Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 6, July 2021: 6451-6465

Research Article

Water Table Rise in Sloping Aquifer due to Canal Seepage

Manisha M. Kankarej

Abstract

A mathematical model is presented to describe the analytical solutions of the linearized Boussinesq equation of the non-linear Boussinesq equation to study transient and steady-state water rise in a homogeneous, isotropic and incompressible unconfined sloping aquifer where recharge is applied from two canals. The rise in the water table was due to seepage from two canals located at different elevations above the sloping impermeable barrier. Proposed analytical solutions were verified with analytical solutions for a horizontal aquifer and were found in close agreement. Thus, the time varying recharge rate is approximated of different slopes depending on the nature of the recharge rate. Application of the prediction of the water table fluctuation and sensitivity analysis of various controlling parameter on the aquifer response is demonstrated.

Keywords:*Water table, aquifers, boussinesq equations, seepage, slopes*

Dr. Manisha M. Kankarej, Assistant Professor, Dept. of Mathematics and Statistics, Zayed University, Dubai, UAE,manisha.kankarej@gmail.com

List of symbols

D = Average saturated depth of the aquifer

h(x, t) = Water head height measured from sloping bed

h_0= Initial water levels in the drain

$\hat{h}(x, t)$ = Variable water head height measured from horizontal datum

K = Hydraulic conductivity

L = Lateral extant of the unconfined aquifer

q = Flow rate per unit area of the aquifer

R(x, t) = Resource term in the domain

S = Specific yield

t = Time

x = Horizontal x axis (space coordinate)

z = Werner's transformation

β = Sloping angle measured in radian

$$\alpha = \frac{tan\,\beta}{K\ D\ cos^2\beta}$$

$$\frac{1}{\gamma} = \frac{S}{K\ D\ cos^2\beta}$$

$$Q = \frac{R(x,t)}{K\ D\ cos^2\beta}$$

Introduction

Canal irrigation has been growing efficiently in many parts of the countries. It is not only providing water for irrigation but also, they act as a source of intensive seepage below the ground in highly pervious tract. Due to continuing seepage from canals as well as percolation losses from irrigation field, water table have highly risen in some areas over the years. This water table build up has created the problem of water logging and salinity and ultimately leading to land degradation in many regions.

In the areas where water table is accompanied with salinity it may not be possible to construct wells for irrigation purposes, the only alternate for such areas is subsurface drainage. In order to accompany any subsurface drainage scheme, it is necessary to understand the temporal variation of groundwater table. The spatial and temporal variation of the water table may be obtained by representing the physical situation in mathematical terms and solving the partial differential flow equation with appropriate initial and boundary conditions by applying analytical and numerical method.

Mathematical model play, a key role in assessing the future behavior of the ground water system to various operational schemes of recharging for sustainable development of the ground water system. A number of investigators have studied the effect of recharge due to canal seepage and field irrigation leading to water table rise in the affected areas. Many mathematical models have been developed to predict water table fluctuations in response to recharge from basins and canals.

Werner (1969) studied groundwater flow in an unconfined aquifer receiving recharge and developed expressions that include the effect of different levels of water level in the drains. Maasland (1958) presented a detailed analysis of the effect on the water table of intermittent recharge. Studies have been carried out by Kraijenhoff van de Leur (1958), Schmid and Luthin (1964), Mustafa (1987), Marino (1974), Gill (1884), Rai and Singh (1992, 1995), Ram et. al (1994). Evolution and stabilization of the free surface in unconfined semi-infinite aquifer subject to rectangular recharge at uniform rate was analyzed by Hantush (1967). Mangalik et. al (1997) proposed a new method for simulating the discontinuous cycles of recharge operation by a sequence of line segment of varying length and slope. Most of these studies relate to obtaining analytical solutions of the 1D linearized Boussinesq equation with certain initial and boundary conditions to describe the water table fluctuation due to replenishment and through seepage from canals overlying a horizontal aquifer.

In some situations, aquifers are located on the ridge line, on the side slopes below the ridge or on the inclined planes. Upadhyay and Chauhan (1998, 1999, 2000, 2002) reviewed and presented the theories related to drainage and water table fluctuations in sloping aquifer. There are also chances of overestimations and under estimation of water table rise in the process of linearization of nonlinear Boussinesq equation. The effect of linearization of the flow governing equation may be studied by obtaining numerical solutions to nonlinear Boussinesq equation and comparison of results with those obtained from an analytical solution of the linearized Boussinesq equation.

