Turkish Online Journal of Qualitative Inquiry (TOJQI)

Volume 12, Issue 3, June 2021: 4200-4209

Research Article

# Inverse Majority Neighborhood number for $2D$ −Lattice of $T \cup C_4 C_8[a, b]$ Nanotori

Paulraj Jayasimman [1] , T. Dhivya [2] and J. Joseline Manora[3]

## Abstract

In this article we found inverse majority neighborhood number $n_M^{-1}(G)$ of a Nanotori topological network. This number helps to identify the majority data communication nodes.

## Introduction

Nanotubes have hexagonal atoms that are bonded to three other carbon atoms. The discovery of nanotubes led researchers to conclude that carbon nanotorus molecules, or carbon molecules made by glueing the two ends of a nanotube together, could exist as well. An experimental proof of such molecules appeared soon after.

Further investigation revealed that these carbon nanotorus molecules possess a diverse range of properties. Certain carbon nanotori species have unusual magnetic properties, such as persistent magnetic moments at near zero flux and massive paramagnetic moments, as well as a wide range of electric properties: some nanotori are conductors, while others are semiconducting or insulators.

The properties of carbon nanotori are strongly related to their geometrical parameters, temperature, and the parameters of the nanotube used for their production. Carbon nanotori's properties are highly influenced by their geometrical parameters, temperature, and the parameters of the nanotube used to make them [10,32,33],[21,22,23].

[1]Associate Professor, Department of Mathematics,AMET Deemed to be University, Kanathur, Chennai, India
[2]Assistant Professor, Department of Mathematics,AMET Deemed to be University, Kanathur, Chennai, India
[3]PG & Research Department of Mathematics, T.B.M.L College, Porayar, India
Email Address: [1]ipjayasimman@gmailcom,[2] dhivyasmile94@gmailcom, [3]joseline_manora@yahoo.co.in

Nanotubical graphs (also known as nanotubes) are graph representations of nanotubular molecules. Nanotubes are thus 3-connected, infinite, cubic planar graphs with a tubular form when expressed in space. A nanotube is created by defining objects (vertices, corners, and faces) lying on two parallel lines in a planar hexagonal grid, i.e. a hexagonal grid is rolled into a tube.

The topology of a network is represented using graphs. Multiprocessors are shown as graphs, with vertices representing processors and edges representing connections between them. The topology of an interconnection network is critical since it determines the network's efficiency. Meshes and tori are two of the most popular multiprocessor networks on the market today. In this research work we found Graph theoretic parameter are analyzed that is inverse majority neighborhood number of a $T \cup C_4 C_8[a, b]$. This number identify the nodes which are communicated with majority nodes.

**Theorem 1.1**

For the graph G be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 2$ then $n_m^{-1}(G) = \left\lceil \frac{a(6b-1)}{6} \right\rceil$

**Proof:**

Let G be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 2$ where a is the number of squares in a row and b is the number of rows of square. $|V(G)| = 4ab$ and $|E(G)| = 6ab - a$.

Let $S_M = \{v_{31}, v_{32}, \dots, v_{3a}, v_{51}, v_{52}, v_{54}, v_{56}, \dots v_{5(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{11a-2}{6} \right\rceil$. Let $S'_M$ be the inverse neighborhood set with respect to $S_M$. The vertex set $S'_M = \{v_{41}, v_{42}, \dots, v_{4a}, v_{21}, v_{22}, v_{24}, v_{26}, \dots v_{2(j-2)}\}$ where $j = 2a$. $S'_M$ covers the edges $|\langle N[S'_M]\rangle| = 3\left(a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{6ab-a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S'_M]| = 4a + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b + 2) > \left\lceil \frac{4ab}{2} \right\rceil = 2ab = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{a(6b-1)}{6} \right\rceil - 1$ then $|\langle N[S'_M]\rangle| = 3\left(a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b+1) < \left\lceil \frac{6ab-a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$. Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{a(6b-1)}{6} \right\rceil$.

