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Research Article

Solving the system of nonlinear equations in vibrations of triple walled carbon nanotubes: Variational iteration method

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Abstract

Mathematical modelling of nonlinear vibrations of triple-walled carbon nanotubes is discussed. This model is based on the system of nonlinear second-order equations. This paper presents the variational iteration method (VIM) to evaluate the nonlinear vibrations of triple-walled nanotubes embedded in an elastic medium. This method is a very effective and efficient method for solving various forms of linear and nonlinear differential equations in different fields. The analytical results are compared with simulation results (Matlab program), and satisfactory agreement is noted.

Keywords: Mathematical modelling, Nonlinear vibration, Carbon nanotube, Variational iteration method.

1. Introduction

One of the important carbon nanotubes (CNTs) applications is as nano pipes conveying fluids [1]. Different types of fluid flows such as water, transient oil flows, dynamic flow of methane, ethane and ethylene molecules, and the diffusive transport of light gases had been reported. The effects of two types of nonlinearities are geometric nonlinearity and nonlinearity of the van der Waals force on the transverse vibration in CNTs [2].

It is well known that most scientific phenomena are modelled by ordinary or partial differential equations. Analytical solutions of these equations may well describe the various phenomena in science and nature, such as vibrations, solutions and propagation with a finite speed. Khader et al. [3] investigated a multiple-beam model with the Van der Waals interlayer powers.

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The governing equations of each layer are coupled with those of its adjacent ones. The amplitude-frequency curves of single-walled, double-walled, and triple-walled carbon nanotubes for large-amplitude vibrations are obtained. Khader et al. [3] studied the effect of changes in geometrical parameters of the nanotube using HAM.

Fu et al. [4] studied the continuum mechanics and a multiple-elastic beam model for nonlinear free vibration of embedded multiwall carbon nanotubes. By using the incremental harmonic balance method, the iterative relationship of nonlinear amplitude and frequency for the double-wall nanotube are expressed. Fang et al. [5] examined the nonlinear free vibration of double-walled carbon nanotubes based on the principle of nonlocal elasticity. The nonlinear equations of motion of double-walled carbon nanotubes are derived using Euler beam theory and the Hamilton principle, with geometric nonlinearity of the von Kármán form and nonlinear van der Waals forces taken into account.

Siddiqui et al. [10] discussed analytic approximations of a nonlinear problem that occurs in the thin film flow of a third-grade fluid using the variational iteration method and the Adomain decomposition method. Our work in this paper focuses primarily on the recently developed variational iteration method [6-9].For applied sciences, this method that accurately measures the solutions in a series form or in an accurate form is of great interest. The key benefit of this method is that the approaches are capable of significantly reducing the size of computational work while still preserving the numerical solution's high accuracy. Khader et al. [3] studied the effect of changes in geometrical parameters of the nanotube using HAM.In this paper, we present the variational iteration method (VIM) to evaluate the nonlinear vibrations of triple-walled nanotubes embedded in an elastic medium.

2. Mathematical formulation of the problem

Consider the TWNT of length l, Young's modulus E, density ρ, cross-sectional area A_i, and cross-sectional inertia moment I_i, embedded in an elastic medium with material constant k. The nonlinear vibration equation for this TWNT is in the following form [2,3]

$$\frac{d^2W_1(t)}{dt^2}+\left(\frac{\pi^4 EI_1}{\rho A_1 l^4}+\frac{c_1}{\rho A_1}\right)W(t)_1+\frac{\pi^4 E}{4\rho l^4}\left(W_1(t)\right)^3-\frac{c_1}{\rho A_1}W_2(t)=0, \tag{1}$$

$$\frac{d^2W_2(t)}{dt^2}+\left(\frac{\pi^4 EI_2}{\rho A_2 l^4}+\frac{c_1}{\rho A_2}+\frac{c_2}{\rho A_2}\right)W_2(t)+\frac{\pi^4 E}{4\rho l^4}\left(W_2(t)\right)^3-\frac{c_1}{\rho A_2}W_1(t)-\frac{c_2}{\rho A_2}W_3(t)=0, \tag{2}$$

$$\frac{d^2W_3(t)}{dt^2}+\left(\frac{\pi^4 EI_3}{\rho A_3 l^4}+\frac{c_1}{\rho A_3}+\frac{c_2}{\rho A_3}+\frac{k}{\rho A_3}\right)W_3(t)+\frac{\pi^4 E}{4\rho l^4}\left(W_3(t)\right)^3-\frac{c_2}{\rho A_3}W_2(t)=0. \tag{3}$$

