).

**Example 2.3.**

Let  $X = [0, 1]$  with the usual metric  $d(x, y) = |x - y|$ .

Define  $S, T : [0, 1] \rightarrow [0, 1]$  by

$$Sx = \begin{cases} x, & x \in [0, 1/2) \\ 1, & x \in [1/2, 1] \end{cases} \quad \text{and} \quad Tx = \begin{cases} 1 - x, & x \in [0, 1/2) \\ 1, & x \in [1/2, 1] \end{cases}$$

Then  $S$  and  $T$  are not continuous at  $t = 1/2$ . Now, we assert that  $S$  and  $T$  are not compatible but they are compatible of type (A). To see this, suppose that  $\{x_n\} \subset [0, 1]$  and that  $Tx_n, Sx_n \rightarrow t$ . By definition of  $S$  and  $T$ ,  $t \in \{1/2, 1\}$ . Since  $S$  and  $T$  agree on  $[1/2, 1]$ , we need only consider  $t = 1/2$ . So we can suppose that  $x_n \rightarrow 1/2$  and that  $x_n < 1/2$  for all  $n$ . Then  $Tx_n = 1 - x_n \rightarrow 1/2$  from the right and  $Sx_n = x_n \rightarrow 1/2$  from the left. Thus, since  $1 - x_n > 1/2$ , for all  $n$ ,

$$STx_n = S(1 - x_n) = 1$$

and,  $x_n < 1/2,$

$$\begin{aligned} & \text{since} \\ TSx_n &= Tx_n = 1 - x_n \rightarrow 1/2 \end{aligned}$$

Consequently,

$$d(STx_n, TSx_n) \rightarrow 1/2,$$

but

$$d(STx_n, TTx_n) = |STx_n - TTx_n| = |1 - T(1 - x_n)| = |1 - 1| \rightarrow 0$$

and

$$d(TSx_n, SSx_n) = |TSx_n - SSx_n| = |(1 - x_n) - 1| = |1 - 2x_n| \rightarrow 0$$

as  $x_n \rightarrow 1/2$ . Therefore,  $S$  and  $T$  are compatible mappings of type (A) but they are not compatible.

**Example 2.4.**

Now we define

$$Sx = \begin{cases} \frac{1}{x^3}, & x > 1 \\ 1, & 0 \leq x \leq 1 \\ 0, & x < 0 \end{cases} \quad \text{and} \quad Tx = \begin{cases} -\frac{1}{x^3}, & x > 1 \\ 1, & 0 \leq x \leq 1 \\ 0, & x < 0. \end{cases}$$

Observe that the restriction of S and T on  $(-\infty, 1]$  are equal.

Thus we take a sequence  $\{x_n\}$  in  $(1, \infty)$ . Then  $\{Sx_n\} \subset (0, 1)$  and  $\{Tx_n\} \subset (-1, 0)$ .

Thus for every n,  $TTx_n = 0$ ,  $TSx_n = 1$ ,  $STx_n = 0$ ,  $SSx_n = 1$ . So that

$d(STx_n, TTx_n) = 0$ ,  $d(TSx_n, TTx_n) = 0$  for every  $n \in N$ .

This shows that the pair (S, T) is compatible of type (A). Now let  $x_n = n$ ,  $n \in N$ . Then  $Tx_n \rightarrow 0$ ,

$Sx_n \rightarrow 0$  as  $n \rightarrow \infty$  and  $STx_n = 0$ ,  $TSx_n = 1$  for every  $n \in N$  and so

$d(STx, TSx) \neq 0$  as  $n \rightarrow \infty$ . Hence the pair (S, T) is not compatible.

Very recently concept of **weakly compatible** obtained by Jungck-Rhoades [10] stated as the pair of mappings is said to be weakly compatible if they commute at their coincidence point.

**Example 2.5 .**

Let  $X = [0, 3]$  be equipped with the usual metric space  $d(x, y) = |x - y|$ .

Define S, T :  $[0, 3] \rightarrow [0, 3]$  by

$$Sx = \begin{cases} x, & x \in [0,1) \\ 3, & x \in [1, 3] \end{cases} \quad \text{and} \quad Tx = \begin{cases} 3 - x, & x \in [0,1) \\ 3, & x \in [1, 3] \end{cases}$$

Then for any  $x \in [1, 3]$ ,  $STx = TSx$ , showing that S and T are weakly compatible maps on  $[0, 3]$

**Example 2.6 .**

Let  $X = R$  and define S, T :  $R \rightarrow R$  by  $Sx = x/3$ ,  $x \in R$  and  $Tx = x^2$ ,  $x \in R$ . Here 0 and 1/3 are two coincidence points for the maps S and T. Note that S and T commute at 0, i.e.  $ST(0) = TS(0) = 1/27$ , but  $ST(1/3) = S(1/9) = 1/27$  and  $TS(1/3) = T(1/9) = 1/81$  and so S and T are not weakly compatible maps on R.