**Theorem 1.2**

For the graph G be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 3$ then $n_m^{-1}(G) = \left\lceil \frac{17a}{6} \right\rceil$

**Proof:**

Let G be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 3$ where a is the number of squares in a row and b is the number of rows of square. $|V(G)| = 12a$ and $|E(G)| = 17a$.

Let $S_M = \{v_{31}, v_{32}, \dots, v_{3a}, v_{61}, v_{62}, \dots, v_{6a}, v_{81}, v_{82}, v_{84}, v_{86}, \dots v_{8(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{17a}{6} \right\rceil$. Let $S'_M$ be the inverse neighborhood set with respect to $S_M$. The vertex set $S'_M = \{v_{41}, v_{42}, \dots, v_{4a}, v_{71}, v_{72}, \dots, v_{7a}, v_{21}, v_{22}, v_{24}, v_{26}, \dots v_{2(j-2)}\}$ where $j = 2a$. $S'_M$ covers the edges $|\langle N[S'_M]\rangle| = 3\left(a(b-1) + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{17a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S'_M]| = 4(2a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b+1) > \left\lceil \frac{12a}{2} \right\rceil = 6a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{17a}{6} \right\rceil - 1$ $|\langle N[S'_M]\rangle| = 3\left(a(b-1) + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - b < \left\lceil \frac{17a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$. Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = a(b-1) + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{17a}{6} \right\rceil$.

**Theorem 1.3**

For the graph G be the $T \cup C_4\, C_8[a, b]$ nanotori then $a \geq 2, b = 4$ then $n_m^{-1}(G) = \left\lceil \frac{23a}{6} \right\rceil$

**Proof:**

Let G be the $T \cup C_4\, C_8[a, b]$ nanotori then $a \geq 2, b = 4$ where a is the number of squares in a row and b is the number of rows of square. $|V(G)| = 16a$ and $|E(G)| = 23a$.

Let $S_M = \{v_{31}, v_{32}, \dots, v_{3a}, v_{61}, v_{62}, \dots, v_{6a}, v_{91}, v_{92}, \dots, v_{9a}, v_{11\,1}, v_{11\,2}, v_{11\,4}, v_{11\,6}, \dots v_{11(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{17a}{6} \right\rceil$. Let $S_M'$ be the inverse neighborhood set with respect to $S_M$. The vertex set $S_M' = \{v_{41}, v_{42}, \dots, v_{4a}, v_{71}, v_{72}, \dots, v_{7a}, v_{10\,1}, \dots, v_{10\,a}, v_{21}, v_{22}, v_{24}, v_{26}, \dots v_{2(j-2)}\}$ where $j = 2a$. $S_M'$ covers the edges $|\langle N[S_M']\rangle| = 3\left(3a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{23a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S_M']| = 4(3a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + b > \left\lceil \frac{16a}{2} \right\rceil = 8a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S_M'$ is inverse majority neighborhood set. Suppose $|S_M'| - 1 = \left\lceil \frac{23a}{6} \right\rceil - 1$ $|\langle N[S_M']\rangle| = 3\left(3a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b-1) < \left\lceil \frac{23a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ . Therefore $S_M'$ is not inverse majority neighborhood set. Hence $|S_M'| = 3a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{23a}{6} \right\rceil$ .

**Theorem 1.4**

For the graph G be the $T \cup C_4\, C_8[a, b]$ nanotori then $a \geq 2, b = 5$ then $n_m^{-1}(G) = \left\lceil \frac{29a}{6} \right\rceil$

**Proof:**

Let G be the $T \cup C_4\, C_8[a, b]$ nanotori then $a \geq 2, b = 5$ where a is the number of squares in a row and b is the number of rows of square. $|V(G)| = 20a$ and $|E(G)| = 29a$.

