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Research Article

A Note On Unary Operatıons On Graphs And Theır Acyclıc Colorıng

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Abstract

An acyclic coloring is a proper vertex coloring in which no cycle in the graph is bichromatic or the subgraph induced by any two colors is acyclic. The acyclic chromatic number a(G) of a graph G is the least number of colors in an acyclic coloring of G. In this paper, acyclic coloring of middle and central graph of some graphs are investigated. For any graph G, a relation between the acyclic chromatic number and acyclic chromatic index of its line graph is derived, and using this relation, acyclic chromatic index of line graph of n -dimensional graphs like hypercube, mesh and torus are analyzed.

Keywords: *Acyclic coloring, acyclic edge coloring, acyclic chromatic number, middle graph, line graph, central graph*
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Introduction

Proper coloring is an assignment of colors to the vertices (or edges), such that no adjacent vertices (or edges) receive the same color. The least number of colors required for the proper coloring is termed as the chromatic number (index). A proper vertex (or edge) coloring of a graph G, is said to be acyclic if it does not contain any bichromatic cycles in it. The minimum number of colors required for an acyclic coloring is referred as the acyclic chromatic number $a(G)$ (in case of a vertex coloring), and acyclic chromatic index $a'(G)$ (in case of an edge coloring). The concept of acyclic coloring, acyclic chromatic number, and star coloring was introduced by Grunbaum [4] in 1973 and mainly studied by Albertson [1], Borodin [2], and amongst others. In 1978 A.V. Kostochka [6] proved in his thesis that, deciding whether if the acyclic chromatic number of G is at most k is an NP-complete problem for a given G and k.
There exist numerous types of operations on graphs, like graph union, graph intersection, graph join, graph sum, graph product, etc., which are generally named as binary operations on graphs. While there are some other types of operations, called unary operations on graph. Some examples for unary operations on a graph are the complement of a graph, power of a graph, line graph of a graph, middle graph of a graph, total graph of a graph, splitting graph of a graph, central graph of a graph, etc. Other operations of this kind can be found in Harary and Wilcox [5]. T. Hamada and I. Yoshimura in 1974 [8] introduced the concept of middle graph. In [17] M. Behzad has introduced the notions of the total chromatic number and the total graph of a graph. In 1932 H. Whitney introduced the concept of a line graph. Acyclic coloring of middle, total and line graph of some particular graphs have been studied by some authors, but no work has been done in the case of a general graph. Note that the middle and total graph of G are generalization of line graph of G. Vernold Vivin. J et al. [14] introduced the concept of central graph.
Let $G = (V, E)$ be a graph. The middle graph [8] $M(G)$ of G, is the graph with the vertex set $V(G) \cup E(G)$ in which two vertices $x, y \in V(MG)$ are adjacent in $M(G)$, only if (i) $x, y \in E(G)$ and x, y are adjacent in G. (ii) $x \in V(G)$, $y \in E(G)$ and x, y are incident in G. Let G be a finite undirected graph with no loops and multiple edges. A graph obtained by partitioning each edge of G exactly once and connecting all the non-adjacent vertices of G using edges is called a central graph [14] $C(G)$ of G. The line graph [15] $L(G)$ of G is the intersection graph, where the points of $L(G)$ are the lines of G, with two points of $L(G)$ are adjacent whenever the corresponding lines of G are. Consider the graph G, with vertex set $V(G) =$

$\{v_1, v_2, \dots, v_p\}$ and the edge set $E(G) = \{e_1, e_2, \dots, e_q\}$. To the graph G, we add p new vertices and p new edges $\{u_i, v_i\}(i = 1,2, \dots, p)$, where $u_i's$ are different from any vertex of *G* and from each other. Then we a obtain new graph called Endline graph [8] of *G*, denoted by $G^+ = (2p, p+q)$ with vertex set $V(G^+) = \{u_1, u_2, \dots, u_p, v_1, v_2, \dots, v_p\}$ and edge set $E(G^+) = e_1, e_2, \dots, e_q, \{u_1, v_1\}, \{u_2, v_2\}, \dots, \{u_p, v_p\}\}$. An *n*-wheel W_n [16] is the graph obtained by connecting all the vertices of an *n*- cycle C_n to one other vertex, called the hub. The newly added edges are called spokes. The wheel graph W_n is isomorphic to the graph sum K_1+C_n.The wheel graph W_n has $n^2 - n + 1$ graph cycles in it. The helm graph H_n, is the graph we received from an *n*-wheel by adding a pendant edge at each vertex of the cycle. The Flower graph F_n is the graph obtained from the helm graph H_n, by adding edges between all the pendant vertices and the central vertex. Note that both the flower graph and helm graph are defined only for $n \geq 3$.