The objective of present paper is to study an analytical solution of the linearized Boussinesq equation and a numerical solution of the nonlinearized Boussinesq equation. These two solutions are used to describe the transient and steady state water table rise in the sloping unconfined aquifer lying between two parallel canals located at different elevations above the impermeable barrier. Application of the prediction of the water table fluctuation and sensitivity analysis of various controlling parameter on the aquifer response is also demonstrated in this paper.

Mathematical formulation

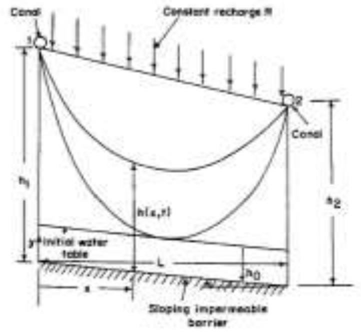

Fig. 1. Definition sketch for water table rise between two canals overlying sloping aquifer due to seepage from canals and recharge from land surface

Initially the water table was at the elevation h_0 above the impermeable barrier. Due to seepage from two canals separate by a distance L and located at elevations $h_1$and h_2 respectively, the groundwater table gradually rises and assumes a steady state condition after some time. The assumptions considered for the formulation of the mathematical problem are given below:

1. The aquifer is unconfined, homogenous, isotropic and incompressible, overlying an impermeable barrier that is inclined with a small slope β.
2. The aquifer is being replenished by two parallel canals acting as a line source. The heights of the canals are $h_1$and h_2. h represents the height of the groundwater table above the horizontal axis.
3. Effects due to capillary rise and evaporation due to water table are ignored.
4. Water through deep percolation moves vertically downward until it joins the groundwater.
5. Flow is characterized by a 1D continuity equation as derived by Boussinesq (1904) using Darcy's law and Dupuit's assumption.

An unconfined aquifer of lateral extent L overlaying an impermeable bed with an upward slope tan β is considered. With Dupuit- Forchheimer assumptions that the streamline is nearly parallel to the sloping impervious bed, the discharge rate per unit width of the aquifer along x axis can be approximated by following relation (Chapman 1980):

$$q = -Kh\, cos^2\beta \frac{\partial}{\partial x}\hat{h} \tag{1}$$

where $\hat{h}$ is the variable water head height measured in vertical direction from horizontal datum and $h(x,t)$ is the water head height measured in the vertical direction from the impermeable sloping bed. K is the hydraulic conductivity, $\tan\beta$ is the bed slope, t = time and x = space coordinate. Applying the principle of mass balance across a vertical slice, the equation of subsurface seepage flow over sloping bed is given by:

$$\frac{\partial q}{\partial x} + S\frac{\partial}{\partial x}\hat{h} = R(x,t) \tag{2}$$

where S is the specific yield of the aquifer, and $R(x,t)$ are spatial and temporal varying net rates of the vertical accretion to the free surface. If the spatial variations in K and S are neglected, then eqns. (1) and (2) imply that

$$Kcos^2\beta\ \frac{\partial}{\partial x}\{h\ \frac{\partial}{\partial x}\hat{h}\} + R(x,t) = S\ \frac{\partial \hat{h}}{\partial t} \tag{3}$$

Since $\hat{h} = h + x\ tan\,\beta$ after rearranging eqns. (1) and (3) can be written as,

$$q = -Kh\, cos^2\beta \left\{\frac{\partial h}{\partial x} + tan\,\beta\right\} \tag{4}$$

$$Kcos^2\beta\ \{\frac{\partial}{\partial x}(h\ \frac{\partial h}{\partial x}) + tan\,\beta\ (\frac{\partial h}{\partial x})\}\ + R(x,t) = S\ \frac{\partial h}{\partial t} \tag{5}$$

The initial and boundary conditions are

$$h(x, t = 0) = h_0;\ 0 < x < L \tag{6}$$

$$h(0,\ t\,) = h_1;\ t > 0 \tag{7}$$

$$h(L,\ t\,) = h_2;\ t > 0 \tag{8}$$

Eqn. (5) is a second order parabolic partial differential equation, analytic solution of which is not tractable. However approximate analytic solutions can be obtained by taming the non-linearity around some mean saturated depth *D*. Rewrite eqn. (5) as

$$\frac{\partial^2 h}{\partial x^2} + \frac{tan\,\beta}{K\,D\,cos^2\beta}\frac{\partial h}{\partial x} + \frac{R(x,t)}{K\,D\,cos^2\beta} = \frac{S}{K\,D\,cos^2\beta}\frac{\partial h}{\partial t}, 0 \le\ x\ \le L \tag{9}$$

In fact, this technique invokes linearization of the flow rate *q* around *D*, given by

$$q = -K\,cos^2\beta\left\{D\frac{\partial h}{\partial x} + h\,tan\,\beta\right\} \tag{10}$$

The average saturated depth *D* of the aquifer is approximated using an iterative formula $D = \frac{h_0 + h_t}{2}$, where h_0 is the initial water table height and h_t is the varying water table height at time *t*, at the end of which *D* is approximated (Marino 1974).