Let $S_M = \{v_{31}, v_{32}, \dots, v_{3a}, v_{61}, \dots, v_{6a}, v_{91}, \dots, v_{9a}, v_{12\,1}, v_{12\,2}, \dots, v_{12\,a}, v_{14\,1}, v_{14\,2}, v_{14\,4}, \dots v_{14(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{29a}{6} \right\rceil$. Let $S_M'$ be the inverse neighborhood set with respect to $S_M$. The vertex set

$S_M' = \{v_{41}, , \dots, v_{4a}, v_{71}, , \dots, v_{7a}, v_{10\,1}, \dots, v_{10\,a}, v_{13\,1}, \dots, v_{13\,a}, v_{21}, v_{22}, v_{24}, v_{26}, \dots v_{2(j-2)}\}$ where $j = 2a$. $S_M'$ covers the edges $|\langle N[S_M']\rangle| = 3\left(4a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{29a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S_M']| = 4(4a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b-1) > \left\lceil \frac{20a}{2} \right\rceil = 10a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S_M'$ is inverse majority neighborhood set. Suppose $|S_M'| - 1 = \left\lceil \frac{29a}{6} \right\rceil - 1$ $|\langle N[S_M']\rangle| = 3\left(3a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b-2) < \left\lceil \frac{29a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ . Therefore $S_M'$ is not inverse majority neighborhood set. Hence $|S_M'| = 4a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{29a}{6} \right\rceil$ .

**Theorem 1.5**

For the graph G be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 6$ then $n_m^{-1}(G) = \left\lceil \frac{35a}{6} \right\rceil$

**Proof:**

Let G be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 6$ where a is the number of squares in a row and b is the number of rows of square. $|V(G)| = 24a$ and $|E(G)| = 35a$.
$S_M = \{v_{31}, \dots, v_{3a}, v_{61}, \dots, v_{6a}, v_{91}, \dots, v_{9a}, v_{12\,1}, \dots, v_{12\,a}, v_{15\,1}, \dots, v_{15\,a}, v_{17\;1}, v_{17\;2}, v_{17\;4}, \dots v_{17(j-2)}\}$
where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{35a}{6} \right\rceil$. Let $S'_M$ be the inverse neighborhood set with respect to $S_M$.
$S'_M = \{v_{41}, \dots, v_{4a}, v_{71}, \dots, v_{7a}, v_{101}, \dots, v_{10a}, v_{13\,1}, \dots, v_{13\,a}, v_{16\,1}, \dots, v_{16\,a}, v_{21}, v_{22}, v_{24}, \dots v_{2(j-2)}\}$
where $j = 2a$. $S'_M$ covers the edges $|\langle N[S'_M] \rangle| = 3\left(5a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{35a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S'_M]| = 4(5a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b-2) > \left\lceil \frac{24a}{2} \right\rceil = 12a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{35a}{6} \right\rceil - 1$ $|\langle N[S'_M] \rangle| = 3\left(5a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b-3) < \left\lceil \frac{35a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ . Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = 5a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{35a}{6} \right\rceil$.

**Theorem 1.6**

For the graph $G$ be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 7$ then $n_m^{-1}(G) = \left\lceil \frac{41a}{6} \right\rceil$

**Proof:**

Let $G$ be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 7$ where $a$ is the number of squares in a row and $b$ is the number of rows of square. $|V(G)| = 28a$ and $|E(G)| = 41a$.
$S_M = \{v_{31}, \dots, v_{3a}, v_{61}, \dots, v_{6a}, v_{91}, \dots, v_{9a}, v_{12\,1}, \dots, v_{12\,a}, v_{15\,1}, \dots, v_{15\,a}, v_{18\;1}, \dots, v_{18\;a}, v_{20\;1}, v_{20\,2}, v_{20\;4}, \dots v_{20(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{41a}{6} \right\rceil$. Let $S'_M$ be the inverse neighborhood set with respect to $S_M$. $S'_M = \{v_{41}, \dots, v_{4a}, v_{71}, \dots, v_{7a}, v_{101}, \dots, v_{10a}, v_{13\,1}, \dots, v_{13\,a}, v_{16\,1}, \dots, v_{16\,a}, v_{19\,1}, \dots, v_{19\,a}, v_{21}, v_{22}, v_{24}, \dots v_{2(j-2)}\}$ where $j = 2a$. $S'_M$ covers the edges $|\langle N[S'_M] \rangle| = 3\left(6a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{41a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S'_M]| = 4(6a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b-3) > \left\lceil \frac{28a}{2} \right\rceil = 14a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{41a}{6} \right\rceil - 1$ $|\langle N[S'_M] \rangle| = 3\left(6a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b-4) < \left\lceil \frac{35a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ . Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = 6a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{41a}{6} \right\rceil$.