The concept of acyclic edge coloring of a graph was introduced by B. Grunbaum [4]. Even for most common class of well-known graphs, the estimation of $a'(G)$ is not yet decided exactly. Alon, Sudakov, and Zaks [13] proved that $a'(G) \leq \Delta + 2$ for almost all Δ - regular graphs. Nesetril and Wormald [12] improved this result and showed that for a random Δ-regular graph $a'(G) \leq \Delta + 1$. Alon et al. [21] designed an algorithm that can acyclically edge color the complete graph K_p, even though finding the exact values of $a'(K_n)$ for every *n* seems hard. Through this work, they constructively showed that $a'(K_p) = p$. An n-dimensional partial torus [20] is a connected graph *G* whose unique prime factorisation is of the form G= $G_1 \boxdot G_2 \boxdot G_3 \dots \boxdot G_n$, where $G_i \in$ PATHS $\cup$ CYCLES, for each $i \geq n$. Let $\mathcal{P}_n$ represents the class of such graphs. Then, *G* is an *n*-dimensional hypercube K_2^n, if each prime factor of G $\in \mathcal{P}_n$ is a P_2. G is an *n*-dimensional mesh $\mathcal{M}_n$, if each prime factor of G $\in \mathcal{P}_n$ is from PATHS. G is an *n*-dimensional torus $\mathcal{T}_n$, if each prime factor of G $\in \mathcal{P}_n$ is from CYCLES.

In this paper, acyclic coloring of middle and central graph of some graphs are examined and acyclic chromatic numbers of these graph operations are obtained. With respect to a coloring *c*, a new concept called acyclically feasible set is introduced, and aslo, some close bounds are attained using endline graph of a graph. The edge coloring of *G* and the vertex coloring of *L*(*G*) are always equivalent. Hence while taking all types of coloring into account, the chromatic index of *G* will be equal to the corresponding version of the chromatic number of *L*(*G*). But the situation is different when we consider the chromatic index of the line graph of *G*. Here the isomorphism need not exists in general, including for the case of acyclic coloring. In [19], it has been proved that, in the case of *3* consecutive coloring, the vertex coloring of G and the corresponding edge coloring of $L(G)$ coincide. That is, $\chi_{3c}(\mathrm{G}) = \chi'_{3c}(\mathrm{L(G)})$. It is a subtle coincidence, as it does not hold for most of the other types of colorings. Here in the case of acyclic coloring, we will prove an inequality.

Throughout this paper graphs means simple connected graphs. In figures the symbol *i* represents the color c_i.

Preliminaries

Theorem 2.1. [7] For any graph G, $a'(G) \leq 4\Delta(G) - 4$.

Theorem 2.2. [8] Let G be any graph. Then $L(G^{+})$ is isomorphic to the middle graph $M(G)$.

Theorem 2.3. [9] For a flower graph F_n, $n \geq 5$, the star chromatic number of $M(F_n)$ is $\chi_s(\mathrm{M}(\mathrm{F}_n)) = 2n + 1$.

Theorem 2.4. [22] For the graph $\mathrm{K}_{1,n}$,

(*i*). $a(\mathrm{M}(\mathrm{K}_{1,n})) = n + 1$.

(*ii*). $a(\mathrm{T}(\mathrm{K}_{1,n})) = n + 1$.

Proposition 2.5. [4] For an arbitrary graph G, $\chi(\mathrm{G}) \leq a(G) \leq \chi_s(\mathrm{G})$.

Proposition 2.6. [8] Let $M(W_k)$ be the middle graph of the wheel graph W_k. Then in $M(W_k)$, the hub together with the spokes constitute a clique of order n +1.