**Example 2.7 .**

Let  $X = [2, 20]$  with usual metric. Define

$$Tx = \begin{cases} 2, & \text{if } x = 2 \\ 12 + x, & \text{if } 2 < x \leq 5 \\ x - 3, & \text{if } 5 < x \leq 20 \end{cases} \quad \text{and} \quad Sx = \begin{cases} 2 & \text{if } x \in \{2\} \cup (5,20] \\ 8 & \text{if } 2 < 5 \leq 5, \end{cases}$$

S and T are weakly compatible mappings which is not compatible. To see that S and T are not compatible of Type (A).

Let us consider a decreasing sequence  $\{x_n\}$  such that

$$\lim x_n = 5,$$

then

$$Tx_n = x_n - 3 \rightarrow 2; Sx_n = 2; STx_n = S(x_n - 3) = 8 \text{ and}$$

$$TTx_n = T(x_n - 3) = 12 + x_n - 3 \rightarrow 14, \text{ that is}$$

$$\lim d(STx_n, TTx_n) = 6 \neq 0.$$

Hence S and T are not compatible of type (A).

**Proposition 2.2** Let S and T be compatible mappings from a d-metric space (X, d) into itself. Suppose that

$$\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = x \text{ for some } x \in X.$$

If S is continuous then  $\lim_{n \rightarrow \infty} TSx_n = Sx$ .

**Theorem 2.1** Let (X, d) be a complete d-metric space and let  $T: X \rightarrow X$  be a contraction mapping, then T has a unique fixed point.

**3 MAIN RESULTS:-**

Now, we prove a unique common fixed point theorem by improving the rational contractive conditions.

The following Lemma is the key in proving our result. Its proof is similar to that of Jungck [7]

**Lemma 3.1:** Let  $\{y_n\}$  be a sequence in a complete dislocated metric space  $(X, d)$ . If there exists a  $k \in (0,1)$  such that

$$d(y_{n+1}, y_n) \leq kd(y_{n+1}, y_n), \text{ for all } n, \text{ then } \{y_n\} \text{ converges to a point in } X.$$

Motivated by the contractive condition given by contractive condition of Jeong and Rhoades [3] we prove the following theorem.

**Theorem 3.1:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, B, S, T, I, J: X \rightarrow X$  satisfying

$$(i) \quad AB(X) \subset J(X), ST(X) \subset I(X), \quad \text{-----(3.1)}$$

$$(ii) \quad d(ABx, STy) \leq \alpha_1 \left[ \frac{d(Jy, STy)d(Ix, ABx)}{d(Ix, Jy)} \right] + \alpha_2[d(ABx, Jy) + d(Jy, STy)] + \alpha_3d(Ix, ABx)$$

..... (3.2)

for all  $x, y \in X$  and  $\alpha_1, \alpha_2, \alpha_3 \geq 0, 0 \leq 2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$ . If either (iii)  $(AB, I)$  are compatible,  $I$  or  $AB$  is continuous and  $(ST, J)$  are weakly compatible or (iv)  $ST, J$  are compatible,  $J$  or  $ST$  is continuous and  $(AB, I)$  are weakly compatible, then  $AB, ST, I$  and  $J$  have a unique common fixed point. Furthermore, if the pairs  $(A, B), (A, I), (B, I), (S, T), (S, J)$  and  $(T, J)$  are commuting mappings then  $A, B, S, T, I$  and  $J$  have a unique common fixed point.

**Proof :**

We construct a sequence as follows. Let  $x_0$  be an arbitrary point in  $X$ . Since  $AB(X) \subseteq J(X)$  we can choose a point  $x_1$  in  $X$  such that  $ABx_0 = Jx_1$ .

Again since  $ST(X) \subseteq I(X)$  we can choose a point  $x_2$  in  $X$  such that  $STx_1 = Ix_2$ , construct a sequence  $\{z_n\}$  be repeatedly using this argument,  $z_{2n} = ABx_{2n} = Jx_{2n+1}, z_{2n+1} = STx_{2n+1} = Ix_{2n+2}, n = 0, 1, 2, \dots$

If  $z_{2n} = z_{2n+1}$  for some  $n$ , then  $STx_{2n+1} = Jx_{2n+1}$ . Therefore,  $x_{2n+1}$  is a coincident point of  $ST$  and  $J$ . Also, if  $z_{2n+1} = z_{2n+2}$  for some  $n$ , then  $ABx_{2n+2} = Ix_{2n+2}$ . Hence,  $x_{2n+2}$  is a coincident point of  $AB$  and  $I$ .

