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Research Article

Stress Intensity Factors For Two Griffith Cracks Opened By A Symmetrical System Of Body Forces In A Stressfree Strip

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ABSTRACT

The problem of determining the stress and the displacement fields in the neighbourhood of two Griffith cracks is reduced to the second king of Fredholm integral equation by using Fourier transform. The solution of this integral equation is obtained by expanding the unknown function in terms of (a^{-1}) -a being half of the width of the strip. The partial closure of the crack is also considered. The numerical results for special point body force are show graphically.

Introduction

Many problems of crack opening due to forces applied at crack faces are solved in the literature [1]. The crack opening due to general system of body forces in infinite isotropic and homogeneous solids has been solved with Fourier transforms by Sneddon and Tweed [2, 3]. The similar problems were solved for finite boundaries (rigidly lubricated) by Parihar and Kushwaha [4] and Kushwaha [5]. In the present paper we are extending the analysis of [4] to the title problem.

Thus we are solving the problem of cracks $y=0$, $b<x<c$, $-c<x<-b$ opened by symmetrical system of body forces $[X,Y]$ in homogeneous isotropic stress-free strip having symmetrically placed cracks normal to edges (see figure 1). Splitting into two displacement boundary value problems, namely, problem I and Problem II, we solve these separately. Therefore, of concern is the problem.

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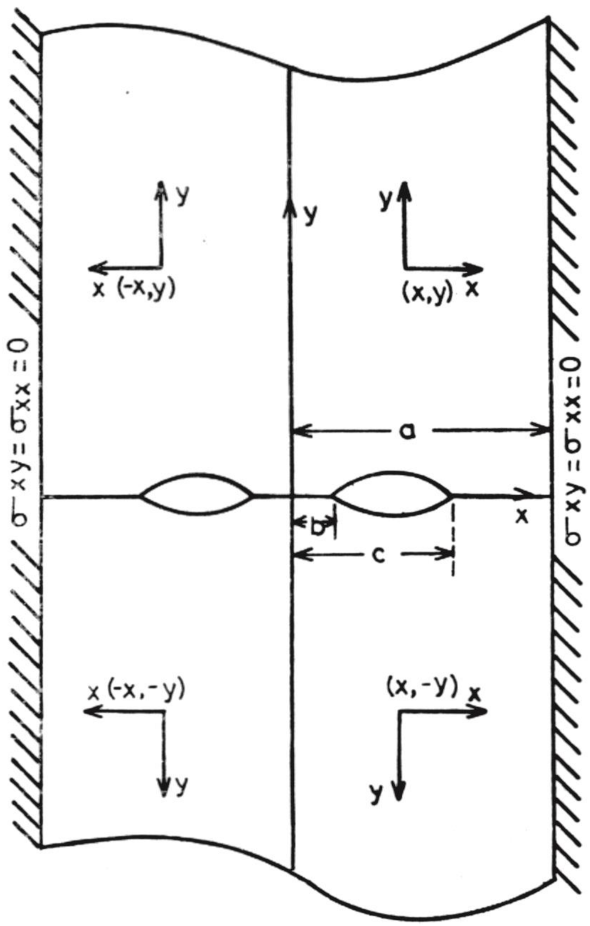

Fig. 1

$$\sigma_{xx}(\pm a, y) = \sigma_{xy}(\pm a, y) = 0,\ 0<|y|<\infty \qquad (1.1)$$

$$\sigma_{xy}(x, 0)=0,\ 0\leq|x|\leq a \qquad (1.2)$$

$$\sigma_{yy}(x, 0)=0,\ b<|x|<c \qquad (1.3)$$

$$u_y(x, 0)=0,\ c<|x|\leq a,\ 0\leq|x|<b, \qquad (1.4)$$

Where $(\sigma_{xx}, \sigma_{xy}, \sigma_{yy},)$ and (u_x, u_y) are the components stress-tensor and of displacement vector, respectively. Following the analysis of [4], we get,