Proposition 2.7. [10, 11] Let G be any graph and $L(G)$ denotes line graph of G. Then,

1. $L(G)$ is connected whenever G is connected
2. If $\upsilon(\mathrm{G}) = n$, $\varepsilon(\mathrm{G}) = m$ and vertex degree are $d_G(\mathrm{v})$, then $\upsilon(\mathrm{L(G)}) = m$ and $\varepsilon(\mathrm{L(G)}) = \frac{1}{2}\sum_{i=1}^{n} \mathrm{d}_G^2(\mathrm{v}) - m$
3. $\chi'(\mathrm{G}) = \chi(\mathrm{L(G)})$
4. $\mathrm{G}_1 \cong \mathrm{G}_2 \Rightarrow \mathrm{L}(\mathrm{G}_1) \cong L(\mathrm{G}_2)$, but the converse is not true
5. Cycles are the only graphs, for which $L(G) \cong G$
6. For path graphs P_n, $n \geq 1$ the line graph $L(P_n) = P_{n-1}$
7. The line graph of a regular graph is regular; however the converse is not true. (Eg $K_{1,3}$ is not regular, but $L(K_{1,3})$ is regular)
8. The line graph of a Eulerian graph is both Eulerian and Hamiltonian

Theorem 2.8. [12] $a'(\mathrm{G}) \leq d + 1$ for almost every *d*-regular graph.

Conjecture 2.9. [13] For any graph G, $a'(G) \leq \Delta(G) + 2$.

Theorem 2.10. [20] (*i*) For an n-dimensional hypercube K_2^n, $n \geq 2$ the acyclic chromatic index $a'(\mathrm{K}_2^n) = \Delta(\mathrm{K}_2^n) + 1$. (*ii*) For an n-dimensional torus T_n, $n \geq 1$ the acyclic chromatic index $a'(T_n) = \Delta(T_n) + 1$. (*iii*) For an n dimensional mesh M_n, $n \geq 1$ the acyclic chromatic index $a'(M_n) = \Delta(M_n)$.

Proposition 2.11. For any graph G, $a'(G) = a(L(G)$.

Acyclic chromatic number of middle and central graphs

Definition 3.1. Let $G = (V, E)$ be a graph and $N_G(v)$ be the neighbor set of the vertex v ∈ V in *G*. Let $c: V(G) \to C$ be a proper vertex coloring of *G*, where *C* is the set of colors. If the function c satisfies $c(x) \neq c(y)$ for every $x,$y ∈ $N_G(v)$, then we say that the vertex *v* is an acyclically feasible vertex with respect to the coloring *c*. The set of all such vertices is called an acyclically feasible set w. r. t the coloring *c*.

Example 3.2. For a complete graph K_n, every set $S \subseteq V(K_n)$ is an acyclically feasible set w. r. t any proper coloring c.

Proposition 3.3. Let $G = (V, E)$ be a graph and $S \subseteq V$ be an acyclically feasible set w. r. t a coloring *c* of *G*. Then no bichromatic cycle is possible through any member of *S*.

Proof. The vertex *v* has a chance to become a part of a possible bichromatic cycle w. r. t a coloring *c*, only if we are able to enter *v* from a vertex *x* and exit *v* to another vertex *y* with $c(x) = c(y)$. But, by the definition of acyclically feasible set, no elements of *S* possess this property.

Theorem 3.4. For any non-empty graph *G*, $a\big(M(G)\big) \leq 4\Delta(G) - 4$.

Proof. Let $M(G)$ denotes the middle graph of the graph G. Let us define an acyclic edge coloring $c: E(G) \to C$ for *G*, by using at most $\Delta(G) + 2$ colors. Let $v \in V(G)$ with $deg_G(v) \leq \Delta(G)$, then all the edges incident with the vertex *v* can be acyclically edge colored by at most $\Delta(G)$ colors. Form the endline graph $G^+$of G. Then the degree of each vertex of *G* will be increased by one. That is $deg_{G^+}(v) \leq \Delta(G) + 1$. Now we choose a color c_1 or c_2 $\in C$, different from the $\Delta(G)$ colors and assign it properly to all the endlines of G^+. Then it results in an acyclic edge coloring of $G^+$with at most $\Delta(G) + 2$ colors, as no bichromatic cycle is possible through the endlines. Now by using Theorem 2.1 and the inequality $\Delta(G) + 2 \leq 4\Delta(G) - 4$ we have $a'(G^+) \leq 4\Delta(G) - 4$, that is, $a(\mathrm{L}((G^+) \leq 4\Delta(G) - 4$. Hence by Theorem 2.2, $a\big(M(G)\big) \leq 4\Delta(G) - 4$.