**Theorem 1.7**

For the graph $G$ be the $T \cup C_4\, C_8[a,\mathrm{b}]$ nanotori then $a \geq 2, b = 8$ then $n_m^{-1}(G) = \left\lceil \frac{47a}{6} \right\rceil$

**Proof:**

Let $G$ be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 8$ where $a$ is the number of squares in a row and $b$ is the number of rows of square. $|V(G)| = 32a$ and $|E(G)| = 47a$.
$S_M =$
$\{v_{31}, \dots, v_{3a}, v_{61}, \dots, v_{6a}, v_{91}, \dots, v_{9a}, v_{12\,1}, \dots, v_{12\,a}, v_{15\,1}, \dots, v_{15\,a}, v_{18\,1}, \dots, v_{18\,a}, v_{21\,1}, \dots, v_{21\,a}, v_{23\,1}, v_{23\,2}, v_{23\,4}, v_{23\,6}, \dots v_{23(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{47a}{6} \right\rceil$. Let $S'_M$ be the inverse neighborhood set with respect to $S_M$. $S'_M = \{v_{41}, \dots, v_{4a}, v_{71}, \dots, v_{7a}, v_{101}, \dots, v_{10a}, v_{13\,1}, \dots,$
$v_{13\,a}, v_{16\,1}, \dots, v_{16\,a}, v_{19\,1}, \dots, v_{19\,a}, v_{22\,1}, \dots, v_{22\,a}, v_{21}, v_{22}, v_{24}, v_{26} \dots v_{2(j-2)}\}$ where $j = 2a$. $S'_M$ covers the edges $|\langle N[S'_M]\rangle| = 3\left(7a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{47a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S'_M]| = 4(7a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b-4) > \left\lceil \frac{32a}{2} \right\rceil = 16a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{47a}{6} \right\rceil - 1$ $|\langle N[S'_M]\rangle| = 3\left(7a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b-5) < \left\lceil \frac{47a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ . Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = 7a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{47a}{6} \right\rceil$.

**Theorem 1.8**

For the graph $G$ be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 9$ then $n_m^{-1}(G) = \left\lceil \frac{53a}{6} \right\rceil$

**Proof:**

Let $G$ be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b = 9$ where $a$ is the number of squares in a row and $b$ is the number of rows of square. $|V(G)| = 36a$ and $|E(G)| = 53a$.
$S_M =$
$\{v_{31}, \dots, v_{3a}, v_{61}, \dots, v_{6a}, v_{91}, \dots, v_{9a}, v_{12\,1}, \dots, v_{12\,a}, v_{15\,1}, \dots, v_{15\,a}, v_{18\,1}, \dots, v_{18\,a}, v_{21\,1}, \dots, v_{21\,a}, v_{24\,1}, \dots, v_{24\,a}, v_{26\,1}, v_{26\,2}, v_{26\,4}, \dots v_{26(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{53a}{6} \right\rceil$. Let $S'_M$ be the inverse neighborhood set with respect to $S_M$. $S'_M = \{v_{41}, \dots, v_{4a}, v_{71}, \dots, v_{7a}, v_{10\,1}, \dots v_{10\,a}, v_{13\,1}, \dots, v_{13\,a},$ $v_{16\,1}, \dots, v_{16\,a}, v_{19\,1}, \dots, v_{19\,a}, v_{22\,1}, \dots, v_{22\,a}, v_{25\,1}, \dots, v_{25\,a}, v_{21}, v_{22}, v_{24}, \dots, v_{2(j-2)}\}$ where $j = 2a$. $S'_M$ covers the edges $|\langle N[S'_M]\rangle| = 3\left(8a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{53a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S'_M]| = 4(8a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b-5) > \left\lceil \frac{36a}{2} \right\rceil = 18a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{53a}{6} \right\rceil - 1$ then $|\langle N[S'_M]\rangle| = 3\left(8a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b-6) < \left\lceil \frac{53a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$. Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = 8a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{53a}{6} \right\rceil$.