Where W_1, W_2 $W_3$are vibrations of the i[th] tube on the neutral axis, where c_i is the coefficient of the Van der Waals force between the (i[th]) tube and the (i-1[th]) tube. By substituting the following dimensionless parameters.

$$r=\sqrt{\frac{I_1}{A_1}},x=\frac{W_1}{r},y=\frac{W_2}{r},z=\frac{W_3}{r},\omega_l=\frac{\pi^2}{l^2}\sqrt{\frac{EI_1}{\rho A_1}},\omega_k=\sqrt{\frac{k}{\rho A_1}},$$

$$\omega_c=\sqrt{\frac{c}{\rho A_1}},\tau=\omega t,\ \beta=\frac{A_1}{A_2},\ \gamma=\frac{l_1}{l_2},\eta=\frac{A_1}{A_3},\ \zeta=\frac{I_1}{I_3},\alpha=0.25 \tag{4}$$

Eqs. (1)-(3) can be transformed to the following dimensionless nonlinear system.

$$\frac{d^2x(\tau)}{d\tau^2}+AB_1x(\tau)+\alpha A(x(\tau))^3-AB_2y(\tau)=0, \tag{5}$$

$$\frac{d^2y(\tau)}{d\tau^2}+AB_3y(\tau)+\alpha A(y(\tau))^3-A\beta B_2x(\tau)-A\beta B_2z(\tau)=0, \tag{6}$$

$$\frac{d^2z(\tau)}{d\tau^2}+AB_4z(\tau)+\alpha A(z(\tau))^3-A\eta B_2y(\tau)=0 \tag{7}$$

where

$$B_1=1+\left(\frac{\omega_c}{\omega_l}\right)^2,\ B_2=\left(\frac{\omega_c}{\omega_l}\right)^2,\ B_3=\beta\left(\frac{1}{\gamma}+2\left(\frac{\omega_c}{\omega_l}\right)^2\right),$$

$$B_4=\eta\left(\frac{1}{\zeta}+2\left(\frac{\omega_c}{\omega_l}\right)^2+\left(\frac{\omega_k}{\omega_l}\right)^2\right), A=\left(\frac{\omega_l}{\omega}\right)^2. \tag{8}$$

The initial conditions are

$$x(0)=X_1,\ \ y(0)=X_2,\ \ z(0)=X_3,$$
$$x'(0)=0,\ \ y'(0)=0, z'(0)=0. \tag{9}$$

3. Analytical expression of vibration of the string using variational iteration method

To illustrate the basic idea of He's variational iteration method [6-8], we consider the following nonlinear functional equation:

$$Lx(\tau)+Nx(\tau)=g(\tau) \tag{10}$$

where $Lx(\tau)$ is a linear operator, $Nx(\tau)$ a nonlinear operator and $g(\tau)$ an in homogeneous term. He et al.[9] suggested a method of general Lagrange multiplier. Then, we can construct a correct functional as follows:

$$x_{n+1}(\tau)=x_n(\tau)+\int_0^\tau \lambda(s)\left(Lx_n(\tau)+N\tilde{x}_n(\tau)-g(s)\right)ds, \tag{11}$$

where $\lambda(s)$ is a Lagrange multiplier that can be identified optimally via the variational theory [7-9]. The subscript n denotes the nth approximation, and $x_n(\tau)$ is considered to be restricted variation, that is, $\delta\tilde{x}_n(\tau)=0$. In this method the Lagrange multiplier $\lambda(s)$ is first determined optimally. The successive approximation $x_{n+1}(\tau), n\geq 0,$ of the solution $x(\tau)$ can be readily obtained by using this determined Lagrange multiplier with any selective function $x_0(\tau)$. Consequently, the solution is given by $x(\tau)=\lim_{n\to\infty} x_n(\tau).$ for the convergence criteria and error estimates of the VIM we refer the reader to [7-12]. According to the variational iteration method, we can construct correction functional of Eqns.(5)-(7) as follows:

$$x_{n+1}(\tau)=x_n(\tau)+\int_0^\tau \lambda(s)(x_n''(s)+AB_1x_n(s)+\alpha A(x_n(s))^3-AB_2y_n(s))ds \tag{12}$$

$$y_{n+1}(\tau)=y_n(\tau)+\int_0^\tau \lambda(s)(y_n''(s)+AB_3y_n(s)+\alpha A(y_n(s))^3-A\beta B_2x_n(s)-A\beta B_2z_n(s))ds \qquad (13)$$

$$z_{n+1}(\tau)=z_n(\tau)+\int_0^\tau \lambda(s)(z_n''(s)+AB_4z_n(s)+\alpha A(z_n(s))^3-A\eta B_2y_n(s))ds \qquad (14)$$

With $\lambda(s)=(s-t)$ .We start with the initial guess $x_0(\tau)=X_1$ in the above iteration formula and obtain the following approximate solutions:

$$x_0(\tau)=X_1 \qquad (15)$$

Put n =0 in Eq. No.(12)

$$x_1(\tau)=x_0(\tau)+\int_0^\tau (s-t)(x_0''(s)+AB_1x_0(s)+\alpha A(x_0(s))^3-AB_2y_0(s))ds$$

$$=x_0(\tau)+\int_0^\tau (s-t)(AB_1X_1+\alpha A(X_1)^3-AB_2X_2)ds$$

$$=X_1-\frac{1}{2}\left(AB_1X_1+\alpha AX_1^{\,3}-AB_2X_2\right)\tau^2 \qquad (16)$$

Put n =1 in Eq. No.(12)

$$x_2(\tau)=x_1(\tau)+\int_0^\tau (s-t)(x_1''(s)+AB_1x_1(s)+\alpha A(x_1(s))^3-AB_2y_1(s))ds$$

$$=x_1(\tau)+\frac{1}{2}\left(l-AB_1X_2-\alpha AX_2^3l+AB_2X_2\right)\tau^2$$

$$+\frac{1}{24}\left(AB_1l+3\alpha AX_2^2l-AB_2\right)\tau^4-\frac{\alpha AX_2l^2}{40}\tau^6+\frac{\alpha Al^3}{448}\tau^8 \qquad (17)$$

Similarly, from the initial conditions (9) we get

$$y_0(\tau)=X_2 \qquad (18)$$

Put n =0 in Eq. No.(13)

$$y_1(\tau)=y_0(\tau)+\int_0^\tau \lambda(s)(y_0''(s)+AB_3y_0(s)+\alpha A(y_0(s))^3-A\beta B_2x_0(s)-A\beta B_2z_0(s))ds$$

$$=X_2-\frac{1}{2}\left(AB_3X_2+\alpha AX_2^{\,3}-A\beta B_2X_1-A\beta B_2X_3\right)\tau^2 \qquad (19)$$

Put n =1 in Eq. No.(13)

$$y_2(\tau)=y_1(\tau)+\int_0^\tau \lambda(s)(y_1''(s)+AB_3y_1(s)+\alpha A(y_1(s))^3-A\beta B_2x_1(s)-A\beta B_2z_1(s))ds$$

$$=y_1(\tau)+\frac{1}{2}\left(m-AB_3X_2-\alpha AX_2^3+A\beta B_2X_2\right)\tau^2$$

$$+\frac{1}{24}\left(AB_3m+3\alpha AX_2^2m-A\beta B_2(n-l)\right)\tau^4-\frac{\alpha AX_2m^2}{40}\tau^6+\frac{\alpha Am^3}{448}\tau^8 \qquad (20)$$

Using the initial conditions (9) we get

$$z_0(\tau) = X_3 \tag{21}$$

Put n =0 in Eq. No(14)

$$z_1(\tau) = z_0(\tau) + \int_0^{\tau} \lambda(s)(z_0^{''}(s) + AB_4 z_0(\tau) + \alpha A(z_0(\tau))^3 - A\eta B_2 y_0(\tau))ds$$

$$= X_3 - \frac{1}{2}\left(AB_4 X_3 + \alpha A X_3^{\ 3} - A\eta B_2 X_2\right)\tau^2 \tag{22}$$