Problem I: is the solution of equations of equilibrium in the presence of body forces *[X,Y]* and the following boundary conditions -

$$\sigma^{(1)}_{xy}(x, 0)=0,\ 0\leq|x|\leq a, \qquad (1.5)$$

$$\sigma^{(1)}_{xy}(a, y)=0,\ 0\leq y<\infty, \qquad (1.6)$$

$$u_y^{(1)}(x, 0)=0,\ 0\leq|x|\leq a, \qquad (1.7)$$

and the problem two as

Problem II: is the solution of equations of equilibrium in the absence of [X,Y] and the following boundary conditions –

$$\sigma^{(1)}_{xx}(\pm a,y) + \sigma^{(2)}_{xx}(\pm a,y) = 0,\ 0 \le |y| < \infty, \quad (1.8)$$

$$\sigma^{(2)}_{xy}(x,0) = 0,\ 0 \le |x| \le a\}, \quad (1.9)$$

$$\sigma^{(2)}_{xy}(\pm a,y) = 0,\ 0 \le |x| < \infty,$$

$$\sigma^{(1)}_{yy}(x,0) + \sigma_{yy}(x,0 = 0,\ b < |x| < c, \quad (1.10)$$

$$u_y^{(2)}(x,0) = 0,\ 0 < |x| < b,\ c < |x| \le a, \quad (1.11)$$

where the superscript (1) and superscript (2) over functions represent the functions obtained for problem I and problem II respectively. Upto section 4 of the paper the condition

$$u_y^{(2)}(x,0) > 0,\ b < |x| < c, \quad (1.12)$$

(see Burniston [6]) is being observed. The following notations for transform are being used

$$\frac{fcs}{sc}(a_n,\ p) = \int_0^{\infty} \left(\begin{matrix}\cos a_n x \\ \sin a_n x\end{matrix}\right) dx \int_0^{\infty} f\,(x,y) \left(\begin{matrix}\sin p\,y \\ \cos p\,y\end{matrix}\right) dy,$$

with the usual inversion formula and $a_n, = n\pi | a = nq$.

The outlet of the paper is as follows: section 2 deals with the formulation of the problem. Section 3 deals with the solution of Fredholm integral equation along with the general expressions for physical quantities like, crack shape, normal component of stress at *y=0* and then the stress-intensity factor. Section 4 gives one example. Section 5 deals with the condition of partial closing of the crack at the centre.

2. Formulation

Problem I : To solve this problem with the boundary conditions (1.5)-(1.7) we take appropriate Fourier transform of equations of equilibrium and of stress-strain relations, and substitute for transformed stress components in the transformed equations of equilibrium and invert, we get as

$$u_x^{(1)}(x,y) = v \sum_{n=1}^{\infty} \int_0^{\infty} \sin(a_n x)\,[w_1 X_{sc} - w_2 Y_{cs}]\,d\rho\,\cos\rho y, \quad (2.1)$$

$$u_y^{(1)}(x,y) = \frac{1}{2}\,u^{(1)}_{yc}(c,y) + \sum_{n=1}^{\infty} u^{(1)}_{yc}(a_n y)\cos a_n x, \quad (2.2)$$

$$u^{(1)}_{yc}(a_n,y) = -v \int_0^{\infty} [w_2 X_{sc} - w_3 Y_{cs}]\,d\rho \sin\rho y, \quad (2.3)$$

and

$v=8(1+\eta)\rho/(\pi a\beta^2E),\ \beta^2=2(1-\eta)/(1-2\eta)$ (2.4)

$w_1=\frac{a_{n^2}+\beta^2}{(a_{n^2}+\beta^2)^2},\ w_2=\frac{(\beta^2-1)\alpha_n}{(a_{n^2}+\beta^2)^2},\ w_3=\frac{\beta^2+a_{n^2}}{(a_{n^2}+\beta^2)^2}$ (2.5)

where ρ and η are mass density and Poisson ratio of the medium, respectively.

Problem II: The solution of problem II is obtained through the similar method of Sneddon and Srivastav [7] and written as

$$u_x^{(2)}(x,y)=\frac{2(1+\eta)}{E}\sum_{n=1}^{\infty}\frac{\sin a_n x}{\alpha_n}\left\{(1-\eta)\phi_{1,yy}+\eta_{a_{n^2}}\phi_1\right\}+\int_0^{\infty}(1-\eta)\phi_{2,xx}$$

$$+(\eta-2)\rho^2\phi_{2,}\,x\frac{\cos\rho\,y}{\rho^2}d\rho], \quad (2.6)$$

$$u_y^{(2)}(x,y)=\frac{1}{2}u_{yc}^{(2)}(0,y)+\sum_{n=1}^{\infty}\quad u_{yc}^{(2)}(\alpha_n,y)\cos\alpha_n x+\frac{2(1+\eta)}{\pi E}$$

$$\int_0^{\infty}\{(1-\eta)\phi_{2,xx}+\eta^2\phi_{2,}\}\frac{\sin\rho y}{\rho}d\rho], \quad (2.7)$$

with

$$u_y^{(2)}(\alpha_n,y)=\frac{2(1+\eta)}{aEa_{n^2}}[(1-n)\phi_{1,yyy}+(\eta-2)\,a_n^2\phi_{2,}y], \quad (2.8)$$