Assume that  $z_{2n} \neq z_{2n+1}$  for all  $n$ . Then, we have

$$\begin{aligned} d(z_{2n+1}, z_{2n+2}) &= d(STx_{2n+1}, ABx_{2n+2}) \\ &\leq \alpha_1 \left[ \frac{d(Jx_{2n+1}, STx_{2n+1}) \cdot d(Ix_{2n+2}, ABx_{2n+2})}{d(Ix_{2n+2}, Jx_{2n+1})} \right] + \alpha_2[d(ABx_{2n+2}, Jx_{2n+2}) \\ &\quad + d(Jx_{2n+1}, STx_{2n+1})] + \alpha_3d(Ix_{2n+2}, Jx_{2n+1}) \\ &= \alpha_1 \left[ \frac{d(z_{2n}, z_{2n+1}) \cdot d(z_{2n}, z_{2n+2})}{d(z_{2n+1}, z_{2n})} \right] + \alpha_2[d(z_{2n+2}, z_{2n+1}) + d(z_{2n}, z_{2n+1})] + \alpha_3d(z_{2n+1}, z_{2n}) \\ &\leq \alpha_1[d(z_{2n}, z_{2n+1}) + d(z_{2n+1}, z_{2n+2})] + \alpha_2[d(z_{2n+2}, z_{2n+1}) + d(z_{2n}, z_{2n+1})] + \alpha_3d(z_{2n+1}, z_{2n}) \end{aligned}$$

which implies that

$$d(z_{2n+1}, z_{2n+2}) \leq \frac{\alpha_1 + \alpha_2 + \alpha_3}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) \leq td(z_{2n}, z_{2n+1})$$

where,  $t = \frac{\alpha_1 + \alpha_2 + \alpha_3}{(1 - \alpha_1 - \alpha_2)} < 1$

Similarly, we can conclude that

$$d(z_{2n}, z_{2n+1}) \leq td(z_{2n-1}, z_{2n})$$

Thus for every  $n$ , we have

$$d(z_n, z_{n+1}) \leq td(z_{n-1}, z_n) \quad \dots (3.3)$$

By Lemma 3.1  $\{z_n\}$  converges to some  $z \in X$ . Hence, the sequences  $ABx_{2n} = Jx_{2n+1}$  and  $STx_{2n+1} = Ix_{2n+2}$ , which are subsequences also converges to the some point  $z$ .

Let us now assume that  $I$  is continuous so that the sequence  $\{I^2x_{2n}\}$  and  $\{IABx_{2n}\}$  converges to the same point  $Iz$ . Also  $(AB, I)$  are compatible, so sequence  $\{ABIx_{2n}\}$  also converges to  $Iz$ .

Now

$$\begin{aligned} d(ABIx_{2n}, STx_{2n+1}) &\leq \alpha_1 \left[ \frac{d(Jx_{2n+1}, STx_{2n+1})d(I^2x_{2n}, ABIx_{2n})}{d(I^2x_{2n}, Jx_{2n+1})} \right] \\ &\quad + \alpha_2[d(ABIx_{2n}, I^2x_{2n}) + d(Jx_{2n+1}, STx_{2n+1})] + \alpha_3d(I^2x_{2n}, Jx_{2n+1}) \end{aligned}$$

which on using (3.2) and letting  $n \rightarrow \infty$  reduces to

$$d(Iz, z) \leq \alpha_1 \left[ \frac{d(z, z)d(Iz, Iz)}{d(Iz, z)} \right] + \alpha_2 [d(Iz, Iz) + d(z, z)] + \alpha_3 d(Iz, z)$$

$$\leq \alpha_1 \left[ \frac{\{d(z, Iz) + d(Iz, z)\}\{d(Iz, z) + d(z, Iz)\}}{d(Iz, z)} \right] + \alpha_2 [\{d(Iz, z) + d(z, Iz)\} + \{d(z, Iz) + d(Iz, z)\}] + \alpha_3 d(Iz, z)$$

or,  $d(Iz, z) \leq 4\alpha_1 d(Iz, z) + 4\alpha_2 d(Iz, z) + \alpha_3 d(Iz, z)$

which implies that

$$d(Iz, z) \leq (4\alpha_1 + 4\alpha_2 + \alpha_3)d(Iz, z)$$

which is a contradiction, since  $4\alpha_1 + 4\alpha_2 + \alpha_3 \neq 1$

Yielding thereby  $Iz = z$ .