Remark 3.5. If $a(L(G)) \geq \Delta(G) + 2$, then $a(M(G)) \leq a(L(G))$.

Theorem 3.6. For a flower graph F_n, $a(M(F_n)) = \Delta(F_n) + 1$.

Proof. From the definition of flower graph, we have $\Delta(F_n) = 2n$.

Case 1. $n \geq 5$

From Theorem 2.3 and Proposition 2.5, we have

$a(M(F_n)) \leq 2n + 1$, for $n \geq 5$ (3.1)

Since the flower graph F_n contains the star graph $K_{1,2n}$ as subgraph, the corresponding middle graphs satisfy the relation $M(F_n) \supseteq M(K_{1,2n})$. Therefore $a(M(F_n)) \geq a(M(K_{1,2n}))$. By Theorem 2.4(1), we have, $a\left(M(K_{1,2n})\right) = 2n + 1$.

That is,

$a(M(F_n)) \geq 2n + 1$ (3.2)

Hence we can conclude that, $a(M(F_n)) = 2n + 1 = \Delta(F_n) + 1$.

Case 2**:** for $n = 3,4$

Since the equation (3.1) is true only for $n \geq 5$, we have to prove the theorem for $n = 3,4$ seperatly. Here we define a coloring *c* for $M(F_n)$ using the color set $C = \{c_1, c_2, \ldots, c_{2n+1}\}$ $n = 3,4$ as follows. In the middle graph $M(F_n)$, the central vertex and its neighboring 2*n* vertices constitute a clique of order 2*n*+1, which need exactly 2*n*+1 colors for its proper coloring and it is the minimum requirement also. Now the color assigned to the central vertex is given to all the pendant vertices corresponding to the original graph H_n. (See the Figure 1, a helm graph, flower graph and middle graph of the flower graph are shown). Let *S* denotes the set of vertices in $M(F_n)$ other than the above mentioned vertices. These vertices are colored using the above 2*n* + 1 colors properly such that the adjacent vertices in the set *S* receive different colors. Thus all the vertices are properly colored with the color set *C*. By Proposition 3.3, we can remove the acyclically feasible sets from the middle graphs, which lead to the conclusion that the coloring *c* is acyclic. Thus $a(M(F_n)) = 2n + 1$.

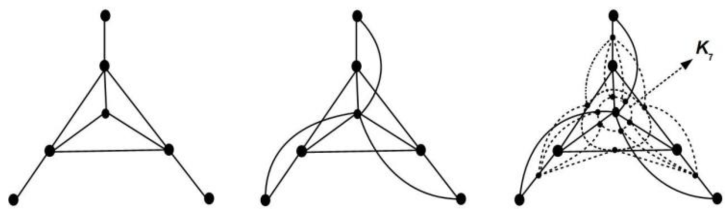

Figure 1: A helm graph H_3, flower graph F_3, and $M(F_3)$)

Theorem 3.7. For a wheel graph W_n, $a(M(W_n)) = \Delta(W_n) + 1$, for $n \geq 4$.

Proof. Consider a wheel graph W_n having $n+1$ vertices.

Let the vertex set be $V(W_n) = \{v_1, v_2, v_3, \ldots v_n, v_{n+1}\}$ which are pointed in counter clockwise direction as depicted in Figure 2, with v_{n+1} as the hub. Then the hub of the wheel graph is the maximum degree vertex with $\Delta = n$. In the middle graph $M(W_n)$, n-new vertices, say v_{ij}, $1 \leq i, j \leq n$ are formed on the edges of the cycle, and another n-vertices on the spokes. Note that by Proposition 2.6 the hub together with its neighboring spoke vertices constitute a clique of order $n + 1$, which requires exactly $n + 1$ colors for its proper coloring so that $a(M(W_n)) \geq n+1$. Let the n-spoke vertices corresponding to the vertices v_i be colored using c_i, $1 \leq i \leq n$ continuously in anti-clockwise direction and the hub by the color cn+1. Now we color the remaining vertices using the above used $n + 1$ colors itself as follows. The n-vertices in $M(W_n)$ corresponding to the n-cycle of W_n are assigned the same color as that of the hub. Finally the colors c_i, $1 \leq i \leq n$ are assigned continuously in anti-clockwise direction for the remaining vertices $v_{i,j}$. Now it can be observed that the subgraph induced by any two color is a forest. Also the coloring is minimum. Thus $a(M(W_n)) = n+1 = \Delta + 1$, for $n \geq 4$.