$$\phi_{1,}=A_n(1+a_n y)e^{-\alpha n y},\phi_2=A(\rho)\,[\cosh\rho x-\tanh\rho\,\alpha\,\sinh\rho\,x], \quad (2.9)$$

where An and A(p) are respectively arbitrary constant and function to be determined. From the expressions (2.6)-(2.9) we see that the boundary conditions (1.9) are satisfied identically. The boundary condition (1.8) gives

$$A(\rho)=\left[-\frac{4}{a}\sum_{n=1}^{\infty}(-1)^n\frac{a_n a_{n^2}\rho^2}{(a_{n^2}+\rho^2)^2}+\frac{\pi}{2}P_1(\rho)\right]\frac{\cosh\rho\,a}{\rho^2} \quad (2.10)$$

and

$$P_1(\rho)=\frac{1}{2}\sigma^{(1)}_{xxc}(a,\rho)+\sum_{n=1}^{\infty}(-1)^n\sigma^{(1)}_{xxc}(nx,\rho). \quad (2.11)$$

Thus we are left with one arbitrary constant and the mix boundary conditions (1.10)-(1.11). Three boundary conditions yield the following system of triple series relations

$$\frac{A_o}{2}+\sum_{n=1}^{\infty}a_n A_n\cos a_n x=0,\ 0\le x\le b,\ c\le x\le a \quad (2.12)$$

$$\sum_{n=1}^{\infty}a_n^2A_n\cos a_n x=2/\pi\sum_{n=1}^{\infty}(-1)^n a_n^3A_n\int_0^{\infty}(\cosh\rho x-\tanh\rho\,a\sinh\rho x)$$

$$(a_n^2+\rho^2)^{-2}d\rho+\int_0^{\infty}P_1(\rho)\cosh\rho\,(a-x)d\rho-a/2\,\sigma_{yy}^{(1)}(x,0),$$

$$b<x<c. \quad (2.13)$$

Reduction to Fredholm Integral Equation

The solution of triple series relations (2.12)-(2.13) is obtained by the method of Parihar [8] and given as

$$g(t) + \frac{4}{a^2}\int_b^c g(y)k(y,t)dy = \frac{2g}{t}\left[\int_b^c \frac{sin(gx)\delta(x)F(x)dx}{G(x,t)} + D\right], \qquad (2.14)$$

$$F(x) = -\frac{a}{2}\sigma_{yy}^{(1)}(x,0) + \int_0^\infty P_1(\rho)\, cosh\, \rho\, (a-x)d\rho, \qquad (2.15)$$

$$K(y,t)=[e(t)]^{-1}\int_b^c \frac{\delta(x)sin\,(gx)}{G(x,t)}\sum_{n=1}^\infty (-1)^n a_n\, sin\, a_n b \int_0^\infty$$

$$\frac{\rho^2 \cosh\rho\,(a-x)}{\sinh\rho a}\,[\rho^2 + a_{n^2}]^{-2}d\rho, \qquad (2.16)$$

$$\delta(x) = [|G(b,x)\; G(x,c)|]^{1/2},\; G(x,y)= cos(qx)-cos(qy), \qquad (2.17)$$

$$a_n^2 A_n= \int_b^c g(t) \sin a_n t\, dt,\; A_0= \int_b^c tg\,(t)\, dt, \qquad (2.18)$$

$$\int_b^c g(t)\, dt=0, \qquad (2.19)$$

D is an arbitrary constant which will be determined through (2.19).

3. Solution of Fredholm Integral Equation

We first approximate the singular kernel *K(y, t)*. Having expanded the hyperbolic functions in terms of exponentials and then using the relation, (see {9} page 23,

$$\sum_{n=1}^\infty (-1)^n/(a^2+n^2) = \frac{1}{2a}[\pi cosech\pi a - a^{-1}], \qquad (3.1)$$

we got, after evaluating integrals,

$$F(x,y)= \sum_{m=0}^\infty \sum_{l=0}^\infty [(y-a)\sum_{i=1}^6 \beta_i^{-1}+(y+a)\sum_{l=7}^{12} \beta_i^{-1}] \qquad (3.2)$$

with

$$\beta_1=\beta^2(a,2m,2l,-y),\ \beta_2=\beta^2(2a,2m,2l,-y),\ \beta_3=\beta^2(2a,2m,2l,-y,-x)$$

$$\beta_4=\beta^2(3a,2m,2l,y),\ \beta_5=\beta^2(4a,2m,2l,y),\ \beta_6=\beta^2(4a,2m,2l,y,-x)$$