Now,

$$d(ABz, STx_{2n+1})$$

$$\leq \alpha_1 \left[ \frac{d(Jx_{2n+1}, STx_{2n+1})d(Iz, ABz)}{d(Iz, Jx_{2n+1})} \right] + \alpha_2 [d(ABz, Iz) + d(Jx_{2n+1}, STx_{2n+1})]$$

$$+ \alpha_3 d(Iz, Jx_{2n+1})$$

which on using (1) and letting  $n \rightarrow \infty$  reduces to

$$d(ABz, z) \leq \alpha_1 \left[ \frac{d(z, z)d(Iz, ABz)}{d(Iz, z)} \right] + \alpha_2 [d(ABz, Iz) + d(z, z)] + \alpha_3 d(Iz, z)$$

Using  $Iz = z$ , we get

$$d(ABz, z) \leq \alpha_1 \left[ \frac{d(z, z)d(z, ABz)}{d(z, z)} \right] + \alpha_2 [d(ABz, z) + d(z, z)] + \alpha_3 d(z, z)$$

$$\leq \alpha_1 d(ABz, z) + \alpha_2 [d(ABz, z) + d(z, ABz) + d(ABz, z)] + \alpha_3 [d(z, ABz) + d(ABz, z)]$$

$$= (\alpha_1 + 3\alpha_2 + 2\alpha_3)d(ABz, z)$$

which is a contradiction, since  $\alpha_1 + 3\alpha_2 + 2\alpha_3 \neq 1$

Implying thereby  $ABz = z$ .

Since  $AB(X) \subset J(X)$ , there always exist a point  $z'$  such that  $Jz' = z$  so that  $STz = ST(Jz')$ .

Now, using (3.2), we have

$$d(z, STz') = d(ABz, STz')$$

$$\leq \alpha_1 \left[ \frac{d(Jz', STz')d(Iz, ABz)}{d(Iz, Jz')} \right] + \alpha_2 [d(ABz, Iz) + d(Jz', STz')] + \alpha_3 d(Iz, Jz')$$

$$= \alpha_1 \left[ \frac{d(z, STz')d(z, z)}{d(z, z)} \right] + \alpha_2 [d(z, z) + d(z, STz')] + \alpha_3 d(z, z)$$

$$\leq \alpha_1 d(z, STz') + \alpha_2 [\{d(z, STz') + d(STz', z)\} + d(z, STz')] + \alpha_3 [d(z, STz') + d(STz', z)]$$

$$= (\alpha_1 + 3\alpha_2 + 2\alpha_3)d(z, STz'),$$

which is a contradiction, since  $\alpha_1 + 3\alpha_2 + 2\alpha_3 \neq 1$ .

Therefore  $STz' = z = Jz'$ .

It shows that  $(ST, J)$  have a coincidence point  $z'$ . Now using the weak compatibility of  $(ST, J)$ , we have

$$STz = ST(Jz') = J(STz') = Jz,$$

which shows that  $z$  is also a coincidence point of the pair  $(ST, J)$ .

Now

$$d(z, STz) = d(ABz, STz)$$

$$\leq \alpha_1 \left[ \frac{d(Jz, STz)d(Iz, ABz)}{d(Iz, Jz)} \right] + \alpha_2 [d(ABz, Iz) + d(Jz, STz)] + \alpha_3 d(Iz, Jz)$$

$$= \alpha_1 \left[ \frac{d(STz, STz)d(z, z)}{d(z, STz)} \right] + \alpha_2 [d(z, z) + d(STz, STz)] + \alpha_3 d(z, STz)$$

$$\begin{aligned} &\leq \alpha_1 \left[ \frac{\{d(STz, z) + d(z, STz)\}\{d(z, STz) + d(STz, z)\}}{d(z, STz)} \right] \\ &+ \alpha_2 [\{d(z, STz) + d(STz, z)\} + \{d(STz, z) + d(z, STz)\}] + \alpha_3 d(z, STz) \\ &\leq (4\alpha_1 + 4\alpha_2 + \alpha_3) d(z, STz) \end{aligned}$$

which is a contradiction, since  $4\alpha_1 + 4\alpha_2 + \alpha_3 \neq 1$

Yielding thereby  $z = STz$ .

Hence,  $z = STz = Jz$ , which shows that  $z$  is a common fixed point of  $AB, I, ST$  and  $J$ .

