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Research Article

Fixed point theorems using simulation function in modular metric space

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ABSTRACT

In this manuscript, we introduce the concept of $(\beta - \mu)$ contraction with the help of a simulation function and use this concept to establish some fixed-point theorems in modular metric space.

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KEYWORDS AND PHRASES: Modular metric space, β - admissible mapping, generalized $(\beta - \mu)$ contractive map, fixed point.

1. Introduction

In 1950, Nakano [8] introduced the theory of modular spaces. The notion of modular metric space, being a natural generalization of classical modulars over linear spaces, was recently introduced. In 2012, Wardowski [10] introduced and studied a new contraction known as F-contraction to establish some fixed point results as a generalization of the Banach contraction principle. In 2014, Abdou and Khamsi [1] proved some fixed point results for multi valued contraction mappings in the frame of modular metric spaces. In recent years, there was a strong interest to study the fixed point property in modular function spaces.

On the other hand, in 2015, Khojasteh et al. [6] introduced the mapping known as the simulation function and the perception of Z-contraction with regard to simulation function. Thereafter, Roldan-Lopez-de Hierroet et al. [4] modified the notion of simulation functions and proved some coincidence and common fixed point theorems utilizing the newly larger class of simulation functions. In 2019, Kumar et al. [7] established some fixed point results via simulation functions.

Recently, Arora et al. [2] extended the results for alpha-admissible contraction mapping with the assistance of simulation function. Throughout this paper, $\mathbb{Q}, \mathbb{Z}+, \mathbb{R}+, \mathbb{R}$ denote the set of all rational numbers, the set of all positive integers, the set of all positive real numbers, and the set of all real numbers, respectively.

2. Preliminaries

In 2010, Chistyakov [3] introduced the notion of modular metric spaces as follows:

Definition 2.1. [4] A function $\omega_\lambda: (0,\infty) \times \mathcal{H} \times \mathcal{H} \longrightarrow [0,\infty]$ is said to be modular metric on $\mathcal{H}$, if it satisfies the following axioms:

(i) $x = y$ if and only if $\omega_\lambda(x,y) = 0$, for all λ > 0;

(ii) $\omega_\lambda(x,y) = \omega_\lambda(y,x)$, for all λ > 0 and $x, y \in \mathcal{H}$;

(iii) $\omega_\lambda(x,y)$= $\omega_\lambda(x,z)$ + $\omega_\lambda(z,y)$, for all λ > 0 and $x, y \in \mathcal{H}$.

Definition 2.2. Let $f: X \rightarrow X$ and $\alpha: X \times X \longrightarrow [0,+\infty)$. Then, f is said to be α- admissible if $\alpha(x,y) \geq 1 \Rightarrow \alpha(fx; fy) \geq 1$, for each $x, y \in X$.

Definition 2.3.[9] Let $f: X \rightarrow X$ and $\alpha: X \times X \longrightarrow [0,+\infty)$. Then, f is said to be β- admissible mapping with respect to μ if $\beta(x,y) \geq \mu(x,y) \Rightarrow \beta(fx; fy)) \geq \mu(fx,fy)$, for each $x, y \in X$.

The class of simulation functions was introduced by Khojasteh et al. in [6] as follows:

Definition 2.4.[6] The function $\zeta : [0,\infty) \times [0,\infty) \rightarrow \mathbb{R}$ is said to be a simulation function, if the following properties hold:

(ζ_1) $\zeta(0,0) = 0$;

$(\zeta_2)\zeta(a,b) < a - b$for all $a, b > 0$;

(ζ_3) if$\{a_n\}, \{b_n\}$are sequences in $(0,\infty)$ such that $\lim_{n\to\infty}\{b_n\} = \ell$, then

$$\lim_{n\to\infty} sup\zeta(a_n, b_n) < 0.$$

The authors in [6] utilized the above class of auxiliary functions to define Z-contractions as follows:

Definition 2.4. Let (X,d) be a metric space, $T: X \rightarrow X$and $\zeta \in Z$. Then T is called a Z-contraction with respect to ζ if the following condition is satisfied:

$$\zeta(d(Tx,Ty); d(x,y) \geq 0, \text{ for all } x, y \in X.$$

Khojasteh et al. [6] proved the following result.