**Theorem 1.8**

For the graph $G$ be the $T \cup C_4\, C_8[a,\text{b}]$ nanotori then $a \geq 2, b = 10$ then $n_m^{-1}(G) = \left\lceil \frac{59a}{6} \right\rceil$

**Proof:**

Let $G$ be the $T \cup C_4\, C_8[a,b]$ nanotube then $a \geq 2, b = 10$ where $a$ is the number of squares in a row and $b$ is the number of rows of square. $|V(G)| = 40a$ and $|E(G)| = 59a$.
$S_M =$
$\{v_{31}, \dots, v_{3a}, v_{61}, \dots, v_{6a}, v_{91}, \dots, v_{9a}, v_{12\,1}, \dots, v_{12\,a}, v_{15\,1}, \dots, v_{15\,a}, v_{18\,1}, \dots, v_{18\,a}, v_{21\,1}, \dots, v_{21\,a}, v_{24\,1}, \dots, v_{24\,a}, v_{27\,1}, \dots, v_{27\,a}, v_{29\,1}, v_{29\,2}, , v_{29\,4}, \dots v_{29(j-2)}\}$ where $j = 2a$ with minimum cardinality $|S_M| = \left\lceil \frac{59a}{6} \right\rceil$. Let $S'_M$ be the inverse neighborhood set with respect to $S_M$. $S'_M = \{v_{41}, \dots, v_{4a}, v_{71}, \dots, v_{7a},$

$v_{10\,1}, \dots v_{10\,a}, v_{13\,1}, \dots, v_{13\,a}, v_{16\,1}, \dots, v_{16\,a}, v_{19\,1}, \dots, v_{19\,a}, v_{22\,1}, \dots, v_{22\,a},$
$v_{25\,1}, \dots, v_{25\,a}, v_{28\,1}, \dots v_{28\,a}$

$, v_{21}, v_{22}, v_{24}, \dots, v_{2(j-2)}\}$ where $j = 2a$. $S'_M$ covers the edges $|\langle N[S'_M]\rangle| = 3\left(9a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) \geq \left\lceil \frac{59a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ and the vertices $|N[S'_M]| = 4(9a) + 3\left(\left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) - 2\right) + (b - 6) > \left\lceil \frac{40a}{2} \right\rceil = 20a = \left\lceil \frac{p}{2} \right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil \frac{59a}{6} \right\rceil - 1$ then $|\langle N[S'_M]\rangle| = 3\left(9a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right)\right) - (b - 7) < \left\lceil \frac{53a}{2} \right\rceil = \left\lceil \frac{q}{2} \right\rceil$ . Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = 9a + \left(a - \left\lfloor \frac{a}{6} \right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil \frac{59a}{6} \right\rceil$.

**Theorem 1.9**

For the graph $G$ be the $T \cup C_4\, C_8[a,b]$ nanotori then $a \geq 2, b \geq 11$ then $n_m^{-1}(G) = \left\lceil \frac{6ab-a}{6} \right\rceil$