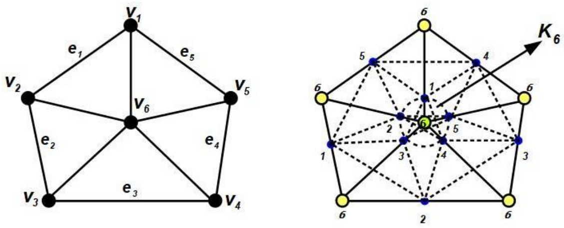

Figure 2: A wheel graph W_5 and an cyclic coloring of $M(W_5)$

Remark 3.8. $a(M(W_3)) = 5$.

The coloring in the proof of Theorem 3.7, is not good for $M(W_3)$ as it constitute bichromatic cycles in $M(W_3)$. Also no acyclic 4−coloring can be defined for $M(W_3)$. The Figure 3 explains an acyclic 5− coloring of $M(W_3)$.

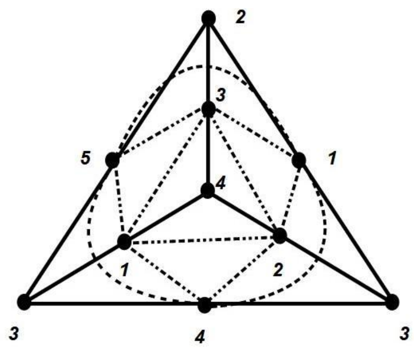

Figure 3: An cyclic coloring of $M(W_3)$

Corollary 3.9. For the helm graph H_n, $a(L(H_n)) \le \Delta(H_n) + 1$, for $n \ge 3$.

Proof. When $n \ge 4$, by Theorems 2.2 and 3.7, we have $a(L(W_n^+)) = a(M(W_n)) = \Delta(H_n) + +1$ (since $\Delta(W_n) = \Delta(H_n)$ for $n \ge 4$). Also by definition, the helm graph $H_n \subseteq W_n^+$, hence we get $a(L(H_n)) \le \Delta(H_n) + 1$. When $n = 3$, by Remark 3.8, we have $a(L(H_3)) \le a\big(L(W_3^+)\big) = 5$ and also $\Delta(H_3) = 4$. Thus for $n \ge 3$, $a(L(H_n)) \le \Delta(H_n) + 1$.

Example 3.10. Two Counter Examples. In [18], it was proved that, for the central graph of a complete graph K_n, n ≥ 3, $a\big(C(K_n)\big) = \frac{n}{2} + 1$, if n is even and $a\big(C(K_n)\big) = \left\lfloor \frac{n}{2} \right\rfloor + 2$, if n is odd. But here we give two counter examples for the above results with an acyclic coloring and then in Theorem 3.11 we will prove that $a\big(C(K_n)\big)$ is a fixed number for any n ≥ 3.

Case 1. When n is odd.

Consider the central graph of K_5. We have given one acyclic coloring c of $C(K_5)$ using 3 colors as depicted in Figure 4. Let S be the acyclically feasible set of veritces, with respect to the coloring c, which are marked inside squares in Figure 4. Then by Proposition 3.3, no bichromatic cycle is possible through vertices of S. Let $C(K_5) - S$ denotes the graph obtained by removing each vertex of S and all associated incident edges from $C(K_5)$. From the Figure 4 it is clear that in $C(K_5) - S$, all the subgraphs induced by any two colors is a forest. Thus $a\big(C(K_5)\big) = 3 \ne \left\lfloor \frac{5}{2} \right\rfloor + 2$.

Case 2. When n is even.

Consider the central graph of K_6, we have given one acyclic coloring c of $C(K_6)$ using 3 colors as depicted in Figure 5. As explained in case 1, we can conclude that, $a\big(C(K_6)\big) = 3 \ne \frac{6}{2} + 1$.