$$\beta_7=\beta^2(a,2m,2l,y),\ \beta_8=\beta^2(2a,2m,2l,y),\ \beta_9=\beta^2(2a,2m,2l,y,x)$$

$$\beta_{10}=\beta_4,\ \beta_{11}=\beta_5,\ \beta_{12}=\beta^2(4a,2m,2l,y,x)$$

$$\beta^2(a_1,a_2,a_3,a_4,a_5)= (\sum_{i=1}^5 a_i)^2 \qquad (3.3)$$

Using above relations and (2.16) we get

$$K(y,t)= \frac{2}{\pi a^2\delta(t)}\sum_{m=0}^\infty \sum_{l=0}^\infty (y-a)(\beta_1^{-1} + \beta_2^{-1} - \beta_4^{-1} - \beta_5^{-1})+$$

$+(y+a)\,(\beta_7^{-1} + \beta_8^{-1} - \beta_{10}^{-1} - \beta_{11}^{-1} + K_{ml}^{-1}(y, t) + + K_{ml}^{2}(y, t)$

$(y-a) + (y+a) + K^2_{ml}(y, t) - K^4_{ml}(y, t)],$ (3.4)

where

$K^i_{m1} = \int_b^a \frac{\sin(gx)\delta(x)}{G(x,t)} F(x, y)\, dx, \; i=1,2,3,4$ (3.5)

and δ and G are given by (2.17). We assume the solution g(t) of the form

$g(t) = \sum_{m=0}^{\infty} a^{-m-2} g_m(t),$ (3.6)

Substituting from equations (3.4)-(3.6) into equation (2.14) and comparing the coefficient of a^{-m} on the sides of the equations, we get

$g_n(t)=F(t)/\delta(t),$

$g_n(t)= \frac{2}{\pi^2\delta(t)} [\int_b^c y g_{n-2}(y)\{M_1(y)+M_2(y, t)\}\, dy +$

$\int_b^c g_{n-1}(y)\{M_{11}(y)+M_{22}(y, t)\}\, dy, \; n\geq 1,$

with

$g_{-1}(t)=0$ (3.7)

and

$F(t)=(\int_b^c \frac{\sin(gx)\delta(x) f(x) dx}{G(x,\; t)} - D)$ (3.8)

$M_1(y)=[\sum_{m=0}^{\infty}\sum_{l=0}^{\infty} [a(\beta_1^{-1} + \beta_2^{-1} + \beta_7^{-1} + \beta_8^{-1})] + \sum_{n=1}^{4}\int_b^c \frac{\sin\,(gx)}{\delta(x)} \beta_{3i}^{-1}(x, y)\, dx)]$
(3.9)

$M_2(y, t)=\delta^2(t)\sum_{n=1}^{\infty}\int_b^c \frac{\sin(qx)\beta_{3i}^{-1}(x,y)dx}{\delta(x)G\,(x,t)}$ (3.10)

$M_{11}(y)= \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} [a(\beta_7^{-1}+ \beta_8^{-1} - 2\beta_4^{-1} + 2\beta_5^{-1} - \beta_1^{-1} + \beta_2^{-1})$

$+\int_b^c \frac{\sin\,(qx)}{\delta(x)} (\beta_3^{-1} + \beta_6^{-1} + \beta_{12}^{-1} + \beta_9^{-1})\, dx],$ (3.11)

$M_{22}(y+t)=\delta^2(t)\int_b^c \frac{\sin(qx)}{\delta(x)G(x,t)} \{-\beta_3^{-1} + \beta_6^{-1} + \beta_{12}^{-1} + \beta_9^{-1}\}\, dx$ (3.12)

Thus the solution of Fredholm integral equation (2.14) will be given by (3.6)-(3.12).

Physical Quantities

Crack Shape: Evaluating the value of the series (2.12) for $b<x<c$ through equations (2.13)-(2.19), we get

$$u_y^{(2)}(x, 0)=\frac{2(1-\eta^2)}{\pi E}\int_x^c g(t)dt, \quad b<x<c, \tag{3.13}$$

where g(t) will be given by equations (3.6)-(3.12).

Normal-stress component at y=0 : The evaluation of $\sigma_{yy}^{(2)}$ (x, 0) through the equations (2.6)-(2.11), alongwith stress-strain relations, we get

$$\sigma_{yy}^{(2)} (x, 0) = 2/a\int_b^c \frac{\sin(\sigma t)}{G(x,\ t)}\, g(t)dt + \frac{2}{\pi a^2}\int_b^c g(y)\, d\rho + \int_0^\infty \frac{p\cosh\rho\,(a-x)}{\sinh\rho\pi}$$

$$[y \cosh \rho y \sinh \rho\pi - \pi \sinh \rho y \cosh \rho\pi]\, d\rho - 2\int_0^\infty F(\rho)\cosh\rho\,(a-x)d\rho \tag{3.14}$$

It is being assumed that there is no singularity in $\sigma_{yy}^{(1)}$ (x, 0) ay crack tips.

