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L-Supermerotopic Spaces

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Abstract

The current paper contributes towards the investigation of L-supermerotopic spaces. It is a generalization of merotopy as far as the axiomatic structure is concerned, and proximity, both in terms of axioms and number of elements. Lattice structures of L-supermerotopic spaces are investigated and it is searched through L-superpre-merotopy. Some basic and desirable results are also obtained..

Keywords: Supermerotopy; Nearness spaces; Proximity spaces

1. Introduction

A large part of our genuine life problems in engineering, and so forth deals with different uncertainties. Of all the theories, the idea of fuzzy sets [8] emerged as a powerful tool to manage uncertainties. The concept of nearness introduced in [6], has been explored broadly widely in fuzzy settings. The idea of grills in conjunction to merotopies may be seen in [9, 10]. Near structures have imparted hugely to understand and comprehend of various extension problems [5]. Nearness of two sets and more than two sets in soft set theory may be seen in [11, 12, 13, 14]. Katsaras [2] presented fuzzy proximity in [0,1]-fuzzy set theory. Afterwards, Wang and Liu [3], further extended the proximities into L-fuzzy set theory. Ward [1] studied nearness of finite order by giving a restriction on the cardinality. Chattopadhyay and Njastad [7] discussed the completion of Riesz merotopic space.

The concept of supernearness is introduced in [4] and motivated by this we extended the concept of supernearness in L-fuzzy set theory in the present paper.

2. Preliminaries

Suppose U be a non-empty set and L be a distributive and complete lattice with largest element 1, smallest element 0 and an order reversing involution: $L \to L$. An $L-$ fuzzy subset in U is an element of the family L^U of every functions from U to L. An $L-$ fuzzy point $x_s \in L^U$, $s \in L - \{0\}$, is defined by $x_s(u) = s$, $x_s(v) = 0$ when $u \neq v$.

2.1 Definition

Let $\mathfrak{C}, \mathfrak{I}$ be subsets of L^U then we say:

(i) $sec\ \mathfrak{C} = \{k \in L^U : k \wedge l \neq 0\ for\ all\ l \in L^U\}$

(ii) $stack\ \mathfrak{I} = \{k \in L^U : k \geq l, for\ some\ l \in L^U\}$

2.2 Definition

Suppose U be a nonempty set and $P(L^U)$ denotes the power set. Then ς is said to be an $L-$merotopy on U provided, for subsets $\mathcal{C}, \mathcal{M} \in L^U$, (M1) $\mathcal{C}$ corefines $\mathcal{M}$ and $\mathcal{M} \in \varsigma$ implies $\mathcal{C} \in \varsigma$,

(M2) $\wedge \mathcal{C} \neq 0$ implies $\mathcal{C} \in \varsigma$,

(M3) $\emptyset \neq \varsigma \neq P(L^U)$,

(M4) $\mathcal{C} \vee \mathcal{M} \in \varsigma$ implies $\mathcal{C} \in \varsigma$ or $\mathcal{M} \in \varsigma$

The pair (U, ς) is said to be an $L-$merotopic space. For an $L-$merotopic spaces (U, ς):

$cl_{\varsigma}k = \vee \{x_p : \{x_p, k\} \in \varsigma\}, k \in L^U$,

An $L-$merotopy ς on U is said to be an $L-$nearness if the below mentioned condition is satisfied:

(M5) $\{cl_{\varsigma}k , cl_{\varsigma}l \} \in \varsigma$ implies $\{k, l\} \in \varsigma$.