**Proof:**

This $T \cup C_4\, C_8$ nanotori is denoted by $T \cup C_4\, C_8[a,b]$ nanotube where $a$ is the number of squares in a row and $b$ is the number of rows of squares. $|V(G)| = \text{p} = 4ab$ and $|E(G)| = q = 6ab - a$. $V(G) = Y_1 \cup Y_2$ where $Y_1 = \{v_{21}, v_{22}, \dots v_{2j}, v_{51}, \dots v_{5j}, v_{81}, \dots, v_{8j}, \dots v_{(3b-1)1}, \dots, v_{(3b-1)j}\}$ where $j = 2a$ and $Y_2 = V(G) - Y_1 \ni \{v_{r1}, \dots v_{ra}\}$. We choose the vertex set $S_M = V_m(G) \cup V_n(G)$ where $V_m(G) = \{v_{r1}, \dots v_{ra}, v_{(2r)1}, \dots v_{(2r)a} \dots v_{[r(b-1)]1}, \dots v_{[r(b-1)]a}\}$ where $r = 3$ and $V_n(\text{G}) = \{v_{(3b-1)1}, v_{(3b-1)2}, v_{(3b-1)4}, \dots v_{(3b-1)(j-2)}\}$ where $j = 2a$. Therefore $S_M = \{v_{r1}, \dots v_{ra}, v_{(2r)1}, \dots v_{(2r)a} \dots v_{[r(b-1)]1}, \dots v_{[r(b-1)]a}, v_{(3b-1)1}, v_{(3b-1)2}, v_{(3b-1)4}, \dots v_{(3b-1)(j-2)}\}$ where $r = 3$ and $j = 2a$. $S'_M \subseteq V - S_M$ be the inverse neighborhood set with respect to $S_M$.

$S'_M = V_M(G) \cup V_N(G)$ where

$V_M(G) = \{v_{s1}, \dots v_{sa}, v_{(s+3)1}, \dots v_{(s+3)a}, \dots v_{(s+6)1}, \dots v_{(s+6)a}, v_{[s(b-2)]1}, \dots v_{[s(b-2)]a}\}$ where $s = 4$ and

$V_N(G) = \{v_{21}, v_{22}, v_{24}, \ldots v_{2(j-2)}\}$ where $j = 2a$ .

Therefore

$S'_M = \{v_{s1}, \ldots v_{sa}, v_{(s+3)1}, \ldots v_{(s+3)a}, \ldots v_{(s+6)1}, \ldots v_{(s+6)a}, v_{[s(b-2)]1}, \ldots v_{[s(b-2)]a}, v_{21}, v_{22}, v_{24}, \ldots$ $v_{2(j-2)}\}$ where $s = 4$ and $j = 2a$ . $S'_M$ covers edges $3\left(a(b-1) + \left(a - \left\lfloor\frac{a}{6}\right\rfloor\right)\right) \geq \left\lceil\frac{6ab-a}{2}\right\rceil = \left\lceil\frac{q}{2}\right\rceil$ and the vertices $|N[S'_M]| = 4(a(b-1)) + 3\left(\left(a - \left\lfloor\frac{a}{6}\right\rfloor\right) - 2\right) + 4 > \left\lceil\frac{4ab}{2}\right\rceil = 2ab = \left\lceil\frac{p}{2}\right\rceil$. Therefore $S'_M$ is inverse majority neighborhood set. Suppose $|S'_M| - 1 = \left\lceil\frac{6ab-a}{6}\right\rceil - 1$ then $|\langle N[S'_M]\rangle| = 3\left(a(b-1) + \left(a - \left\lfloor\frac{a}{6}\right\rfloor\right)\right) - 3 < \left\lceil\frac{6ab-a}{2}\right\rceil = \left\lceil\frac{q}{2}\right\rceil$ . Therefore $S'_M$ is not inverse majority neighborhood set. Hence $|S'_M| = a(b-1) + \left(a - \left\lfloor\frac{a}{6}\right\rfloor\right) \Rightarrow n_m^{-1}(G) = \left\lceil\frac{6ab-a}{6}\right\rceil$.