Stree0Intensity factors : We define the stress-intensity factors at crack tips *(b, 0)* and *(c, 0)* as

$$K_b = \lim_{x\to b-} \sqrt{(b-x)}\sigma_{yy}^{(2)}(x, 0),\ K_c=\lim_{x-c+} \sqrt{(x-c)}\sigma_{yy}^{(2)}(x,0), \tag{3.15}$$

substituting from (3.6)-(3.12) into (3.14) and evaluating the integrals and then using the definitions (3.15) we get

$$K_b = -m(b)[F(b)+\frac{4}{\pi a^2}\sum_{n=1}^{\infty} a^{-n}\, g_n(b)],K_c = m(c)[P(c)+\frac{4}{\pi a^2}\sum_{n=1}^{\infty} a^{-n}\, g_n(c) \tag{3.16}$$

with

$$m(y)=[2q \sin (qy)\, G(b, c)]^{-1/2} \tag{3.17}$$

4. An Example

To make the analysis of sections 1-3 clear we consider one example of point body force. The loading is defined as (see figure 2),

$$X(x, y)=0,\ Y=(x, y)= \frac{Q\delta(x)}{2\rho}[(y-h)-(y+h)], \tag{4.1}$$

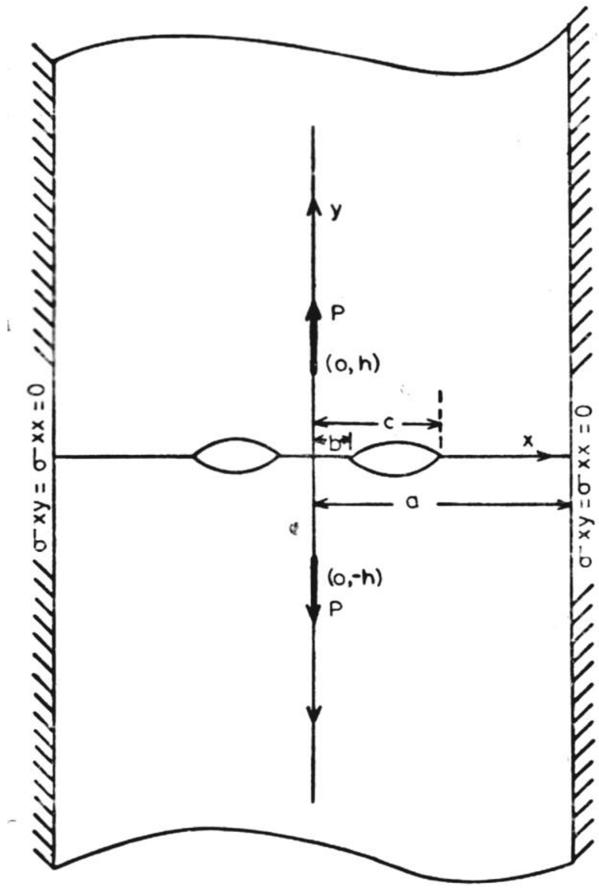

Fig. 2

Where Q is the magnitude of the loading, p is the mass density of the medium. The above loading represents a force at (0, h) in positive y-direction and at (0, -h) in negative$y$-direction. We evaluate the components of stress at y=0 by using the equations (2.1)-(2.5) and the stress-strain relations. To get $g(t)$ through equations (3.6)-(3.12) we need evaluate the integrals for $P(t)$ which involves $\sigma_{yy}^{(1)}$ $(x,0)$ and $\sigma_{xx}^{(1)}$ $(n\pi,y)$. Thus we get

$$P(t)=Q[aT_h\frac{\sin(qh)\,G(t,c)}{Q(h)}(1-\frac{G(b,t)}{R(h,t)})]+D+P_3(t), \tag{4.2}$$

where

$$Q(h)=[|(\cos qh-\cos qb)\,(\cos qh-\cos qc)|]^{1/2},$$

$$R(h,t)=\cos qh-\cos qt,$$

$$P_3(t)=\int_b^c \sin(qx)\delta(x)P_2(x)\,dx|G(x,t), \tag{4.3}$$

$$P_2(x)=Q\sum_{m=0}^{\infty}\sum_{l=0}^{\infty}(\beta^2-1)q\pi[(\delta_1+\delta_2-\delta_3-\delta_4)+a[(\delta_1^2+\delta_2^2+\delta_3^2+\delta_4^2)]-$$

$$-\frac{q\pi\beta^2}{2}\left\{\frac{1}{t+2ma}+\frac{1}{4a-t+2ma}\right\}, \tag{4.4}$$