Analytical solution for transient water table rise in sloping aquifer

Applying Werner's (1946) transformation $z = h^2 - h_0^2$, and replacing h associated with the equation by *D*, average depth of flow equations can be written as

$$\frac{\partial^2 z}{\partial x^2} - \frac{tan\,\beta}{K\,D\,cos^2\beta}\frac{\partial z}{\partial x} + Q = \frac{S}{K\,D\,cos^2\beta}\frac{\partial z}{\partial t}$$

$$\frac{\partial^2 z}{\partial x^2} - \alpha\frac{\partial z}{\partial x} + Q = \frac{1}{\gamma}\frac{\partial z}{\partial t} \tag{11}$$

where $\alpha = \frac{tan\,\beta}{K\,D\,cos^2\beta}$, $\frac{1}{\gamma} = \frac{S}{K\,D\,cos^2\beta}$ and $Q = \frac{R(x,t)}{K\,D\,cos^2\beta}$

The corresponding initial and boundary conditions become

$$z(x, t = 0) = 0\,;\ 0 < x < L \tag{12}$$

$$z(0,\ t\,) = z_1;\ t > 0 \tag{13}$$

$$z(L,\ t\,) = z_2;\ t > 0 \tag{14}$$

The solution to this boundary value problem is given as:

$$z(x,t) = \frac{1}{\alpha(e^{L\,\alpha}-1)}(\alpha\,(8e^{L\,\alpha} - 8)\left(\sum_{n=0}^{\infty}(-\frac{1}{(e^{L\,\alpha}-1)(L^2\alpha^2+4\pi^2n^2)}(4(-sinh\left(\frac{(L^2\alpha^2+4\pi^2n^2)\gamma t}{4L^2}\right) + cosh\left(\frac{(L^2\alpha^2+4\pi^2n^2)\gamma t}{4L^2}\right)\right)\pi n\left(cos\,h\left(\frac{x\alpha}{2}\right)+sinh\left(\frac{x\alpha}{2}\right)\right)((-1)^n e^{\frac{-\alpha L}{2}}((\frac{\alpha^2 z_2}{4} + Q)L^2+\pi^2n^2z_2) - (-1)^n e^{\frac{\alpha L}{2}}((\frac{\alpha^2 z_2}{4}+Q)L^2+\pi^2n^2z_2) + (e^{L\,\alpha}-1)((\frac{\alpha^2 z_1}{4} + Q)L^2+\pi^2n^2z_1)sin(\frac{n\,\pi\,x}{L})))) + (Qx\ +\alpha z_1)e^{L\,\alpha} + ((-z_1+z_2)\,\alpha - QL)\,e^{x\alpha} - z_2\,\alpha\ +\ Q(L-x)) \qquad (15)$$

In a specific case of an analytical solution, if the impermeable barrier is horizontal *z(x, t)* can be obtained from eqn. (11) by substituting $\beta = 0$ that implies $\alpha = \frac{tan\,\beta}{K\,D\,cos^2\beta} = 0\ hence\ \ tan\,\beta\ = 0$. So, above equation takes the form,

$$z(x,t) = \left(\sum_{n=0}^{\infty} -\frac{2}{n^3\pi^3}(-sinh\left(\frac{\pi^2n^2\gamma t}{L^2}\right) + cosh\left(\frac{\pi^2n^2\gamma t}{L^2}\right))(\,(QL^2+\pi^2n^2z_1)\sin(\frac{\pi xn}{L}))\right) \qquad (16)$$

$$h^2(x,t) - {h_0}^2(x,t) = \left(\sum_{n=0}^{\infty} -\frac{2}{n^3\pi^3}(-sinh\left(\frac{\pi^2n^2K\,D\,cos^2\beta t}{S\,L^2}\right) + cosh\left(\frac{\pi^2n^2K\,D\,cos^2\beta t}{S\,L^2}\right))(\,(QL^2+\pi^2n^2z_1)\sin(\frac{\pi xn}{L}))\right)$$