Put n =1 in Eq. No. (14)

$$z_2(\tau) = z_1(\tau) + \int_0^{\tau} \lambda(s)(z_1^{''}(s) + AB_4 z_1(\tau) + \alpha A(z_1(\tau))^3 - A\eta B_2 y_1(\tau))ds$$

$$= z_1(\tau) + \frac{1}{2}\left(n - AB_4 X_3 - \alpha A X_3^3 + A\eta B_2 X_3\right)\tau^2$$
$$+ \frac{1}{24}\left(AB_4 n + 3\alpha A X_3^2 n - A\eta B_2 m\right)\tau^4 - \frac{\alpha A X_3 n^2}{40}\tau^6 + \frac{\alpha A n^3}{448}\tau^8 \tag{23}$$

By considering the two iteration we get

$$x(\tau) \approx x_2(\tau),\ y(\tau) \approx y_2(\tau),\ z(\tau) \approx z_2(\tau) \tag{24}$$

4. Validation of analytical results with simulation results

Tables.1-3 represents the comparison between numerical and analytical results. Also, the average relative errors are given in the respective tables. From Tables.1-3 it is confirmed that the variational iteration method is the effective method for obtaining the analytical expressions for the vibration amplitudes in a TWNT. The error percentage is less than 3.

Table 1: Comparison of analytical result (Eq.No.(17)) of vibration x with numerical results when

$\alpha = 0.25, A = 1,\ B_1 = 0.1, B_2 = 0.1, X_2 = 1$

	$X_1 = 1$			$X_1 = 1.1$			$X_1 = 1.2$			$X_1 = 1.5$			$X_1 = 2$		
τ	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error
0	1.000	1.000	0.00	1.100	1.100	0.00	1.200	1.200	0.00	1.500	1.500	0.00	2.000	2.000	0.00
0.2	0.978	0.995	1.70	1.074	1.093	1.78	1.155	1.181	2.25	1.390	1.351	2.81	1.790	1.731	3.30
0.4	0.954	0.980	2.70	1.041	1.073	3.08	1.109	1.144	3.14	1.310	1.264	3.52	1.644	1.584	3.66
0.6	0.931	0.955	2.67	1.007	1.038	3.12	1.080	1.109	2.65	1.255	1.211	3.52	1.547	1.504	2.80

0.8	0.907	0.920	1.43	0.981	0.991	0.98	1.040	1.051	1.00	1.225	1.195	2.45	1.492	1.425	4.49
1	0.884	0.875	1.05	0.955	0.929	2.79	0.992	0.974	1.84	1.214	1.175	3.20	1.475	1.445	2.03
Average error		1.59		1.96			1.81			2.58			2.71		

Table 2 :Comparison of analytical result (Eq. No.(20)) of vibration y with numerical results when $\alpha = 0.25, \beta = 1, A = 1, B_1, B_2 = 1, X_2, X_3 = 1$

	$X_2 = 1$			$X_2 = 1.1$			$X_2 = 1.2$			$X_2 = 1.5$			$X_2 = 2$		
τ	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error
0	1.000	1.000	0.00	1.100	1.100	0.00	1.200	1.200	0.00	1.500	1.500	0.00	2.000	2.000	0.00
0.2	1.061	1.02	3.02	1.142	1.112	2.68	1.222	1.207	1.20	1.530	1.487	2.79	1.899	1.854	2.39
0.4	1.108	1.076	2.91	1.196	1.165	2.56	1.279	1.249	2.36	1.562	1.517	2.90	1.850	1.817	1.80
0.6	1.178	1.145	2.77	1.246	1.212	2.72	1.325	1.286	2.90	1.588	1.533	3.45	1.827	1.763	3.49
0.8	1.275	1.250	1.94	1.324	1.292	2.48	1.364	1.328	2.68	1.605	1.556	3.09	1.816	1.745	3.89
1	1.361	1.375	1.03	1.405	1.389	1.17	1.415	1.384	2.19	1.610	1.587	1.40	1.813	1.728	4.72
Average error		1.94		1.93			1.89			2.27			2.71		

Table 3: Comparison of analytical result (Eq. No.(23)) of vibration z with numerical results when $\alpha = 0.25, \beta = 1, \eta = 1, A = 1, B_1, B_2, B_3 = 2, X_1, X_2 = 1$