$\delta_1^{-1}=t+2a(m+1), \delta_2^{-1}=4a\,t + 2ml,\ \delta_3^{-1}=\delta_1^{-1}+2a,\ \delta_4^{-1}=\delta_2^{-1}+2a,$ (4.5)

and the differential operator T_h is defined as

$T_h=1-\left(\frac{\beta^2-1}{\beta^2}\right)h\frac{d}{dh}$ (4.6)

D can be determined from $\int_b^c g_1(t)dt$ and condition σ_{xy} is finite at $(b, 0)$ so, we get

$$D=\int_b^c F_1(y)+\frac{aTq}{2}\sin Q(y)x\sqrt{\left(\frac{G(y,c)}{G(b,y)}\right)}dy,$$

Therefore, $g(t)$ will be given by equations (3.6)-(3.7), (3.9)-(3.12) and (4.3)-(4.6). Substituting for $P(y)$ and $g_n(y)$ in equations (3.16), we get the stress-intensity factors. We have plotted $(\pi\sqrt{(2c)}/Q)K$, $(K=K_b, K_c)$, against h/c for different values of c/a=0.5, 0.5 in figure 3. We took $\eta=0.25$.

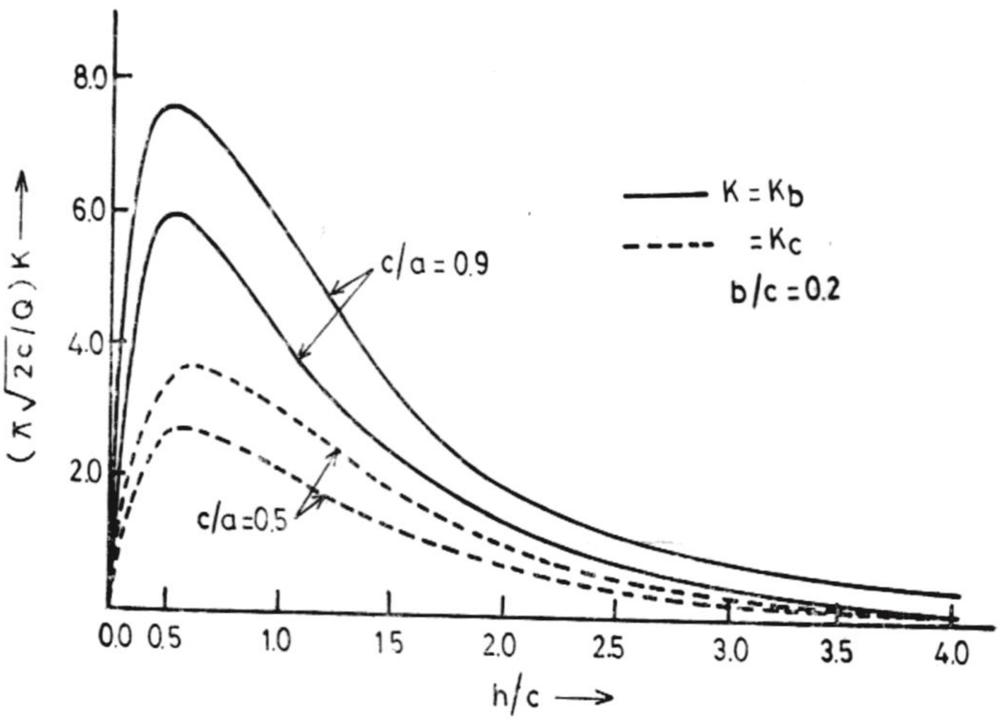

Fig. 3

We truncated the series for $[g_n(t)]$ at n=10, for F_{ml}, 1=15, m=25. Accuracy is of the order of 5%. In figure 4 we plotted $(\pi E/Q)\ u_y^{(2)}\ (x, 0)$ against x/c for h/c=0.5, 2.0 and c/a=0.9, 0.5 We took b/c=0.2.