So,

$$h^2(x,t) = {h_0}^2(x,t) + \left(\sum_{n=0}^{\infty} -\frac{2}{n^3\pi^3}(-sinh\left(\frac{\pi^2n^2K\,D\,cos^2\beta t}{S\,L^2}\right) + cosh\left(\frac{\pi^2n^2K\,D\,cos^2\beta t}{S\,L^2}\right))(\,(QL^2+\pi^2n^2z_1)\sin(\frac{\pi xn}{L}))\right) \qquad (17)$$

Steady State Solution

Steady state solution for eqn. (15) can be obtained by substituting $\frac{\partial z}{\partial t} = 0$ on the right side. The solution of the equation for the boundary conditions in eqns. (13) and (14) is obtained as,

$$z = \frac{(Qx + \alpha z_1)e^{\alpha L} + ((-z_1 + z_2)\alpha - QL)e^{\alpha x} - z_2\alpha + Q(L-x)}{(e^{L\alpha} - 1)\alpha} \tag{18}$$

Thus eqn. (18) gives the steady state water table profile in a sloping aquifer with canals at two ends.

For horizontal aquifer, a steady state solution can be obtained from eqn. (18) after expanding the exponential terms $\boldsymbol{e^{\alpha L}}$ and $\boldsymbol{e^{\alpha x}}$ in the series form. After neglecting the greater than 2nd order term and putting $\boldsymbol{\beta = 0}$, above eqn. reduces to

$$z = z_1\left(1 - \frac{x}{L}\right) + z_2\frac{x}{L} \tag{19}$$

which is similar to the solution obtained by Mustafa (1987) for prediction of the steady-state water table profile in a horizontal aquifer.

Numerical solution of the non-linear flow equation

In order to examine the validity of linearization technique, analytic solution of the linearized Boussinesq equation in compared with the numerical solution of the corresponding non-linear equation. For this purpose, a fully explicit Mac Cormack finitedifference computational scheme (Mac Cormack 1969) is employed. To use this scheme, eqn. (5) is written as

$$C_1\frac{\partial}{\partial x}\left(h\,\frac{\partial h}{\partial x}\right) + C_2\left(\frac{\partial h}{\partial x}\right) + \frac{R(x,t)}{S} = \frac{\partial h}{\partial t} \tag{20}$$

where $C_1 = \frac{K cos^2\beta}{S}$ and $C_2 = \frac{K\,sin\,2\beta}{2\,S}$

R. K. Bansal (2015), gave the corrected value of $h_{k,n+1}$ which is given as an arithmetic mean of $h*_{k,n+1}$and $h**_{k,\ n+1}$ i.e.,

$$h_{k,\ n+1} = \frac{1}{2}[h_{k,\ n} + h*_{k,\ n+1} + C_2\frac{\Delta t}{\Delta x}\left(h*_{k,\ n+1} - h*_{k-1,\ n+1}\right) + C_1\frac{\Delta t}{(\Delta x)^2}\{h*_{k,\ n+1}\left(h*_{k+1,\ n+1} - h*_{k,\ n+1}\right) - h*_{k-1,\ n+1}\left(h*_{k,\ n+1} - h*_{k-1,\ n+1}\right)\}] + \frac{R(x,t)\Delta t}{S} \tag{21}$$

Mac Cormack scheme is conditionally stable. It is not possible to obtain a simple stability criterion for Mac Cormack scheme (Tannehill et al. 1975). Numerical experiments are carried out with various values of Δt and Δx, and it is observed that the stability of the numerical solution requires

$\Delta t \leq 0.06\frac{(\Delta t)^2}{C_1}$; $\Delta t \leq 0.9\frac{\Delta x}{C_2}$.

Analytical Solution for Mustafa

Mustafa (1987) proposed an analytical solution to describe spatial and temporal variation of the water table in a horizontal aquifer considering seepage from two canals and a constant recharge from the land surface. The solution by Mustafa is given as:

$$z(x,t) = Q\,\gamma\,t\{1 - [\sum_{n=0}^{\infty}((-1)^n 4\, i^2 erfc(\frac{nL+x}{\sqrt{4\,\gamma\,t}}) + \sum_{n=0}^{\infty}((-1)^n 4\, i^2 erfc(\frac{nL+L-x}{\sqrt{4\,\gamma\,t}})]\} + z_1 \sum_{n=0}^{\infty}(erfc(\frac{2nL+x}{\sqrt{4\,\gamma\,t}}) - erfc(\frac{2nL+2L-x}{\sqrt{4\,\gamma\,t}})) + z_2 \sum_{n=0}^{\infty}(erfc(\frac{2nL+L-x}{\sqrt{4\,\gamma\,t}}) - erfc(\frac{2nL+L+x}{\sqrt{4\,\gamma\,t}})) \quad (22)$$