	$X_3 = 1$			$X_3 = 1.2$			$X_3 = 1.4$			$X_3 = 1.7$			$X_3 = 2$		
τ	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error	Numerical	VIM	Error
0	1.000	1.000	0.00	1.200	1.200	0.00	1.400	1.400	0.00	1.700	1.700	0.00	2.0000	2.000	0.00
0.2	1.107	1.075	2.88	1.254	1.265	0.73	1.435	1.450	1.05	1.784	1.721	3.57	2.0307	2.001	1.47

0.4	1.278	1.300	1.70	1.429	1.453	1.67	1.583	1.601	1.13	1.855	1.801	2.90	2.0594	2.002	2.79
0.6	1.598	1.645	2.97	1.695	1.751	3.28	1.824	1.853	1.55	1.907	1.847	3.15	2.0790	2.003	3.66
0.8	2.095	2.157	2.97	2.124	2.214	4.21	2.124	2.205	3.81	1.939	1.898	2.14	2.0920	2.004	4.21
1	2.551	2.575	0.94	2.554	2.564	0.38	2.621	2.657	1.36	1.949	1.913	1.87	2.1097	2.005	4.96
Average error		1.91		1.71			1.48			2.29			2.85		

5. Result and discussion

Equations (17), (20) & (23) represent the new analytical expressions **of** the nonlinear vibrations of triple-walled nanotubes embedded in an elastic medium. **Fig.1.** represent the dimensionless vibration of TWNT with for various values of parameters. From Fig.1, it is observed that when the initial value X_1 increases the TWNT vibration increases. An increase in initial value X_2 and X_3 leads to decrease in the dimensionless vibration of TWNT.

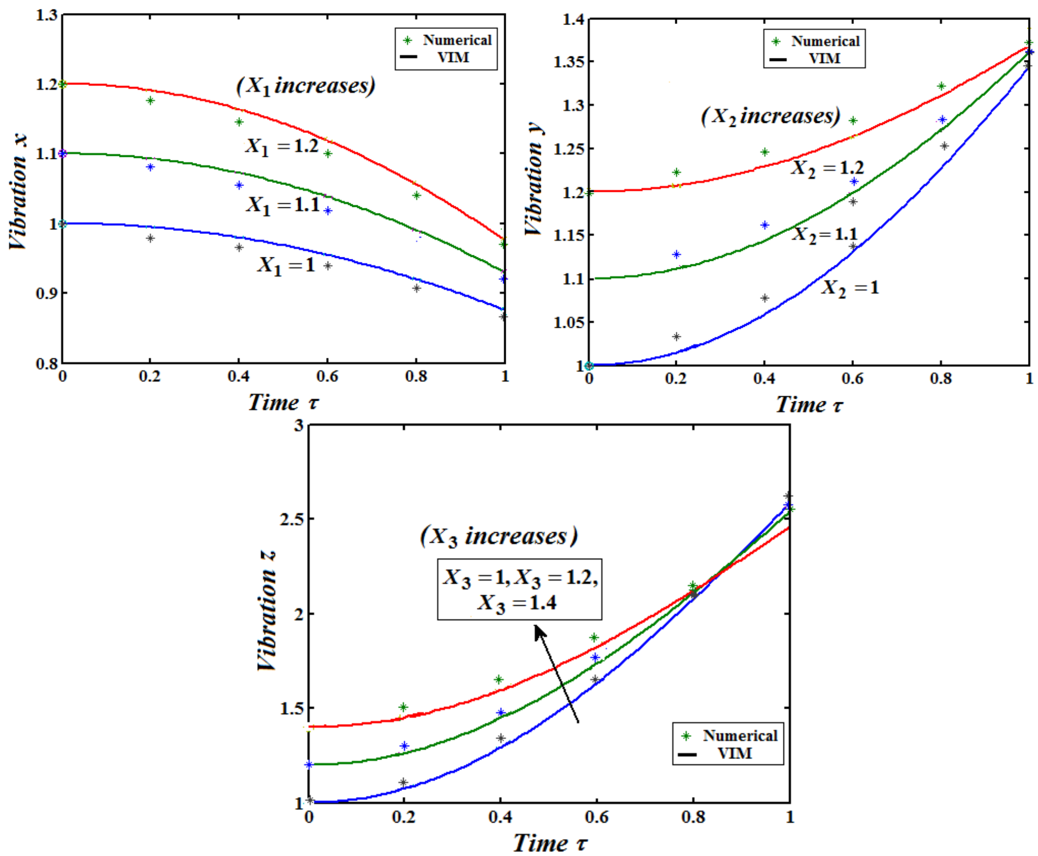

Fig.1. Comparison of dimensionless vibration of TWNT with simulation results for several values of parameters $\alpha,\beta,\eta,A,B_1,B_2,B_3,X_1,X_2$ and X_3.
The geometric parameters used to obtain this figure are,
$\alpha=0.25,\beta=0.6,\eta,A=1,B_1,B_2,B_3=0.1,X_2,X_3=1$