Now, we suppose that  $AB$  is continuous, so that, the sequence  $\{AB^2x_{2n}\}$  and  $\{ABIx_{2n}\}$  converges to  $ABz$ . Since  $(AB, I)$  are compatible, it follows that  $\{IABx_n\}$  also converges to  $ABz$ . Thus,

$$\begin{aligned} d(AB^2x_{2n}, STx_{2n+1}) &\leq \alpha_1 \left[ \frac{d(Jx_{2n+1}, STx_{2n+1}) \cdot d(IABx_{2n}, AB^2x_{2n})}{d(Iz, Jx_{2n+1})} \right] \\ &+ \alpha_2 [d(AB^2x_{2n}, IABx_{2n}) + d(Jx_{2n+1}, STx_{2n+1})] + \alpha_3 d(IABx_{2n}, Jx_{2n+1}) \end{aligned}$$

which on using (1) and letting  $n \rightarrow \infty$  reduces to

$$\begin{aligned} d(ABz, z) &\leq \alpha_1 \left[ \frac{d(z, z)d(ABz, ABz)}{d(Iz, z)} \right] + \alpha_2 [d(ABz, ABz) + d(z, z)] + \alpha_3 d(ABz, z) \\ &\leq \alpha_1 \left[ \frac{d(z, z)\{d(ABz, z) + d(z, ABz)\}}{d(z, z)} \right] \\ &+ \alpha_2 [\{d(ABz, z) + d(z, ABz)\} + \{d(z, ABz) + d(ABz, z)\}] + \alpha_3 d(z, STz) \\ &= (2\alpha_1 + 4\alpha_2 + \alpha_3)d(ABz, z) \end{aligned}$$

which is a contradiction, since  $2\alpha_1 + 4\alpha_2 + \alpha_3 \neq 1$ ,

yielding thereby  $ABz = z$ .

As earlier, there exists a point  $z'$  in  $X$  such that  $ABz = z = Jz'$ . Then,

$$\begin{aligned} d(AB^2x_{2n}, STz') &\leq \alpha_1 \left[ \frac{d(Jz', STz') \cdot d(IABx_{2n}, AB^2x_{2n})}{d(IABx_{2n}, Jz') + d(STz', IABx_{2n})} \right] + \alpha_2 d(AB^2x_{2n}, IABx_{2n}) \\ &+ d(Jz', STz') + \alpha_3 d(IABx_{2n}, Jz') \end{aligned}$$

which on letting  $n \rightarrow \infty$  reduces to

$$\begin{aligned} d(z, STz') &\leq \alpha_1 \left[ \frac{d(Jz', STz')d(ABz, ABz)}{d(ABz, Jz')} \right] + \alpha_2 [d(ABz, ABz) + d(Jz', STz')] + \alpha_3 d(ABz, Jz') \\ &\leq \alpha_1 \left[ \frac{d(z, STz')d(z, z)}{d(z, z)} \right] + \alpha_2 [d(z, z) + d(z, STz')] + \alpha_3 d(z, z) \\ &= \alpha_1 d(z, STz') + \alpha_2 [\{d(z, STz') + d(STz', z)\} + d(z, STz')] + \alpha_3 [d(z, STz') + d(STz', z)] \\ &= (\alpha_1 + 3\alpha_2 + 2\alpha_3)d(z, STz') \end{aligned}$$

which is a contradiction, since  $\alpha_1 + 3\alpha_2 + 2\alpha_3 \neq 1$ ,

yielding thereby  $STz' = z$

This gives  $STz' = z = Jz'$ .

Thus,  $z'$  is a coincidence point of  $ST$  and  $J$ . Since, the view of weakly compatibility of the pair  $(ST, J)$ , one has  $STz = ST(Jz') = J(STz') = Jz$ , which shows that  $STz = Jz$ .

Further,

$$\begin{aligned} d(ABx_{2n}, STz) &\leq \alpha_1 \left[ \frac{d(Jz, STz)d(Ix_{2n}, ABx_{2n})}{d(Ix_{2n}, Jz)} \right] \\ &+ \alpha_2 [d(ABx_{2n}, Ix_{2n}) + d(STz, Jz)] + \alpha_3 d(Ix_{2n}, Jz) \end{aligned}$$

which on using (1), letting  $n \rightarrow \infty$ , and using  $STz = Jz$ , reduces to

$$d(z, STz) \leq \alpha_1 \left[ \frac{d(Jz, STz)d(z, z)}{d(z, Jz)} \right] + \alpha_2 [d(z, z) + d(Jz, STz)] + \alpha_3 d(z, Jz)$$

$$\begin{aligned}
 &= \alpha_1 \left[ \frac{d(z, STz)d(z, z)}{d(z, z)} \right] + \alpha_2 [d(z, z) + d(z, STz)] + \alpha_3 d(z, z) \\
 &\leq \alpha_1 d(z, STz) + \alpha_2 [d(z, STz) + d(STz, z)] + \alpha_3 [d(z, STz) + d(STz, z)] \\
 &= (\alpha_1 + 3\alpha_2 + 2\alpha_3) d(z, STz)
 \end{aligned}$$

which is a contradiction, since  $\alpha_1 + 3\alpha_2 + 2\alpha_3 \neq 1$ .

Hence  $STz = z$ .

This gives,  $STz = z = Jz$ . It follows from the upper part.