Result and Discussion

To demonstrate the combined effects of bed slopes and time varying localized charges, recharge variation is considered here to test the validity of the model and demonstrate its application in the prediction of water table fluctuation due to time varying recharge. The numerical values of various parameters are $L=1{,}000\ m$, $z_0 = 0$, $z_1 = (10.995)^2$, $z_2 = (10)^2$, $\frac{KD}{S} = 12{,}000\,\frac{m^2}{d}$, $D = \sqrt{\frac{(z_1+z_2)}{2}} = 5.244\, m$ and $S = 0.30$. Numerical values of the slopes and intercepts for these linear elements are obtained by fitting elements of different segments of the exponential curves as shown below in fig 2(a, b and c).

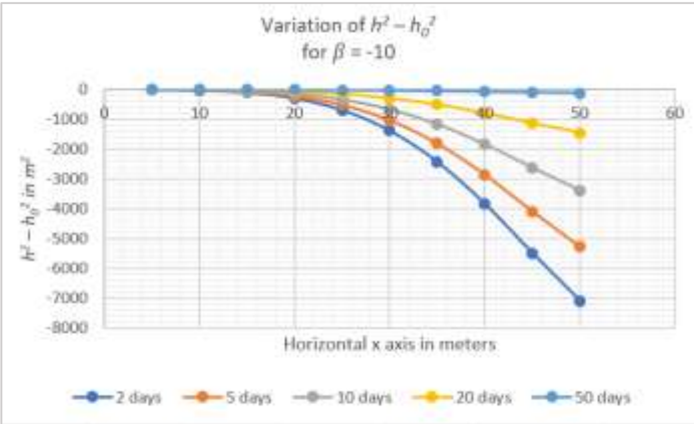

Fig 2(a) Illustrates the comparison of the water table profile for $\beta = -10^o$ $for\ Q = 0$

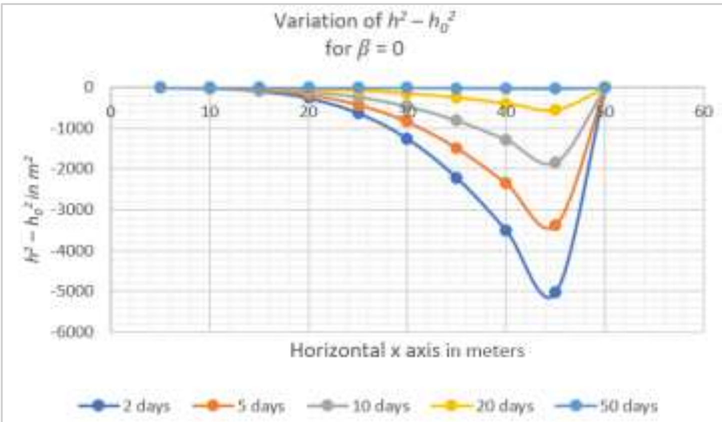

Fig 2(b) Illustrates the comparison of the water table profile for $\beta = 0^o$ $for\ Q = 0$

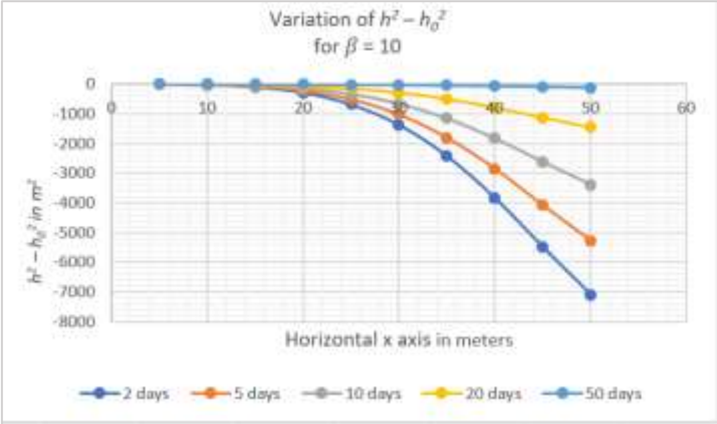

Fig 2(c) Illustrates the comparison of the water table profile for $\beta = 10^o \; for \; Q = 0$

It is observed that for horizontal aquifer the dynamic profile of mound evolved symmetrically about the center of the basins.