Fig. 2 represents the vibrations for different values of B_1 and B_2 . The geometric parameters used to obtain this figure are, $\alpha=0.25,\beta=1,\eta,A=2,B_3=1,X_1,X_2,X_3=1$. It is evident that decrease in B_1 or B_2 , vibration x increases.

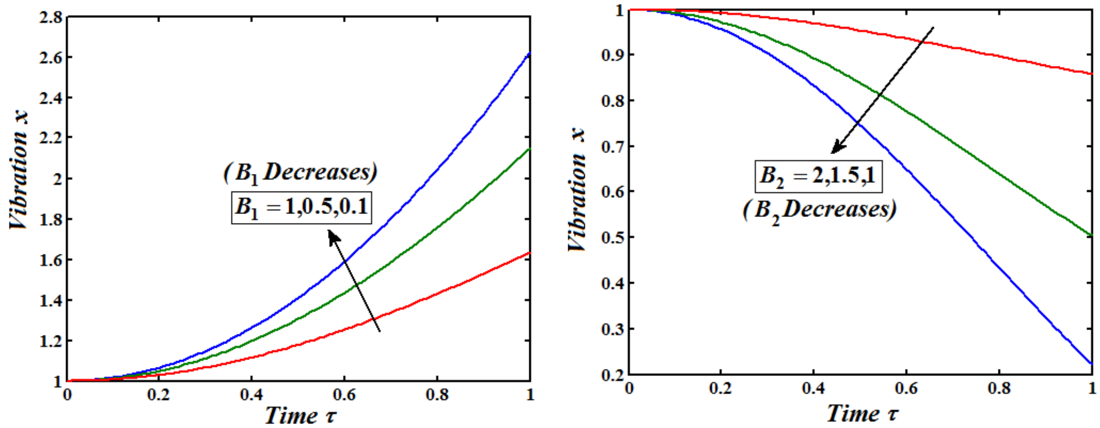

Fig.2. Plot of dimensionless vibration x versus dimensionless time τ for several values of parameters $\alpha,\beta,\eta,A,B_1,B_2,B_3,X_1,X_2$ and X_3 using equation (17).

The effects of the parameter on vibrations y is shown in Figs. 3(a-b). As the parameter B_2 and B_3 increase the vibration also increases.

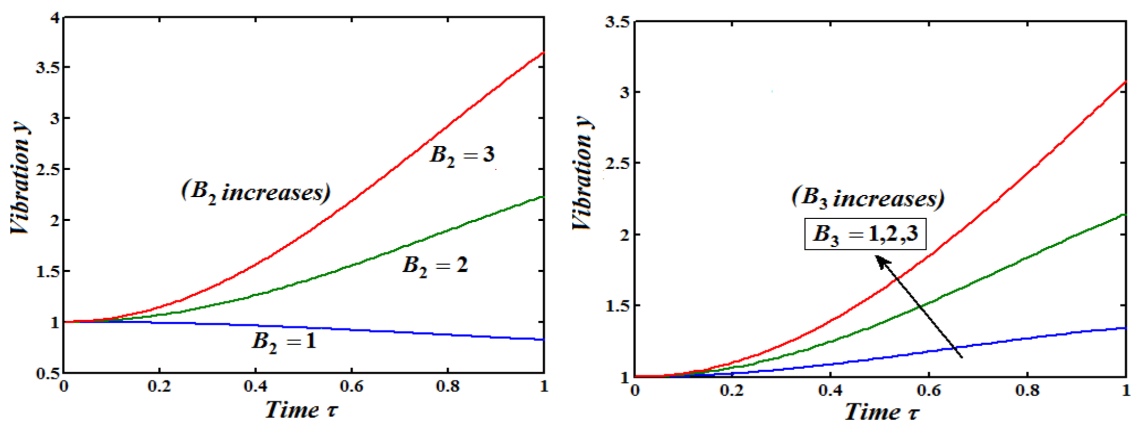

Fig.3. Plot of dimensionless vibration y versus dimensionless time τ for several values of parameters $\alpha,\beta,\eta,A,B_1,B_2,B_3,X_1,X_2$ and X_3 using equation (20). The geometric parameters used to obtain this figure are, $\alpha=0.25,\beta=1,\eta,A=2,B_1=2,X_1,X_2,X_3=1$