Theorem 2.5.[6] Let (X, d) be a complete metric space and $T: X \longrightarrow X$ be a Z-contraction with respect toa certain simulation functionζ, that is,

$$\zeta(d(Tx, Ty); d(x, y) \geq 0, \text{ for all } x, y \in X.$$

Then T has a unique fixed point. Moreover, for every $x_0 \in X$, the Picard sequence $\{T^n x_0\}$ converges to this fixed point.

In 2015, Roldan et al. [4] observed that the third condition (namely:ζ_3) is symmetric in both arguments of ζ but, in proofs, this property is not necessary. In fact, in practice, the arguments of ζhave different meanings and they play different roles. Then, they slightly modify the condition ζ_3 as follows:

($\zeta_3{}'$) if $\{a_n\}, \{b_n\}$are sequence in $(0, \infty)$ such that $\lim_{n\to\infty}\{b_n\} = \ell$, then

$$\lim_{n\to\infty} sup\zeta(a_n, b_n) < 0.$$

Example 2.6.(see[4, 5, 6]) We define the mappings $\zeta_i : [0,\infty) \times [0,\infty) \longrightarrow \mathbb{R}$ for $i = 1, 2, 3, 4, 5$, as follows:

Next, we present some examples of simulation functions:

1. $\zeta_3(a, b) = \lambda b - a, \ \forall a, b \in [0, \infty)$, where $\lambda \in [0,1)$.
2. $\zeta_4(a, b) = \frac{b}{b+1}, \ \forall a, b \in [0, \infty)$.
3. $\zeta_1(a, b) = \psi(b) - \psi(a) \ \forall a, b \in [0, \infty)$, where $\phi, \psi \in [0, \infty) \longrightarrow [0, \infty)$are two continuous functions such that $\psi(a) - \phi(a) = 0$ if and only if $a = 0$ and $\psi(a) < a \leq \phi(a), \ \forall a > 0$.
4. $\zeta_2(a, b) = b - \eta(b) - a, \ \forall a, b \in [0, \infty)$, where $\eta : [0, \infty) \longrightarrow [0, \infty)$ is a lower semi continuous function such that $\eta(a) = 0$ if and only if $a = 0$.
5. $\zeta_5(a, b) = b - \int_0^a \varphi(u)du, \quad \forall a, b \in [0, \infty)$, where $\varphi : [0, \infty) \longrightarrow [0, \infty)$ is a function such that $\int_0^\epsilon \varphi(a)da$, exists and $\int_0^\epsilon \varphi(a)da$, for each $\epsilon > 0$.

3. Main Results

Let Λ_F be family of all functions F: $(0, \infty) \longrightarrow \mathbb{R}$ such that

(F_1) F is strictly increasing, that is, for all $a, b \in [0, \infty)$,

if$a < b$, then $F(a) < F(b)$.

(F_2) For each sequence a_n of positive numbers,

$\lim_{n\to\infty} a_n = 0$if and only if $\lim_{n\to\infty} F(a_n) = -\infty$.

(F_3) There exists $k \in (0,1)$ such that

$$\lim_{a\to 0^+}(a^k F(a)) = 0.$$

Let Λ_J denotes the set of all functions $J\colon \mathbb{R}_+ \to \mathbb{R}_+$ satisfying:

(J) for all $s_1, s_2, s_3, s_4 \in \mathbb{R}_+$ with $s_1.s_2.s_3.s_4 = 0$, there exists $\tau > 0$ such that $J(s_1, s_2, s_3, s_4) = \tau$.

Definition 3.1. Let $(H,\ \omega_\lambda)$ be a modular metric space and $S_1\colon \mathcal{H} \longrightarrow \mathcal{H}$be the self-map on (H, ω). Imagine that $\beta, \mu : \mathcal{H} \times \mathcal{H} \longrightarrow [0, \infty)$ be two mappings. Then, $S_1$is generalized $(\beta - \mu)$ contractive map with respect to ζ if $\mu(x\, Tx) \leq \beta(x, y)$, $\lambda > 0$ and $\omega_\lambda(S_1x, S_1y) > 0 \implies$

$\zeta(J(\omega_\lambda(x, S_1x), \omega_\lambda(y, S_1y), \omega_\lambda(x, S_1y), \omega_\lambda(y, S_1x)) + F(\omega_\lambda(S_1x, S_1y)), F(\omega_\lambda(x, y))) \geq 0$, (3.1)

where $J \in \Lambda_J$ and $F \in \Lambda_F$.