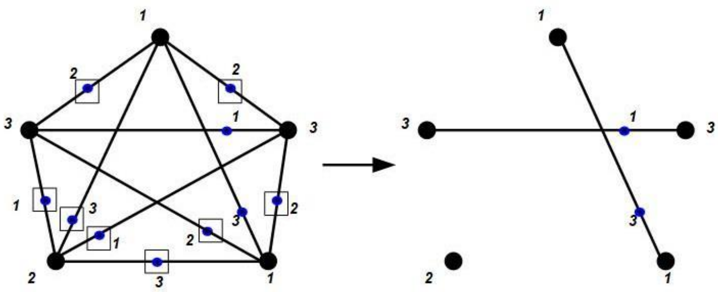

Figure 4: Graph of $C(K_5)$ and $C(K_5) - S$

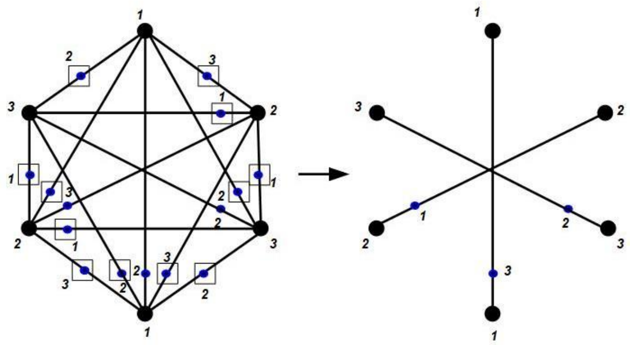

Figure 5: Graph of $C(K_6)$ and $C(K_6) - S$

Theorem 3.11. For the central graph of K_n, $a(C(K_n)) = 3$ for $n \geq 3$.

Proof. Let $V(K_n) = \{v_i | 1 \leq i \leq n\}$. Let v_{hk}, $1 \leq h < k \leq n$ be the newly introduced vertices of $C(K_n)$. Consider a coloring c using the color set $C = \{c_1, c_2, c_3\}$. We color the vertices of $C(K_n)$ in two cases.

Case 1: Coloring of the vertex set *V*.

Assign the colors $c_i \in C$ continuously and cyclically in anti-clockwise direction to the vertex set *V*.

Case 2: Coloring of the newly introduced vertices.

Let S be the acyclically feasible set of vertices in $C(K_n)$. For the elements in S there is only one possible color assignment, which will not cause any bichromatic cycle in $C(K_n)$. Now let us remove the set S from $C(K_n)$. The new subgraph $C(K_n)$-S obtained will be always the union of paths P_3 or even cycles and the maximum length of such cycles will be $2\left\lceil\frac{n}{3}\right\rceil$. Since the newly introduced vertices are not adjacent to each other, each of the even cycle can be acyclically 3 colored. Thus, $a(C(K_n)) = 3$ for $n \geq 3$.

Acyclic edge coloring of graphs

Theorem 4.1. For any connected graph G with the number of vertices at least 4, $a(G) \leq a'(L(G))$.

Proof. Case 1. If *G* is a cycle, then by Proposition 2.7 (5), $a(G) = a'\big(L(G)\big) = 3$.

Case 2. If G is a path P_n, then by Proposition 2.7 (6), the equality $a(G) = a'(L(G))$ holds. If G is a tree having branches and sub branches, then $\upsilon(G) = \varepsilon(G) + 1$ and $a(G) = 2$. Since line graph of a tree can be a tree or a cyclic graph, we have $a'(L(G)) \geq 2$, which gives $a(G) \leq a'(L(G))$.

Case 3. If G is connected, which is neither a tree nor a cycle, we have

$\upsilon(G) \leq \varepsilon(G)$.

$\Rightarrow \upsilon(G) \leq \varepsilon(G) = \upsilon(L(G)) \leq \varepsilon(L(G))$.

Also, $a'(L(G)) = a(L^2(G)) \geq a(G)$.

Thus we can conclude that, for any graph G, $a(G) \leq a'(L(G))$.

Remark 4.2. In the case of the complete graph K_4, the strict inequality holds. Here $a(K_4) = 4$ and $a'(L(K_4)) = 5$.

Now we find upper bound for the acyclic chromatic number of line graphs of n-dimensional hypercube K_2^n ($n \geq 2$), n-dimensional torus T_n ($n \geq 1$), and n-dimensional mesh M_n.