From the fig.4, it is notify that the dimensionless vibration z increases when dimensionless parameter B_4 is decreases. Also vibration z decreases when the parameter η is increases. The geometric parameters used to obtain this figure are, $\alpha = 0.25, \beta = 1, \eta, A = 1, B_1 = 1, B_2, B_3 = 2, X_1, X_2, X_3 = 1$.

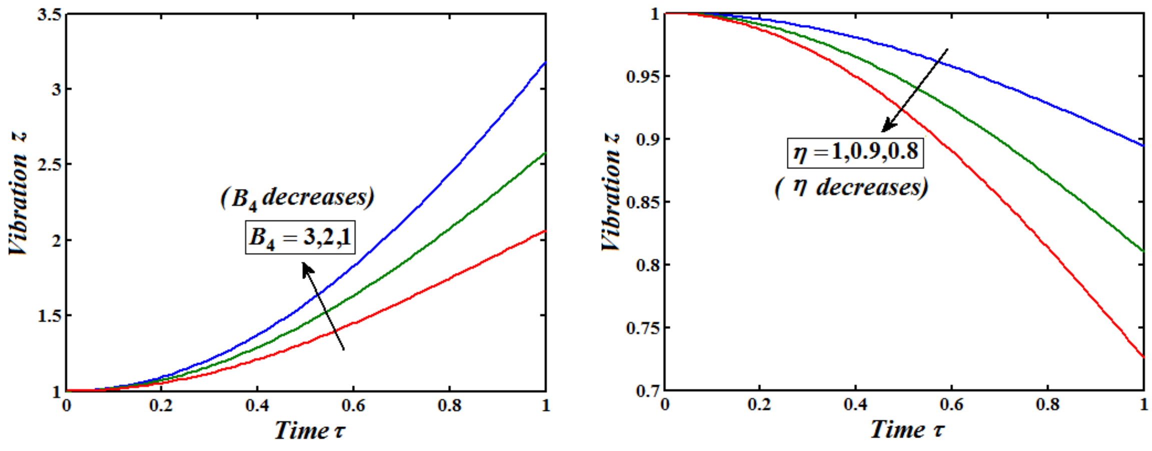

Fig.4. Plot of dimensionless vibration z versus dimensionless time τ for several values of parameters $\alpha, \beta, \eta, A, B_1, B_2, B_3, X_1, X_2$ and X_3 using equation (23).

Conclusion

In this research, we showed how VIM can be used to obtain the approximate analytical solutions of a nonlinear initial value problem in triple walled carbon nanotubes. It is concluded that these techniques are very effective and useful methods for solving different kinds of nonlinear problems that occur in various fields of science and engineering.The technique is powerful and reliable techniques that, if present, this method provides higher accuracy and closed form solutions approximations.

Nomenclature and units

Symbols	Name	Unit
ω	Nonlinearfree vibration frequency	cm^{-1}
k	Spring constant	N/m^2
ρ	Density	kg/m^3
A_1, A_2, A_3	Cross sectional area	$(nm)^2$
E	Young modulus	TPa
I_1, I_2, I_3	Cross sectional interia moment	TPa

W_1, W_2, W_3	Transverse displacements of the ith tube on the neutral axis	TPa
t	Time	s
l_1, l_2, l_3	Length of TWNT	nm
ω_c	Dimensionless constant	none
ω_k	Dimensionless	none
ω_l	Dimensionless	none
X	Dimensionless vibration amplitude	none
$\alpha, \beta, \gamma, \eta, \zeta$	Dimensionless constant	none
τ	Dimensionless time	none
x, y, z	Dimensionless vibration amplitude	none
r	Dimensionless	none

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