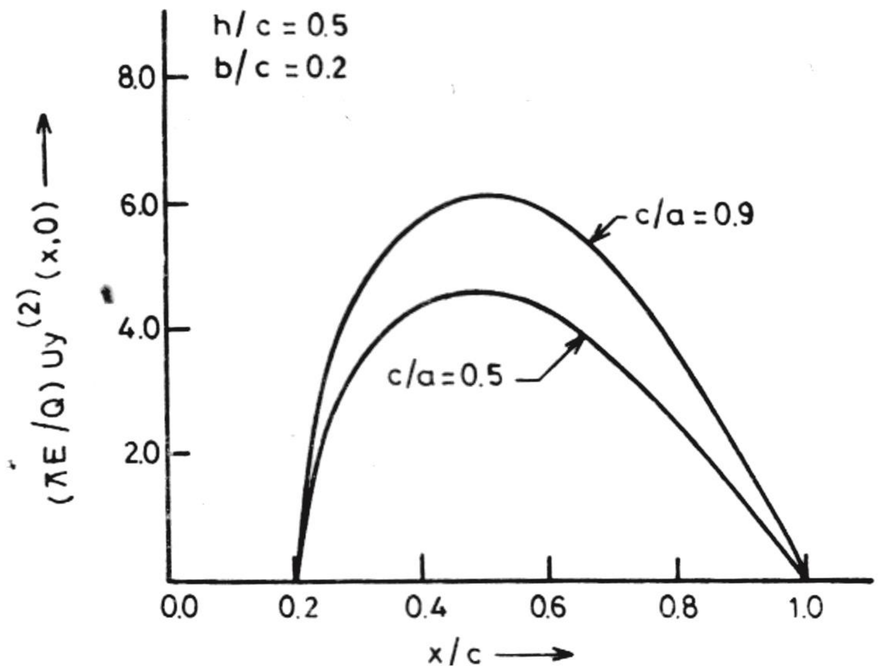

Fig. 4

5. Partial Closing

Now we consider the problem of finding the stress field in the neighbourhood of the Griffith cracks $0\le|x|\le c(y=0)$ in the stress-free strip$0\le|x|\le a(0\le|y|<\infty)$ which is acted upon by a uniform tension *T* at infinity normal to *x*-axis and a system of body forces such that the crack faces meet somewhere near the center of the crack. The corresponding problem for rigidly lubricated strip has been solved by Parihar and Kushwaha [4].

The formulation of the above boundary value problem is exactly the same as in sections 1-2 with the change in boundary condition (1.10) to

$$\sigma_{yy}^{(1)}(x,0)+\sigma_{yy}^{(2)}(x,0)=-T,\ b<|x|<c, \qquad (5.2)$$

where *b* in this case is an unknown parameter to be determined by the conditions of finiteness of stress $\sigma_{yy}^{(1)}(x,0)$ at $(b, 0)$. The solution could be obtained through the equations (2.14)-(2.19) with the change in $F(x)$ as

$$F(x)=\frac{a}{2}\left[\sigma_{yy}^{(1)}(x,0)+T\right]+\int_0^\infty P_1(\rho)\cosh\rho(a-x)\,d\rho. \qquad (5.2)$$

Thus the condition of finiteness of $\sigma_{yy}^{(1)}(x,0)$ at $(b, 0)$ give $K_b$to be zero at $(b, 0)$. Therefore, we get

$$P(b)=\frac{2}{\pi a^2}\sum_{n=1}^{\infty} g_n(b)a^{-n}=0 \qquad (5.3)$$

with

$$P(b) = \int_b^c sin\,(qx)\sqrt{\left(\frac{G(x,c)}{G(b,x)}\right)}\,\sigma yy^{(1)}(x,0) + T - a^{-1}\int_0^{\infty} P_1(\rho)$$

$$\cosh\rho(a\text{-}x)d\rho\; dx + D \qquad (5.4)$$

where D will be obtained from the condition (2.19). To illustrate the use of the general formula one special case is being considered.