3. L −Supermerotopies

3.1 Definition

Let U a non-empty set. A subset $B^U \subset L^U$ is said to be $L-$prebornology or a $B-$structure on U if the followings conditions hold:

(FB1) $f \leq g \in B^U \Rightarrow f \in B^U$

(FB2) $0 \in B^U$

(FB3) $x \in U \Rightarrow \{x_p\} \in B^U$

For a pair of $FB-$structures B^U, B^V on sets U and V, respectively, a map $F: U \to V$ is said to be bounded if and only if it satisfies the followings condition:

$$\{F[b] | b \in B^U\} \subseteq B^V$$

Elements of B^U will be called bounded fuzzy sets

3.2 Definition

For an $FB-$ set B^U, a map $M_U: B^U \to P(P(L^U))$ is said to be $L-$premerotopic operator (denoted as super $L-$premerotopic) if the following axioms holds:

(FNS1) $g \in B^U$and $\mathfrak{C}$ corefines $\mathfrak{J} \in M_U(g) \Rightarrow \mathfrak{C} \in M_U(g)$

(FNS2) $g \in B^U \Rightarrow B^U \notin M_U(g) \neq 0$

(FNS3) $g \in M_U(0) \Rightarrow g = 0$

(FNS4) $\{x_p\} \in U \Rightarrow \{x_p\} \in M_U\{\{x_p\}\}$

(FNS5) $h \subseteq g \in B^U \Rightarrow M_U(h) \subseteq M_U(g)$

If in addition to this,

(FNS6) $g \in B^U$ and $\mathcal{C} \vee \mathcal{M} \in M_U(g) \Rightarrow \mathcal{C} \in M_U(g)$ or $\mathcal{M} \in M_U(g)$

Then the pair (B^U, M_U) is said to be an $L-$ supermerotopic space.

Elements of $M_U(g)$ are known as $B-$near collection.

For a pair merotopic spaces $(B^U, M_U), (B^V, M_V)$ a bounded map $\tau: B^U \to B^V$ is said to be $L-$supermerotopic map if and only if it satisfies $\{\tau[g] | g \in B^U\} \subseteq B^V$.

3.3 Example

(i) Let s be a $L-$superproximity structures on B^U. Then for $g \in B^U$, define:

$M_U(h) = \{\mathcal{C} \,|\mathcal{C} \subset s(g)\}$ such that $s(g) = \{h \in L^U | g \; s \; h\}$.

(ii) For semi nearness space (U, ς) such that $B^U = P(L^U)$, define:

$$M_{\xi}(h) = \begin{cases} \{0\}, if \; f = 0 \\ \{g: \{h\} \cup g \in \varsigma\}, \text{otherwise} \end{cases}.$$

3.4 Remar

For an L −merotopy structures ξ on B^U, for $k \in B^U$

$M_U(k) = \{\mathfrak{C}: \mathfrak{C} \subset \xi(k)\}$ where $\xi(k) = \{l \in B^U: \{k, l\} \in \xi\}$

Then $M_U(k)$ is super L −premerotopy

Let $SM(U) = \{M_U: M_U$ is a $L-$ supermerotopy on $U\}$ and family $SM(U)$ is partially ordered by the relation defined by $M_U \le M_V$ if and only if $U \subset V$, for $M_U, M_V \in SM(U)$

3.5 Proposition

Suppose (B^U, M_U) be a nonempty subfamily of $SM(U)$ (super L −premerotopic spaces). Then

(i) $\cup M_U$ is a super L −premerotopy on U and $\cup M_U = \sup\{M_U: U \in I\}$.

(ii) $\bigcap M_U$ is a super L −premerotopy on U and $\bigcap M_U = \inf\{M_U: U \in I\}$.

Proof: Since $f \in \cup M_U$ implies $f \in M_U$ such that $\mathfrak{C}$ corefines $\mathfrak{I} \in \cup M_U \Rightarrow \mathfrak{C} \in M_U$ which gives $\mathfrak{C} \in \cup M_U$. Suppose $M_U(0) = \{0\}$ such that $M = \cup M_U(0) = \{0\}$ and $B(L^U) \notin M_U(f), \forall f \in B^U$. Since $B^U \notin M_U(f)$ implies $B^U \notin \cup M_U(f)$. Therefore, $0 \notin M_U(f)$. Further $h \le g \in M_U$ implies $M_U(h) \subset M_U(g)$. It can be easily proved that $\cup M_U(g) \subset \cup M_U(f)$. Also $x_p \in U \Rightarrow \{x_p\} \in \cup M_U\left\{\{x_p\}\right\}$. Since $x \vee y \in M(f) = \cup M_U(f)$. Now $x \vee y \in M_{U_s}(f)$, for some s which implies $x \in M_{U_s}(f)$ or $y \in M_{U_s}(f)$ also $x \in \cup M_{U_s}(f)$ or $y \in \cup M_{U_s}(f)$. This gives $x \vee y \in M_U$. Hence proved.