Theorem 3.2. Let $(H,\ \omega_\lambda)$ be a complete modular metric space. Let $S_1\colon \mathcal{H} \longrightarrow \mathcal{H}$ be generalized $(\beta - \mu)$ contractive map with respect to ζ, which fulfills the following conditions:

(i) There exists $x_0 \in \mathcal{H}$ such that $\beta(x_0, S_1x_0) \geq \mu(x_0, S_1x_0)$;

(ii) S_1 is β- admissible with respect to μ;

(iii) S_1 is $\beta - \mu -$ continuous mapping.

Then, S_1 possess a fixed point. In addition to this, S_1 possess a unique fixed point if $\beta(x, y) \geq \mu(x, x) \forall x, y \in \text{Fix}(S_1)$.

Proof. Let us choose a point $x_1 \in \mathcal{H}$ such that $x_1 = S_1x_0$. Continuing this process, we can choose x_{n+1} in $\mathcal{H}$ such that

$$x_{n+1} = S_1x_n. \tag{3.2}$$

Since S_1 is β- admissible w.r.t μ, we have

$$\beta(x_0, x_1) = \beta(x_0, S_1x_0) \geq \mu(x_0, S_1x_0) = \mu(x_0, x_1),$$

which implies that, $\beta(x_0, x_1) =\geq \mu(x_0, x_1)$.

Using induction, we get

$$\beta(x_n, x_{n+1}) \geq \mu(x_n, x_{n+1}),\ \forall\, n = 0,1,2, \ldots \tag{3.3}$$

If $x_{n+1} = x_n$ for some n, then by (3.2), we obtain that S_1 possess a fixed point at $x = x_{n+1}$ and so we have completed the proof. Further, we assume that $\omega_\lambda(S_1x_n, S_1x_{n+1}) > 0$.

Putting $x = x_n$ and $y = x_{n+1}$ in (3.1), we get

$$0 \leq \zeta(J(\omega_\lambda(x_n, S_1x_n), \omega_\lambda(x_{n+1}, S_1x_{n+1}), \omega_\lambda(x_n, S_1x_{n+1}), \omega_\lambda(x_{n+1}, S_1x_n)$$
$$+ F(\omega_\lambda(S_1x_n, S_1x_{n+1}), F(\omega_\lambda(x_n, x_{n+1})))$$
$$= \zeta(J(\omega_\lambda(x_n, x_{n+1}), \omega_\lambda(x_{n+1}, x_{n+2}), \omega_\lambda(x_n, x_{n+2}), \omega_\lambda(x_{n+1}, x_{n+1})$$
$$+ F(\omega_\lambda(x_{n+1}, x_{n+2}), F(\omega_\lambda(x_n, x_{n+1})))$$
$$< F\big(\omega_\lambda(x_n, x_{n+1})\big) - J\big((\omega_\lambda(x_n, x_{n+1}), \omega_\lambda(x_{n+1}, x_{n+2}), \omega_\lambda(x_n, x_{n+2}), \omega_\lambda(x_{n+1}, x_{n+1})\big) +$$
$$F(\omega_\lambda(x_{n+1}, x_{n+2}),$$

which indicates that

$$J\big((\omega_\lambda(x_n, x_{n+1}), \omega_\lambda(x_{n+1}, x_{n+2}), \omega_\lambda(x_n, x_{n+2}), \omega_\lambda(x_{n+1}, x_{n+1})\big) + F(\omega_\lambda(x_{n+1}, x_{n+2}) \leq F\big(\omega_\lambda(x_n, x_{n+1})\big). \tag{3.4}$$

Thus,

$$J(\omega_\lambda(x_n, x_{n+1}), \omega_\lambda(x_{n+1}, x_{n+2}), \omega_\lambda(x_n, x_{n+2}), 0) + F\big(\omega_\lambda(x_{n+1}, x_{n+2})\big) \leq F\big(\omega_\lambda(x_n, x_{n+1})\big).$$

Now,

$$\omega_\lambda(x_n, x_{n+1}), \omega_\lambda(x_{n+1}, x_{n+2}), \omega_\lambda(x_n, x_{n+2}), 0 = 0.$$

From $(\mathcal{H})$, we can find $\tau > 0$ so that

$$J(\omega_\lambda(x_n, x_{n+1}), \omega_\lambda(x_{n+1}, x_{n+2}), \omega_\lambda(x_n, x_{n+2}), 0) = \tau.$$