Again, since  $ST(X) \subset I(X)$  there always exist a point  $z''$  in  $X$ , such that  $Iz'' = z$ .

Thus,

$$\begin{aligned}
 &d(ABz'', z) = d(ABz'', STz) \\
 &\leq \alpha_1 \left[ \frac{d(Jz, STz)d(Iz'', ABz'')}{d(Iz'', Jz)} \right] + \alpha_2 [d(ABz'', Iz'') + d(Jz, STz)] + \alpha_3 d(Iz'', Jz) \\
 &= \alpha_1 \left[ \frac{d(z, z)d(z, ABz'')}{d(z, z)} \right] + \alpha_2 [d(ABz'', z) + d(z, z)] + \alpha_3 d(z, z) \\
 &\leq \alpha_1 d(z, ABz'') + \alpha_2 [d(ABz'', z) + \{d(z, ABz'') + d(ABz'', z)\}] + \alpha_3 [d(z, ABz'') + d(ABz'', z)] \\
 &= (\alpha_1 + 3\alpha_2 + 2\alpha_3) d(z, ABz'')
 \end{aligned}$$

which is a contradiction, since  $\alpha_1 + 3\alpha_2 + 2\alpha_3 \neq 1$ .

Hence  $ABz'' = z$ .

Also, since  $(AB, I)$  are compatible and hence weakly compatible, we obtain

$$\begin{aligned}
 d(ABz, Iz) &= d(AB(Iz), I(ABz)) \\
 &\leq d(Iz, ABz)
 \end{aligned}$$

Therefore,  $ABz = Iz = z$ .

Thus, we have proved that  $z$  is a common fixed point of  $AB, ST, I$  and  $J$ .

Instead of  $AB$  or  $I$ , if mappings  $ST$  or  $J$  is continuous, then the proof that  $z$  is a common fixed point of  $AB, ST, I$  and  $J$  is similar.

To show that  $z$  is unique, let  $v$  be another fixed point of  $I, J, AB$  and  $ST$ . Then,

$$\begin{aligned}
 &d(z, v) = d(ABz, STv) \\
 &\leq \alpha_1 \left[ \frac{d(Jv, STv)d(Iz, ABz)}{d(Iz, Jv)} \right] + \alpha_2 [d(ABz, Iz) + d(Jv, STv)] + \alpha_3 d(Iz, Jv) \\
 &\leq \alpha_1 \left[ \frac{d(v, v)d(z, z)}{d(z, v)} \right] + \alpha_2 [d(z, z) + d(v, v)] + \alpha_3 d(z, v) \\
 &\leq \alpha_1 \left[ \frac{\{d(v, z) + d(z, v)\} \{d(z, v) + d(v, z)\}}{d(z, v)} \right] + \alpha_2 [\{d(z, v) + d(v, z)\} \\
 &\quad + \{d(v, z) + d(z, v)\}] + \alpha_3 d(z, v) \\
 &= (4\alpha_1 + 4\alpha_2 + \alpha_3) d(z, v)
 \end{aligned}$$

which is a contradiction, since  $4\alpha_1 + 4\alpha_2 + \alpha_3 \neq 1$ ,

yielding thereby  $z = v$ .

Finally, we need to show that  $z$  is also a common fixed point of  $A, B, S, T, I$  and  $J$ . For this, let  $z$  be the unique common fixed point of both the pairs  $(AB, I)$  and  $(ST, J)$ . Then,

$$Az = A(ABz) = A(BAz) = AB(Az) \text{ and } Az = A(Iz) = I(Az),$$

$$Bz = B(ABz) = B(A(Bz)) = BA(Bz) = AB(Bz) \text{ and } Bz = B(Iz) = I(Bz),$$

which shows that  $Az$  and  $Bz$  are common fixed points of  $(AB, I)$ , yielding thereby  $Az = z = Bz = Iz = ABz$ , in the view of uniqueness of common fixed point of the pair  $(AB, I)$ .

Similarly, using the commutativity of  $(S, T), (S, J)$  and  $(T, J)$ , it can be shown that  $Sz = z = Tz = Jz = STz$ .