## References

[1] Frank Harry, "Graph Theory", Addision-Wesley Addision Wesley reading MA.
[2] Haynes, T W hedetniemi, S T, and Slater PJ, Fundamentals of Domination in Graphs", Marcel Dekker. Inc., New York, 1998.
[3] Sampathkumar E and Walikar H B, "The Connected Domination of a Graph", Jour. Math.phy.sci. 13.6, 1979.
[4] Sampathkumar E and Prabha S Neeralagi, "Neighborhood Number of a Graph", Indian J. Pure. Appl.Math, 16.2, pp.126-132, 1985.
[5] Sampathkumar E and Prabha S Neeralagi, "Independent, Perfect and Connected Neighborhood Number of a Graph", Journal of combinators Information and System of science.
[6] Joseline Manora J and Swaminathan V, "Majority Dominating Sets"- Published in JA R J 3.2,pp.75- 82, 2006 .
[7] Joseline Manora J and Swaminathan V, "Majority Neighborhood Number of a Graph"- Published in Scientia Magna, Dept. of Mathematics Northwest University, Xitan, P.R China- Vol(6), 2, 20-25, 2010.
[8] Joseline Manora J and Paulraj Jayasimman I, "Neighborhood Sets Polynomial of a Graph", Int Journal of Applied Mathematical Sciences, ISSN 0973-017 6. 1, pp.91-97,2013.
[9] Joseline Manora J and Paulraj Jayasimman I "Independent Mjority Neighborhood Number of a Graph", Int Journal of Applied Computational Science and Mathematics, 4.1, pp. 103-112 , 2014.
[10] Joseline Manora J and Paulraj Jayasimman I, "Results On Neighborhoods Sets Polynomial of a Graph", Int Journal of mathematical sciences with computer applications pp 421-426 , 2015.
[11] Joseline Manora J and Paulraj Jayasimman I, "Majority Neighborhood Polynomial of a

Graph",Vol.7 ,pp 97-102, 2015.
[12] Paulraj Jayasimman I and Joseline Manora J, "Independent Neighborhood Polynomial of a graph", Global Journal of pure and Applied Mathematics, Vol.13,pp.179-181.
[13] Paulraj Jayasimman I and Joseline Manora J, "Independent Majority Neighborhood Polynomial of a graph", Int Journal Mathematical Archive, Vol. 8, pp 109-112.
[14] Paulraj Jayasimman I and Joseline Manora J, "Connected Majority Neighborhood Polynomial of a Graph", Int Journal of Computational and Applied Mathematics ,Vol. 12, pp 208-212, 2017.
[15] Kulli.V.R and Kattimani, "The Inverse Neighborhood Number of a Graph", South.East. Asian.J.Math. and Math. Sc., 6.3,pp. 23-28, 2008.
[16] T Dhivya, I Paulraj Jayasimman , Joseline Manora J, " Majority Domination Polynomial of graph" Int journal of of Research,Vol 7,pp 484-487, Aug 2018.
[17] Paulraj Jayasimman I , Dhivya T , Joseline Manora J , "Inverse Majority neighborhood Number of a Graph", IOP Journal of physics conference series, Vol 1139,ISSN 1742-6596,pp 1-6, 2018.
[18] Paulraj Jayasimman I and Dhivya T, Joseline Manora J "Inverse Majority Vertex Covering Number of Graph", Int Journal of Engineering and Technology, Vol 7, ISSN 2227-524X,pp 2925-2927,2018.
[19] Paulraj Jayasimman I and Joseline Manora J, "The Maximal Majority Neighborhood Number of a Graph", Int journal of of Research,Vol 7,pp 670-673, Aug 2018.
[20] Paulraj Jayasimman I and Joseline Manora J, "Connected Majority Neighborhood Number of a Graph", Int journal of of Research,Vol 7,pp 984-987, Sep.2018.
[21] Renuka J and Balaganesan P,2018,Felicitous Labeling of some Path Related of Graphs, Indian journal of public health research and development, Vol.9,9,pp 389-396, ISSN 0976-0245.