With the assistance of (3.4), we acquire

$$F\big(\omega_\lambda(x_{n+1}, x_{n+2})\big) \leq F\big(\omega_\lambda(x_n, x_{n+1})\big) - \tau.$$

Therefore,

$$\begin{aligned} F\big(\omega_\lambda(x_{n+1}, x_{n+2})\big) &\leq F\big(\omega_\lambda(x_n, x_{n+1})\big) - \tau \\ &\leq F\big(\omega_\lambda(x_{n-1}, x_n)\big) - 2\tau \\ &\leq F\big(\omega_\lambda(x_{n-2}, x_n)\big) - 3\tau \\ &\;\;. \\ &\;\;. \\ &\;\;. \\ &\leq F\big(\omega_\lambda(x_0, x_1)\big) - n\tau, \end{aligned}$$

which implies that

$$F\big(\omega_\lambda(x_{n+1}, x_{n+2})\big) \leq F\big(\omega_\lambda(x_0, x_1)\big) - n\tau \tag{3.5}$$

Letting $n \longrightarrow \infty$ in (3.5), we acquire

$$F\big(\omega_\lambda(x_{n+1}, x_{n+2})\big) \longrightarrow -\infty \tag{3.6}$$

With the assistance of (3.6) and property of $F \in \Lambda_F$, we get

$$\lim_{n\to\infty}\big(\omega_\lambda(x_{n+1},x_{n+2})\big) = 0.$$

For every $\delta > 0$ however small, $\exists\ m \in \mathbb{Z}_+$, so that

$$\omega_\lambda(x_{n+1},x_{n+2}) < \delta, \qquad \forall n \geq m.$$

Let us imagine that $q > n$.

For $\frac{\lambda}{q-n} > 0, \exists\ \frac{n\lambda}{q-n} \in \mathbb{Z}_+$ so that

$\omega_{\frac{\lambda}{q-n}}(x_{n+1},x_{n+2}) < \frac{\delta}{q-n}, \forall n \geq \frac{n\lambda}{q-n}$.

Further, we have

$$\begin{aligned}\omega_\lambda\big(x_n,x_q\big) &\leq \omega_{\frac{\lambda}{q-n}}(x_{n+1},x_{n+2}) + \omega_{\frac{\lambda}{q-n}}(x_{n+2},x_{n+3}) + \cdots + \omega_{\frac{\lambda}{q-n}}(x_{q-1},x_q)\\ &< \frac{\delta}{q-n} + \frac{\delta}{q-n} + \cdots + \frac{\delta}{q-n} = \delta,\end{aligned}$$

for all $q, n \geq \frac{n\lambda}{q-n}$, which implies that $\{x_n\}$ is a Cauchy sequence. Due to completeness property of $(\mathcal{H},\omega_\lambda), \exists u \in \omega_\lambda$, so that $x_n \longrightarrow u$, when $n \longrightarrow \infty$. But S_1 is $\beta - \mu -$continuous and $\mu(x_n,x_{n+1}) \leq \beta(x_n,x_{n+1}), S_1x_{n+1} = x_{n+2} \longrightarrow S_1u$, when $n \longrightarrow \infty$.

Consequently,

$$u = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} S_1x_{n+1} = S_1u,$$

which proves that u is a fixed point of S_1.

Next, we show that S_1 has almost one fixed point.

On the contrary, we suppose that u and v are two fixed points of S_1 such that $S_1u = u \neq v = S_1v$.

$$\begin{aligned}0 &\leq \zeta(J\big(\omega_\lambda(u,S_1x),\omega_\lambda(y,S_1y),\omega_\lambda(x,S_1y),\omega_\lambda(v,S_1x)\big) + F\big(\omega_\lambda(S_1x,S_1y)\big), F(\omega_\lambda(x,y)))\\ &= \zeta(J\big(0,0,\omega_\lambda(x,S_1y),\omega_\lambda(v,S_1x)\big) + F\big(\omega_\lambda(S_1x,S_1y)\big), F(\omega_\lambda(x,y)))\\ &< F\big(\omega_\lambda(x,y)\big) - (J\big(0,0,\omega_\lambda(x,S_1y),\omega_\lambda(v,S_1x)\big) + F\big(\omega_\lambda(S_1x,S_1y)\big),\end{aligned}$$

which indicates that

$$J\big(0,0,\omega_\lambda(x,S_1y),\omega_\lambda(v,S_1x)\big) + F\big(\omega_\lambda(S_1x,S_1y)\big) = \tau,$$

which shows that

$$\tau + F\big(\omega_\lambda(S_1x,S_1y)\big) \leq F\big(\omega_\lambda(x,y)\big),$$

which is contradiction. So, our supposition is wrong.