(ii) $\bigcap M_U$ is a L −premerotopy on U if satisfies: if $k \le l$ and $l \in M_U\ \forall \beta$, then $k \in M_U$ for all β. If $\bigcap k \ne 0$ then $k \in M_U\ \forall \beta$ and so $\bigcap M_U \ne \emptyset$. If $0 \in k$, gives $k \notin M_U$ and $\{1\} \in M_U \forall \beta$.

3.6 Theorem

(i) Consider a super L −supermerotopic space (B^U, M_U^*). Define $\mathfrak{C} \in M_U^*$ if and only if there do not exists finitely many $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n \notin M_U^*$ such that $\mathfrak{I}_1 \vee \mathfrak{I}_2 \vee \mathfrak{I}_3 \vee, \dots \dots, \vee \mathfrak{I}_n$ corefines $\mathfrak{C}$. Then (B^U, M_U^*) is an $L-$ superpremerotopic space.

(ii) Suppose $M_U^* = \bigcap M_U^*$ be a nonempty family of L −merotopic space on the set U. If $M_U^* = \bigcap M_U^*$ then $M_U^* = inf\{M_U^*\}$.

Proof. With axioms (FNS1) $f \in B^U$ and $\mathfrak{I}$ corefines $\mathfrak{C} \in M_U^*$ implies $\mathfrak{I} \in M_U^*$. Suppose $\mathfrak{C} \in M_U^*$ there do not exists finitely many $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n$ does not corefines $\mathfrak{C}$ and $\mathfrak{I} \notin M_U^*$ there exists $\mathfrak{I}_1 \vee \mathfrak{I}_2 \vee \mathfrak{I}_3 \vee, \dots \dots, \vee \mathfrak{I}_n$ corefines $\mathfrak{I}$ and $\mathfrak{I}$ corefines $\mathfrak{C}$ implies there exists $\mathfrak{I}_1 \vee \mathfrak{I}_2 \vee \mathfrak{I}_3 \vee, \dots \dots, \vee \mathfrak{I}_n$ corefines $\mathfrak{C}$ such that $\mathfrak{C} \notin M_U^*$. Now suppose that $0 \notin M_U^*$ there exists finitely many $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n \notin M_U^*(\emptyset)$ and also $\mathfrak{I}_1 \vee \mathfrak{I}_2 \vee \mathfrak{I}_3 \vee, \dots \dots, \vee \mathfrak{I}_n \notin M_U^*(0)$ impies $0 \notin M_U^*(0)$ which is contradiction. So $0 \in M_U^*$. Since $g < f \in B^U$ implies $M_U^* \subset M_U^*(f)$ and $M_U^* \ne B^U$.

Let $\{M_{U_\alpha}: \alpha \in I, where\ I\ is\ an\ index\ set\}$ is the family of super L −premerotopies on a set U, then $\cup M_{U_\alpha}$ is a super L −premerotopy on U.

3.7 Theorem

The family $SM(U)$ of all L −supermerotopies on a universal set U is a complete distributive lattice with respect to the order defined by set inclusion.