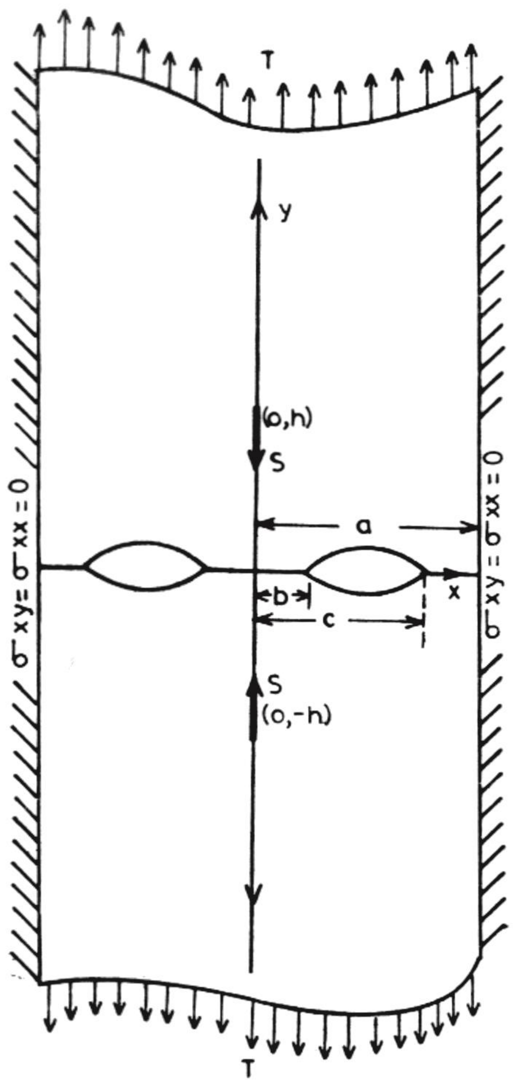

Fig. 5

We consider the case in which the crack is opened by constant uniform tension T at infinity and closed partially due to the point body forces, see figure 5, specified by equation

$$Y(x, y) = -\frac{S\delta(x)}{2p}\,[\delta(y\text{-}h) - \delta(y+h)],\; X(x, y)=0 \qquad (5.5)$$

The function *F(x)* can easily be evaluated, as in section 4 with the change in *Q* by *(-S)*. We have plotted *S/(cT)* against *b/c* for different values of *c/a* and of *h/c*=0.5, 1.0 in figure 6, from equations (5.3)-(5.4).

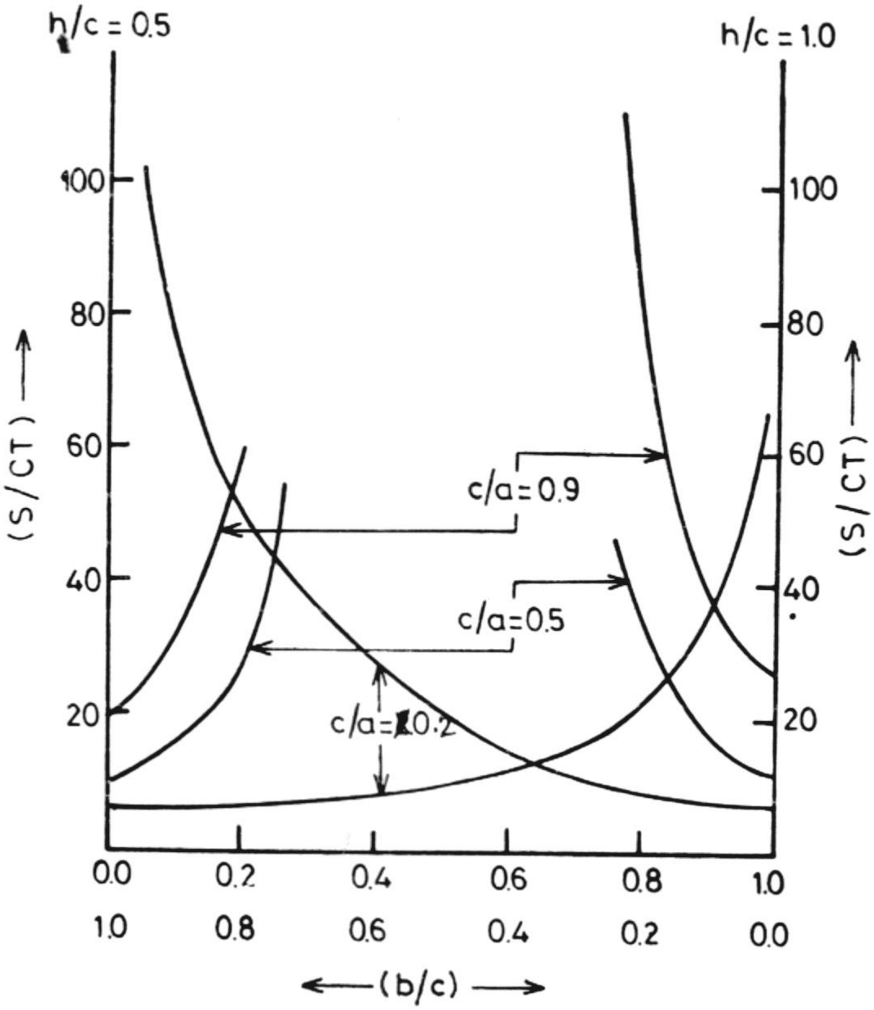

Fig. 6

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