Three different values of the sloping angle are considered, namely, $\beta = 10^o, 0^o \; and \; -10^o$.Analytic values of water head are computed using eqn. (17) in which the mean saturated depth D of the aquifer is successively approximated using an iterative formula $D = \frac{(h_0 + h_t)}{2}$, where h_t is the water table height at time t at the end of which D is approximated. Numerical experiments of various values of aquifer parameters indicate that the infinite sequence in the right-hand side of eqn. (17) converges very fast to a final value. We have taken 50 terms of the sequence to represent the sum of the whole infinite series.

Performance of analytic solution is tested at 200 equally spaced grid points of length 1m for t = 2, 5, 10, 20 and 50 days as shown in fig 3(a, b, c and d). It is observed that the two solutions are in excellent agreement during the initial stages (t = 2, 5, 10, 20 days). As time increases e. g. 50 days, analytic solutions marginally underestimate the numerical solutions. In numerical solution the dimensionless time increment $\Delta t = 0.00001$and $\Delta x = 0.01$.

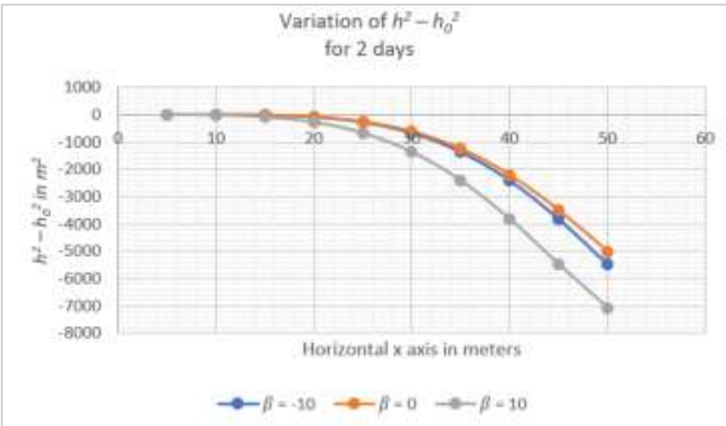

Fig 3(a) Illustrates the comparison of the water table profile for t = 2 days

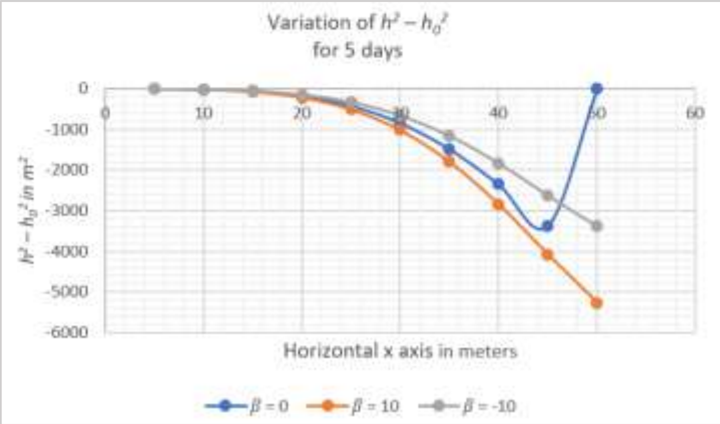

Fig 3(b) Illustrates the comparison of the water table profile for t = 5 days