Proof. As per the previous theorem, $SM(U)$ is a complete lattice and for $M_{U_1}, M_{U_2}, M_{U_3} \in SM(U)$, $M_{U_1} \vee \left(M_{U_2} \vee M_{U_3}\right) = (M_{U_1} \vee M_{U_2}) \wedge (M_{U_1} \vee M_{U_3})$; here $\vee\left\{M_{U_i}: i \in I\right\} = \cup\left\{M_{U_i}: i \in I\right\}, M_{U_i} \in SM(U)$. Suppose $\mathfrak{I} \in M_{U_1} \vee \left(M_{U_2} \vee M_{U_3}\right)$. Then either $\mathfrak{I} \in M_{U_1}$ or $\mathfrak{I} \in \left(M_{U_2} \vee M_{U_3}\right)$. Suppose $\mathfrak{I} \in M_{U_1}$ for any finite number of $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n$ if $\mathfrak{I}_1 \vee \mathfrak{I}_2 \vee \mathfrak{I}_3 \vee, \dots \dots, \vee \mathfrak{I}_n$ corefines $\mathfrak{I}$ then $\mathfrak{I}_i$ belongs to M_{U_i} for some i. Therefore $\mathfrak{I} \in (M_{U_1} \vee M_{U_2}) \wedge (M_{U_1} \vee M_{U_3})$. Second care, Suppose that $\exists$ finitely many element $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n$ belongs to $P(L^U) - (M_{U_1} \cup M_{U_2}) \cap (M_{U_1} \cup M_{U_3})$ such that $\mathfrak{I}_1 \vee \mathfrak{I}_2 \vee \mathfrak{I}_3 \vee, \dots \dots, \vee \mathfrak{I}_n$ corefines $\mathfrak{I}$. Then either $\mathfrak{I}_i \notin (M_{U_1} \cup M_{U_2})$ or $\mathfrak{I}_i \notin (M_{U_1} \cup M_{U_3})$ for all i therefore $\mathfrak{I}_i \notin (M_{U_1} \cap M_{U_3})$.

So $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n \in P(L^U) - (M_{U_2} \cap M_3)$ such that $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n$ corefines $\mathfrak{I}$ implies $\mathfrak{I} \notin (M_{U_2} \wedge M_3)$.

Converse of this theorem , suppose $\mathfrak{I} \in (M_{U_1} \cup M_{U_2}) \wedge (M_{U_1} \cup M_{U_3})$ such that there do not exists finitely many $\mathfrak{I}_1, \mathfrak{I}_2, \mathfrak{I}_3, \mathfrak{I}_4, \dots \dots, \mathfrak{I}_n \notin (M_{U_1} \cup M_{U_2}) \wedge (M_{U_1} \cup M_{U_3})$ such that $\mathfrak{I}_1 \vee \mathfrak{I}_2 \vee \mathfrak{I}_3 \vee, \dots \dots, \vee \mathfrak{I}_n$ corefines $\mathfrak{I}$. Then $\mathfrak{I} \in M_{U_1} \cup (M_{U_2} \wedge M_{U_3})$ which implies $(M_{U_1} \cup M_{U_2}) \cap (M_{U_1} \cup M_{U_3}) \supseteq M_{U_2} \cap M_{U_3}$

3.8 Definition

A non-empty L −merotopic space (B^U, M_U) such that $0 \neq f \in B^U$ being a bounded set. A subset $\mathfrak{C} \subset L^U$ is said to be M_U– clan in f if it follows following axioms:

(cl1) $stack\ \mathfrak{C} = \mathfrak{C}$,

(cl2) $f \vee g \in \mathfrak{C} \Rightarrow f \in \mathfrak{C}\ or\ g \in \mathfrak{C}$,

(cl3) $0 \notin \mathfrak{C}$,

(cl4) $cl_N(f) \in \mathfrak{C} \Rightarrow f \in \mathfrak{C}$,

(cl5) $\mathfrak{C} \in M_U(f)$,

(cl6) $f \in \sec\{cl_N(f) | f \in \mathfrak{C}\}$.

3.9 Theorem

Consider a super L −supermerotopic space (B^U, M_U). Let $C_U = \{\mathfrak{C} \subset L^U \mid \exists\, g \in B^U \setminus \{0\}, \mathfrak{C}\ is\ an\ N-clan\ in\ g\}$ and for each $\mathcal{M} \subset C_U$, we have $cl_{C_U}(\mathcal{M}) = \{\mathfrak{C} \in C_U \mid \cap \mathcal{M} \subset \mathfrak{C}\}$. Then $cl_{C_U}(\mathcal{M})$is a kuratowski closure operator on U which generates a topology.