This proves that the fixed point of S_1 is unique.

Example 3.3. Consider $\mathcal{H} = [0,3]$ associated with the metric

$$\omega_\lambda(x,y) = \frac{1}{\lambda}|x-y|,$$

for all $x, y \in \mathcal{H}$. Define the mappings $S_1\colon \mathcal{H} \longrightarrow \mathcal{H}$ by

$$S_1 x = \begin{cases} \frac{1}{11} e^{-\lambda} x \quad if\ x \in \mathbb{Q} \\ \frac{1}{17} e^{-\lambda} x\ if\ x \in \mathbb{R} - \mathbb{Q} \end{cases}$$

with $\beta(x,y) = x + y$ and $\mu(x,y) = \frac{x+y}{9}$. Let $J\colon\ \mathbb{R}_+{}^4 \longrightarrow \mathbb{R}$ be defined as $J(s_1, s_2, s_3, s_4) = \tau$ and $J\colon\ \mathbb{R}_+ \longrightarrow \mathbb{R}$ be defined as $F(n) = In\ s$. Let $\zeta\colon\ \mathcal{H} \times \mathcal{H} \longrightarrow \mathbb{R}$ be defined as $\zeta(t,s) = s - \frac{t+2}{t+1} t$. It is clear that $\beta(x,y) \geq \mu(x,y) \Longrightarrow \beta(S_1 x, S_1 y) \geq \mu(S_1 x, S_1 y)$, which shows that S_1 is an $\beta -$ admissible mapping with respect to μ.

Case1: When $x, y \in \mathbb{Q}$.

Let $\mu(x, Tx) \leq \beta(x,y)$, then

$$\omega_\lambda(S_1 x, S_1 y) = \frac{1}{11\lambda} e^{-\tau}\,|x-y| \leq \frac{1}{\lambda} e^{-\tau}\,|x-y| = e^{-\tau}\,\omega_\lambda(x,y).$$

$$\begin{aligned} \zeta(\tau + F(\omega_\lambda(S_1 x, S_1 y)), F(\omega_\lambda(x,y))) &= \zeta(\tau + In(\omega_\lambda(S_1 x, S_1 y)), In(\omega_\lambda(x,y))) \\ &= \zeta(\tau + \tfrac{1}{11\lambda} e^{-\tau}\,|x-y|,\ In\tfrac{|x-y|}{\lambda}) \\ &= \zeta(\tau - \tau + In\tfrac{|x-y|}{11\lambda},\ In\tfrac{|x-y|}{\lambda}) \\ &= \zeta(z, 11z) \\ &= 11z - \tfrac{z+2}{(z+1)}\tfrac{z}{2} \\ &= \tfrac{22z(z+1) - z(z+2)}{2(z+1)} \\ &= \tfrac{22z^2 + 22z - z^2 - 2z)}{2(z+1)} \\ &= \tfrac{21z^2 + 20z}{2(z+1)} \geq 0. \end{aligned}$$

Hence, S_1 is generalized (β, μ) contractive map with respect to ζ.

Case 2: When $x, y \in \mathbb{R} - \mathbb{Q}$.

Let $\mu(x, Tx) \leq \beta(x,y)$, then

$$\omega_\lambda(S_1 x, S_1 y) = \frac{1}{17\lambda} e^{-\tau}\,|x-y| \leq \frac{1}{\lambda} e^{-\tau}\,|x-y| = e^{-\tau}\,\omega_\lambda(x,y).$$

Now,

$$\zeta(\tau + F(\omega_\lambda(S_1x, S_1y)), F(\omega_\lambda(x,y))) = \zeta(\tau + In(\omega_\lambda(S_1x, S_1y)), In(\omega_\lambda(x,y)))$$

$$= \zeta(\tau + \frac{1}{17\lambda} e^{-\tau} |x-y|,\ In \frac{|x-y|}{\lambda})$$

$$= \zeta(\tau - \tau + In \frac{|x-y|}{17\lambda},\ In \frac{|x-y|}{\lambda})$$

$$= \zeta(z, 17z)$$

$$= 17z - \frac{z+2}{(z+1)} \frac{z}{2}$$

$$= \frac{34z(z+1) - z(z+2)}{2(z+1)}$$

$$= \frac{34z^2 + 34z - z^2 - 2z)}{2(z+1)}$$

$$= \frac{33z^2 + 32z}{2(z+1)} \geq 0.$$

Hence, S_1 is generalized (β, μ) contractive map with respect to ζ.