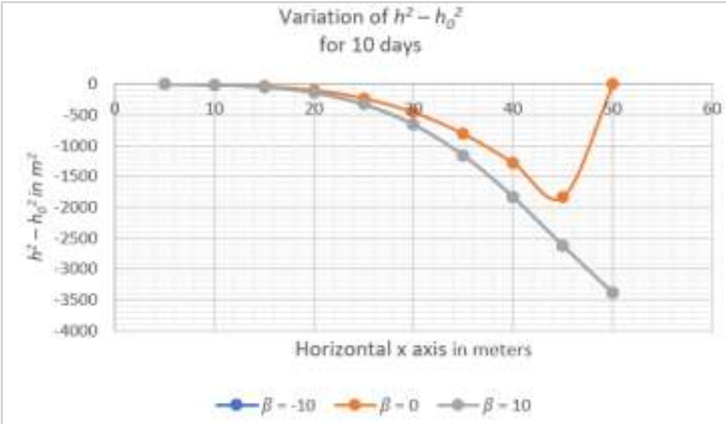

Fig 3(c) Illustrates the comparison of the water table profile for t = 10 days

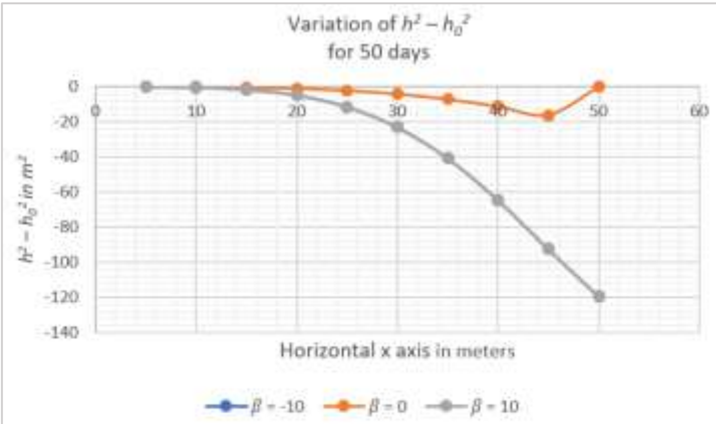

Fig 3(d) Illustrates the comparison of the water table profile for t = 50 days

Distributions of water head for t = 2, 5, 10 and 50 days in $10^{o}, 0^{o}$ and -10^{o}sloping aquifers are plotted against the spatial coordinate x in Fig 2(a, b and c), Fig 3(a, b, c and d), Fig 4(a, b and c) and Fig 5(a, b, c and d). The profiles of free surface develop in the form of groundwater mound which expands with time at varying rates. The water table height $h^2(x,t) - {h_0}^2(x,t)$ at x = 5, 10, 15, 20….50. from the origin at time t = 2, 5, 10and 50is calculated for *Q*= *0* (Fig 2(a, b and c) and Fig 3(a, b, c and d)and *Q* = *2* Fig 4(a, b and c) and Fig 5(a, b, c and d).

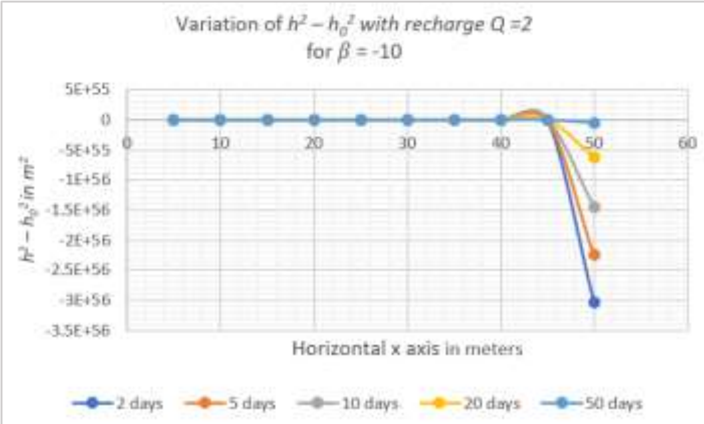

Fig 4(a) Illustrates the comparison of the water table profile for $\boldsymbol{\beta = -10^o\ for\ Q = 2}$

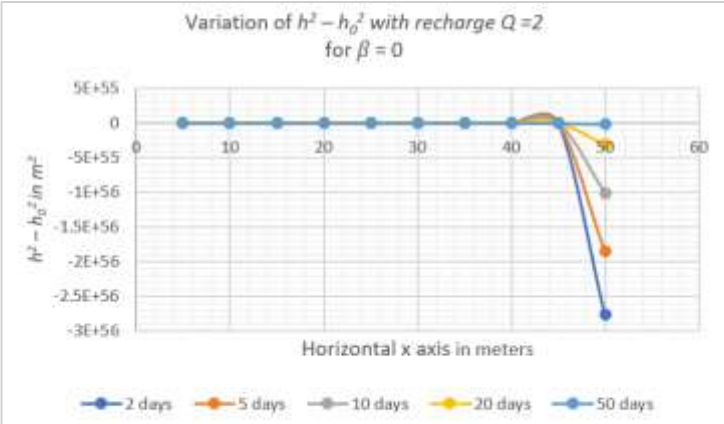

Fig 4(b) Illustrates the comparison of the water table profile for $\boldsymbol{\beta = 0^o\ for\ Q = 2}$

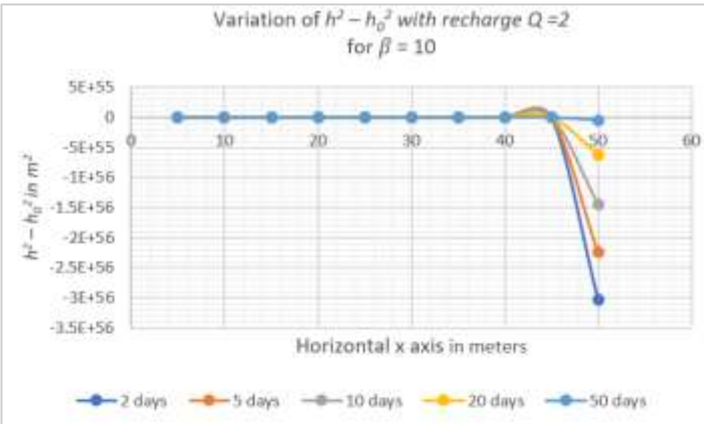

Fig 4(c) Illustrates the comparison of the water table profile for $\boldsymbol{\beta = 10^o\ for\ Q = 2}$