Proof. Since $0 \notin \mathfrak{C}$, $\mathfrak{C} \notin cl_{C_U}(\mathcal{M})$ and so the result follows. Let $\mathcal{K} \in \mathcal{M}$. Then $\cap \mathcal{M} \subset \mathcal{K}$. Therefore $\mathcal{M} \subset cl_{C_U}(\mathcal{M})$. As $\cap \mathcal{K}_2 \subset \cap \mathcal{K}_1$ for every $\mathcal{K}_1 \subset \mathcal{K}_2$, we have, $cl_{C_U}(\mathcal{K}_1) \subset cl_{C_U}(\mathcal{K}_2)$. Let $\mathfrak{C} \notin cl_{C_U}(\mathcal{M}_1) \cup cl_{C_U}(\mathcal{M}_2)$. Then $\cap \mathcal{M}_1 \nsubseteq \mathfrak{C}$ and $\cap \mathcal{M}_1 \nsubseteq \mathfrak{C}$. This gives that there exists, $h_1 \in \cap \mathcal{M}_1$ but $h_1 \notin \mathfrak{C}$. Also, there exists, $h_2 \in \cap \mathcal{M}_2$ but $h_2 \notin \mathfrak{C}$. Hence, $h_1 \vee h_2 \notin \mathfrak{C}$. But, $h_1 \vee h_2 \in (\cap \mathcal{M}_1) \cap (\cap \mathcal{M}_2) = \cap (\mathcal{M}_1 \cup \mathcal{M}_2)$. Therefore, $\mathfrak{C} \notin cl_{C_U}(\mathcal{M}_1 \cup \mathcal{M}_2)$. Lastly, let $\mathfrak{C} \notin cl_{C_U}(\mathcal{M})$. So, $\cap \mathcal{M} \nsubseteq \mathfrak{C}$. The result follows by noting that there exists, $h \in \cap \mathcal{M}$ but $h \notin \mathfrak{C}$.

3.10 Theorem

For L −supermerotopic spaces (B^U, M_U) and (B^V, M_V), let $f: U \to V$ be a L −supermerotopic map. Define a function $f: C_U \to C_V$ by setting for each $\mathfrak{C} \in C_U : \hat{f}(\mathfrak{C}) = \{\mathfrak{D} \subset L^U | f^{-1}[cl_V(\mathfrak{D})] \in \mathfrak{C}\}$. Then $\hat{f}: (C_U, cl_{C_U}) \to (C_V, cl_{C_U})$ is a continuous map.

3.11 Theorem

Let ς be an L −supermerotopy on U. Then every maximal ς −compatible family (with respect to set inclusion) is an L −grill and therefore a maximal ς −clan.

Proof. Suppose that ς is an L −supermerotopy on U, and $\delta \subseteq L^U$ be a maximal ς −compatible family. So, $0 \notin \delta$. Suppose $k \geq l$ and $l \in \delta$. Then $\{k\} \cup \delta$ corefines δ and so $\{k\} \cup \delta$ is an ς −compatible family. But since, δ is a maximal ς −compatible family, $k \in \delta$. Suppose $k \vee l \in \delta$. Then this is our claim that either $\{k\} \cup \delta$ or $\{l\} \cup \delta$ is an ς −compatible family. Let $\gamma = \{\delta \subseteq L^U : \delta \subseteq \delta\}$ and let the above stated the claim is not true. Then there exists δ_1 and δ_2 in γ such that $\{k\} \cup \delta_1 \notin \varsigma$ and $\{l\} \cup \delta_2 \notin \varsigma$, and $(\{k\} \cup \delta_1) \vee (\{l\} \cup \delta_2) \notin \varsigma$

4 Conclusion

We introduce L- supermerotopies, which is a generalization of nearness both in axiomatic structure and fuzzy settings. The study investigates some significant and desirable properties in fuzzy settings. Moreover, the results obtained will be useful when considering extension problems. For future research, it is of interest to extend the results to study L −supermerotopies through categorical viewpoint. Taking into account the generalization, the current work will assist in exploring the ideas of nearness in further studies of topology.

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