Now we need to show that  $Az = Sz$  ( $Bz = Tz$ ). Also remains a common fixed point of both the pairs  $(AB, I)$  and  $(ST, J)$ . For this,

$$\begin{aligned}
 d(Az, Sz) &= d(A(BAz), S(TSz)) \\
 &= d(AB(Az), ST(Sz))
 \end{aligned}$$

$$\begin{aligned} &\leq \alpha_1 \left[ \frac{d(J(Sz), ST(Sz))d(I(Az), AB(Az))}{d(I(Az), J(Sz))} \right] + \alpha_2 [d(AB(Az), I(Az)) \\ &\quad + d(J(Sz), ST(Sz))] + \alpha_3 d(I(Az), J(Sz)) \\ &= \alpha_1 \left[ \frac{d(Sz, Sz)d(Az, Az)}{d(Az, Sz)} \right] + \alpha_2 [d(Az, Az) + d(Sz, Sz)] + \alpha_3 d(Az, Sz) \\ &\leq \alpha_1 \left[ \frac{d\{(Sz, Az) + d(Az, Sz)\} \{d(Az, Sz) + d(Sz, Az)\}}{d(Az, Sz)} \right] \\ &+ \alpha_2 [\{d(Az, Sz) + d(Sz, Az)\} + \{(Sz, Az) + d(Az, Sz)\}] + \alpha_3 d(Az, Sz) \\ &= (4\alpha_1 + 4\alpha_2 + \alpha_3)d(Az, Sz), \end{aligned}$$

which is a contradiction, since  $4\alpha_1 + 4\alpha_2 + \alpha_3 \neq 1$ , yielding thereby  $d(Az, Sz) = 0 \Rightarrow Az = Sz$ . Similarly, it can be shown that  $Bz = Tz$ . Thus,  $z$  is the unique common fixed point of  $A, B, S, T, I$ , and  $J$

The other part is identical to the case (I), hence it is omitted, this completes the proof.

If we put  $AB = A, ST = B$  in Theorem (3.1), we get the following result.

**Corollary 3.1:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, B, S, T: X \rightarrow X$  satisfying

- (i)  $A(X) \subset T(X), B(X) \subset S(X)$ ,
- (ii)  $d(Ax, By) \leq \alpha_1 \left[ \frac{d(Ty, By)d(Sx, Ax)}{d(Sx, Ty)} \right] + \alpha_2 [d(Ax, Ty) + d(Ty, By)] + \alpha_3 d(Sx, Ax)$

for all  $x, y \in X$  and  $\alpha_1, \alpha_2, \alpha_3 \geq 0, 0 \leq 2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$ .

If either (iii)  $(A, S)$  are compatible,  $S$  or  $A$  is continuous and  $(B, T)$  are weakly compatible or (iv)  $(B, T)$  are compatible,  $T$  or  $B$  is continuous. Then  $A, B, S$  and  $T$  have a unique common fixed point.

If we put  $A=B$  in the above Corollary 3.1, we can obtain the following corollary easily.

**Corollary 3.2:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, B, S, T: X \rightarrow X$  satisfying

- (i)  $A(X) \subset T(X), A(X) \subset S(X)$ ,
- (ii)  $d(Ax, Ay) \leq \alpha_1 \left[ \frac{d(Ty, Ay)d(Sx, Ax)}{d(Sx, Ty)} \right] + \alpha_2 [d(Ax, Ty) + d(Ty, Ay)] + \alpha_3 d(Sx, Ax)$

for all  $x, y \in X$  and  $\alpha_1, \alpha_2, \alpha_3 \geq 0, 0 \leq 2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$ .

If  $(A, S)$  are compatible,  $S$  or  $A$  is continuous and  $(A, T)$  are weakly compatible Then  $A, S$  and  $T$  have a unique common fixed point.

If we put  $S=T$  in the above Corollary 3.1, we can obtain the following corollaries easily.

**Corollary 3.3:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, B, I: X \rightarrow X$  satisfying

- (i)  $A(X) \subset S(X), B(X) \subset S(X)$ ,
- (ii)  $d(Ax, By) \leq \alpha_1 \left[ \frac{d(Sy, By)d(Sx, Ax)}{d(Sx, Sy)} \right] + \alpha_2 [d(Ax, Sy) + d(Sy, By)] + \alpha_3 d(Sx, Ax)$

for all  $x, y \in X$  and  $\alpha_1, \alpha_2, \alpha_3 \geq 0, 0 \leq 2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$ .

If either (iii)  $(A, S)$  are compatible,  $S$  or  $A$  is continuous and  $(B, S)$  are weakly compatible or (iv)  $(B, S)$  are compatible,  $I$  or  $B$  is continuous. Then  $A, B$ , and  $S$  have a unique common fixed point.

If we put  $A=B$  and  $S=T$  in the above Corollary 3.1, we can obtain the following corollaries easily.

**Corollary 3.4:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, I: X \rightarrow X$  satisfying

- (i)  $A(X) \subset S(X),$  ,
- (ii)  $d(Ax, Ay) \leq \alpha_1 \left[ \frac{d(Sy, Ay)d(Sx, Ax)}{d(Sx, Sy)} \right] + \alpha_2 [d(Ax, Iy) + d(Sy, Ay)] + \alpha_3 d(Sx, Ax)$

for all  $x, y \in X$  and  $\alpha_1, \alpha_2, \alpha_3 \geq 0, 0 \leq 2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$ .