Case 3: When $x \in \mathbb{Q}, y \in \mathbb{R} - \mathbb{Q}$.

Let $\mu(x, Tx) \leq \beta(x,y)$, then

$$\omega_\lambda(S_1x, S_1y) = \frac{1}{17\lambda} e^{-\tau} |x-y| \leq \frac{1}{\lambda} e^{-\tau} |x-y| = e^{-\tau} \omega_\lambda(x,y).$$

$$\zeta(\tau + F(\omega_\lambda(S_1x, S_1y)), F(\omega_\lambda(x,y))) = \zeta(\tau + In(\omega_\lambda(S_1x, S_1y)), In(\omega_\lambda(x,y))$$

$$= \zeta(\tau + In \frac{1}{\lambda} e^{-\tau} \left|\frac{x}{11} - \frac{y}{17}\right|, In \left|\frac{x-y}{\lambda}\right| \geq 0.$$

In all cases, S_1 is generalized (β, μ) contractive map with respect to ζ.

Consequently, all conditions of Theorem 3.2 fulfilled and note that zero is a fixed point of S_1.

Corollary 3.4. Let $(\mathcal{H}, \omega_\lambda)$ be a complete modular metric space. Let S_1: $\mathcal{H} \longrightarrow \mathcal{H}$ be self mapping with respect to ζ, which fulfills the following conditions:

(i) There exists $x_0 \in \mathcal{H}$ such that $\beta(x_1, S_1x_0) \geq \mu(x_1, S_1x_0)$;

(ii) $S_1$is $\beta -$ admissible with respect to μ;

(iii) $S_1$is $\beta - \mu$ contractive mapping;

(iv) If $\mu(x, Tx) \leq \beta(x,y), \lambda > 0$ and $\omega_\lambda(S_1x, S_1y) > 0 \Longrightarrow$

$$\zeta(\tau + F(\omega_\lambda(S_1x, S_1y)), F(\omega_\lambda(x,y))) \geq 0.$$

where$\tau > 0$ and $F \in \wedge_F$.

Then, S_1 possess a fixed point. In addition to this, S_1 possess a unique fixed point if $\beta(x,y) \geq \mu(x,x), \forall x,y \in Fix(S_1)$.

Proof. By inserting $J(s_1,s_2,s_3,s_4) = \min\{s_1,s_2,s_3,s_4\} + \tau$ in Theorem 3.2, we get the result.

Corollary 3.5. Let $(\mathcal{H},\omega_\lambda)$ be a complete modular metric space. Let S_1: $\mathcal{H} \longrightarrow \mathcal{H}$ be self mapping with respect to ζ, which fulfills the following conditions:

$$\zeta(\tau + F(\omega_\lambda(S_1x,S_1y)), F(\omega_\lambda(x,y))) \geq 0,$$

where$\tau > 0$ and $F \in \wedge_F$. Then S_1 has a unique fixed point.

Proof. By inserting$\beta(x,y) = \mu(x,x) = 1, \forall x,y \in \mathcal{H}$in Theorem 3.2, we deduce the result of Wardowski [10] in the frame of modular metric space.

Corollary 3.6. Let $(\mathcal{H},\omega_\lambda)$ be a complete modular metric space. Let S_1: $\mathcal{H} \longrightarrow \mathcal{H}$ be self mapping with respect to ζ, which fulfills the following conditions:

$$\zeta(\tau + F(\omega_\lambda(S_1x,S_1y)), F(\omega_\lambda(x,y))) \geq 0,$$

Then S_1 has a unique fixed point.

Proof. By inserting $\beta(x,y) = \mu(x,x) = 1,\ Fx = x \ and\ \tau = 0 \forall x,y \in \mathcal{H}$ in Theorem 3.2, we deduce the result of Khojasteh et al. [6] in the frame of modular metric space.

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