[22] Renuka J and Balaganesan P,2018, Odd Harmonious and Strongly odd Harmonious Labeling of some classes of Graphs, Indian journal of public health research and development, Vol.9,9,pp 397-402 ,ISSN 0976-0245.

[23] Renuka J and Balaganesan P,2018, Odd Harmonious Labeling of some Classes of Cycle Related Graphs, Indian journal of public health research and development, Vol.9,9,pp 403- 408, ISSN 0976-0245.

[24] P Gayathri and S Sunantha , 2018, Multiplicative Connectivity Indices of TUC4C8(R) Nanotube ,
International journal of Mathematics and its application , Vol. 6, pp 443-455.
[25] V R Kulli , 2019, Multiplicative Neighborhood Indices , Annals of Pure and Applied Mathematics ,
Vol 19,2, pp 175-181.
[26] B.Basavanagoud, A.P.Barangi and S.M.Hosamani, First neighbourhood Zagreb index of some
nanostructures, Proceedings IAM. 7(2) (2018) 178-193.
[27]. R. Todeshine and V. Consonni, New vertex invariants and descriptors based on functions of vertex degrees, MATCH Commun. Math. Comput. Chem., 64(2010), 359-372.

[28] M. Eliasi, A. Iranmanesh and I. Gutman, Multiplicative versions of first Zagreb index, MATCH Commun. Math. Comput. Chem., 68(2012), 217-230.

[29] V. R. Kulli, Multiplicative hyper-Zagreb indices and coindices of graphs, International Journal of Pure Algebra, 67(2016), 342-347.

[30] V. R. Kulli, Branden Stone, Shaohui Wang and Bing Wei, Multiplicative Zagreb and Multiplicative hyper-Zagreb indices of polycyclic aromatic hydrocarbons, Benzenoidsystems, Preprint.

[31] M. Randic, On characterization of molecular branching, Journal of the American Chemical Society, 97(23)(1975), 6609- 6615.

[32] V. R. Kulli, Multiplicative connectivity indices of certain nanotubes, Annals of Pure and Applied Mathematics, 12(2)(2016), 169-176.

[33] V. R. Kulli, first multiplicative K Banhatti index of certain nanotubes, Annals of Pure and Applied Mathematics, 11(2)(2016), 79-82.

[34] V. R. Kulli, General Multiplicative Zagreb indices of T UC4C8[m, n] and T UC4[m, n] Nanotube, International Journal of Fuzzy Mathematical Archive, (2016), 39-43.

[35] Wei Gao, Muhammad Kamran Jamil et.al, Degree-based Multiplicative Atom-bond Connectivity Index of Nanostructures, IAENG International Journal of Applied Mathematics, 47(4)(2017).

[36] Najmeh Soleimani, Mohammad Javad Nikmehr and Hamid Agha Tavallaee, Computatioon of the different topological indices of nanostructures, J. Natn. Sci. Foundation Sri Lanka, 43(2)(2015), 127-133.

[37] Sakander Hayat and Muhammad Imran, On Degree Based Topological Indices of Certain Nanotubes, Journal of Computational and Theoretical Nanosscience, 12(2015), 1599-1605.

[38] Tomas Vetrik, Degree-based topological indices of hexagonal nanotubes, J. Appl. Math. Comput., 2017(2017).

[39] V. R. Kulli, Multiplicative Connectivity Indices of Nanostructures, Journal of Ultra Scientist of Physical Sciences, 29(1)(2017), 1-10. 454P.

[40] Abbas Heydari and Bijan Taeri, Hyper Wiener index of T UC4C8(R) nanotubes, Journal of computational and Theoritical Nanoscience, 5(2008), 2275-2279.

[41] Tomislav et.al, Topological Compression Factors of 2-Dimensional T UC4C8(R) Lattices and Tori, Iranian Journal of Mathematical Chemistry, 1(2)(2010), 73-80.

[42] Lixin Xu and Hanyuan Deng, The Schultz Molecular Topological Index of C4C8 Nanotubes, MATCH Commun. Math. Comput. Chem., 59(2008), 421-428.