If  $(A, S)$  are weakly compatible and  $S$  or  $A$  is continuous . Then  $A$  and  $S$  have a unique common fixed point.

Putting  $\alpha_2 = 0$ , this will give the following generalization of Jeong-Rhoades [3] in dislocated metric spaces.

**Corollary 3.5:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, B, S, T: X \rightarrow X$  satisfying

- (i)  $A(X) \subset T(X), B(X) \subset S(X)$ ,
- (ii)  $d(Ax, By) \leq \alpha_1 \left[ \frac{d(Ty, By)d(Sx, Ax)}{d(Sx, Ty)} \right] + \alpha_2 d(Sx, Ax)$ ,

for all  $x, y \in X$  and  $\alpha_1, \alpha_2 \geq 0, 0 \leq 2\alpha_1 + \alpha_2 < 1$ .

If either (iii)  $(A, S)$  are compatibles or  $A$  is continuous and  $(B, T)$  are weakly compatible or (iv)  $(B, T)$  are compatible,  $T$  or  $B$  is continuous. Then  $A, B, S$  and  $T$  have a unique common fixed point.

If we put  $A=B$  in the above Corollary 3.5, we can obtain the following corollary easily.

**Corollary 3.6:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, S, T: X \rightarrow X$  satisfying

- (i)  $A(X) \subset T(X), B(X) \subset S(X)$ ,
- (ii)  $d(Ax, Ay) \leq \alpha_1 \left[ \frac{d(Ty, Ay)d(Sx, Ax)}{d(Sx, Ty)} \right] + \alpha_2 d(Sx, Ax)$ ,

for all  $x, y \in X$  and  $\alpha_1, \alpha_2 \geq 0, 0 \leq 2\alpha_1 + \alpha_2 < 1$ .

If  $(A, S)$  are compatible,  $S$  or  $A$  is continuous and  $(A, T)$  are weakly compatible Then  $A, S$  and  $T$  have a unique common fixed point.

If we put  $S=T$  in the above Corollary 3.5, we can obtain the following corollaries easily.

**Corollary 3.7:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, B, S: X \rightarrow X$  satisfying

- (i)  $A(X) \subset S(X), B(X) \subset S(X)$ ,
- (ii)  $d(Ax, By) \leq \alpha_1 \left[ \frac{d(Sy, By)d(Sx, Ax)}{d(Sx, Sy)} \right] + \alpha_2 d(Sx, Ax)$

for all  $x, y \in X$  and  $\alpha_1, \alpha_2 \geq 0, 0 \leq 2\alpha_1 + \alpha_2 < 1$ .

If either (iii)  $(A, S)$  are compatible,  $S$  or  $A$  is continuous and  $(B, S)$  are weakly compatible or (iv)  $(B, S)$  are compatible,  $S$  or  $B$  is continuous. Then  $A, B$ , and  $S$  have a unique common fixed point.

If we put  $A=B$  and  $S=T$  in the above Corollary 3.5, we can obtain the following corollaries easily.

**Corollary 3.8:** Let  $(X, d)$  be a complete dislocated metric space. Suppose mappings  $A, S: X \rightarrow X$  satisfying

- (i)  $A(X) \subset S(X),$  ,
- (ii)  $d(Ax, Ay) \leq \alpha_1 \left[ \frac{d(Sy, Ay)d(Sx, Ax)}{d(Sx, Sy)} \right] + \alpha_2 d(Sx, Ax)$

for all  $x, y \in X$  and  $\alpha_1, \alpha_2 \geq 0, 0 \leq 2\alpha_1 + \alpha_3 < 1$ .

If  $(A, S)$  are weakly compatible and  $S$  or  $A$  is continuous. Then  $A$  and  $S$  have a unique common fixed point.

#### 4 Conclusion

Fixed point theory is a rich, intriguing, and dynamic branch of mathematics. Although it is relatively young, it has become a fully developed area of research. The study of the existence of fixed points spans several domains, including functional analysis, operator theory, and general topology. Fixed points and fixed point theorems have always been essential theoretical tools in diverse fields such as topology, mathematical economics, game theory, approximation theory, and initial and boundary value problems in ordinary and partial differential equations. Moreover, the usefulness of this concept has recently increased enormously due to the development of accurate and efficient techniques for computing fixed points, making fixed point methods a major tool in the arsenal of mathematics. In recent years, several definitions of conditions weaker than commutativity have emerged, significantly facilitating the extension of Jungck's theorem and numerous others. In this paper, we establish some common fixed point results, using the concepts of compatibility, weak compatibility, and commutativity in complete dislocated metric (d-metric) spaces This generalizes several previously known results, demonstrating that our result is superior to many others in the literature.

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