Proposition 4.3. Let K_2^n, T_n and M_n respectively denote the n-dimensional graphs hypercube, torus and mesh. Then

1) For $n \geq 1$, the line graph $L(K_2^n)$ has the number of vertices $\upsilon(L(K_2^n)) = \varepsilon(K_2^n)$ and number of edges $\varepsilon(L(K_2^n)) = 2^{n-1}(n^2 - 1) - 2.\varepsilon(K_2^{n-1})$ and it is a $2(n-1)$ regular graph.
2) For $n \geq 1$, the line graph $L(T_n)$ is an $4n - 2$ regular graph.
3) For each path factor is of length at least four in the mesh M_n, $\Delta(L(M_n)) = 4n - 2$.

Proof. It is familiar that, in a line graph $L(G)$, for each vertex $e \in V(L(G))$, corresponding to the edge uv in G,

$$\deg_{L(G)}(e) = |N_G(u)| + |N_G(v)| - 2 \qquad (4.1)$$

1) Note that K_2^n is an n-regular graph with $\upsilon(K_2^n) = 2^n$ and $\varepsilon(K_2^n) = 2.\varepsilon(K_2^{n-1}) + 2^{n-1}$, for $n \geq 1$. Then by property 2.7(2), we get $\upsilon(L(K_2^n)) = \varepsilon(K_2^n)$ and $\varepsilon(L(K_2^n)) = 2^{n-1}(n^2 - 1) - 2.\varepsilon(K_2^{n-1})$. By equation 4.1, we get $d_{L(K_2^n)}(v) = 2n - 2$. Thus $L(K_2^n)$ is a $2(n-1)$ regular graph.
2) Since T_n is $2n$-regular graph, by equation 4.1, we get $d_{L(T_n)}(v) = 4n - 2$ for every vertex of $L(T_n)$.
3) Since in M_n, each path is of length at least four, there will be at least two vertices of maximum degree $2n$. So by equation 4.1, we get $\Delta(L(M_n)) = 4n - 2$.

Theorem 4.4. Let K_2^n, T_n and M_n respectively denote the n-dimensional graphs hypercube, torus and mesh. Then

1) $a(\mathrm{K}_2^n) \leq 2n-1$, for $n \geq 2$
2) $a(\mathrm{T}_n) \leq 4n-1$, for $n \geq 1$
3) For M_n with each path factor is of length at least four, $a(M_n) \leq 4n$.

Proof.

1) Since $\mathrm{L}(K_2^n)$ is a $2(\mathrm{n}-1)$ regular graph, by Theorem 2.8 we have $a'(\mathrm{L}(\mathrm{K}_2^n)) \leq 2n-1$. By Theorem 4.1, we get $a(\mathrm{K}_2^n) \leq 2n-1$. Note that $a(\mathrm{K}_2^1) = 2$.
2) Since $L(T_n)$ is a $(4\mathrm{n}-2)$ regular graph, by Theorem 2.8 we can write $a'(L(T_n)) \leq 4n-1$. Also by Theorem 4.1, we get $a(T_n) \leq 4n-1$.
3) Since $\Delta\big(L(M_n)\big) = 4n-2$, by Conjecture 2.9, we have $a'(L(M_n)) \leq 4n$. Now by using the Theorem 4.1, we get $a(M_n) \leq 4n$.

The following remark is obtained from Theorem 2.10 and Propositions 2.11 and 4.3.

Remark 4.5. Let $\mathrm{G}_1 = \mathrm{L}(K_2^n)$ with $n \geq 2$, $\mathrm{G}_2 = L(T_n)$ with $n \geq 1$and $\mathrm{G}_3 = L(M_n)$ with each path factor is of length at least four. Then $a(\mathrm{G}_i) = \frac{\Delta(\mathrm{G}_i)}{2} + 2$ for $i = 1,2$ and $a(\mathrm{G}_i) = \frac{\Delta(\mathrm{G_i})}{2} + 1$ for $\mathrm{i} = 3$.

Conclusion

In this chapter, the acyclic chromatic number of the unary operations like middle graph, line graph and central graph of some graphs are studied. A relation betwen the acyclic chromatic number of G and acyclic chromaic index of $L(G)$ is established. The acyclic chromatic index of the line graph of n-dimensional partial torus is obtained.

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