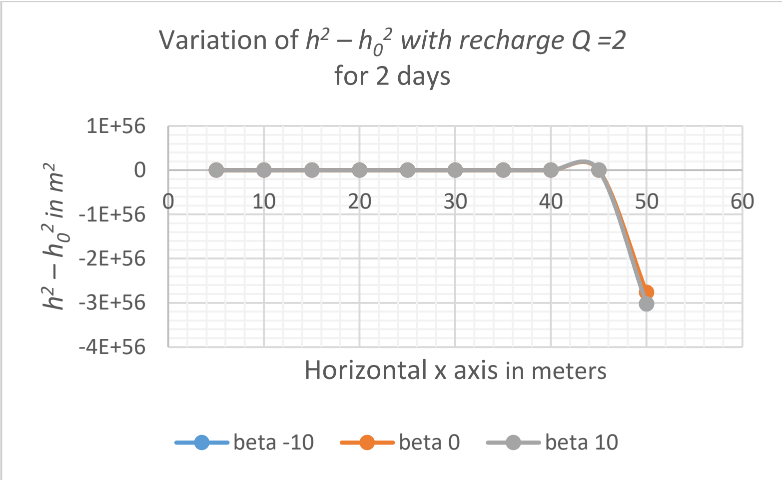

Fig 5(a) Illustrates the comparison of the water table profile for t = 2 days $for\ Q = 2$

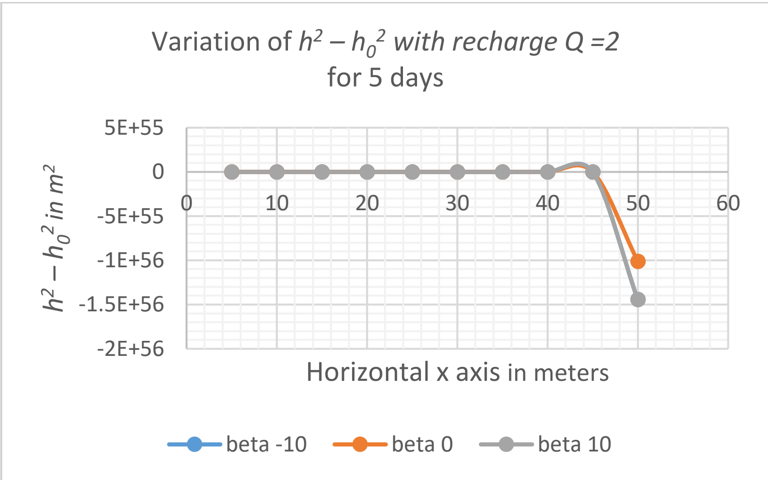

Fig 5(b) Illustrates the comparison of the water table profile for t = 5 days $for\ Q = 2$

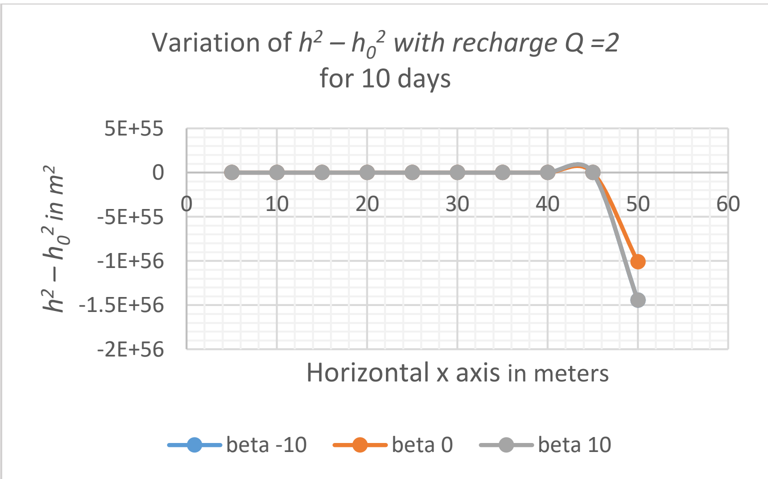

Fig 5(c) Illustrates the comparison of the water table profile for t = 10 days $for\ Q = 2$

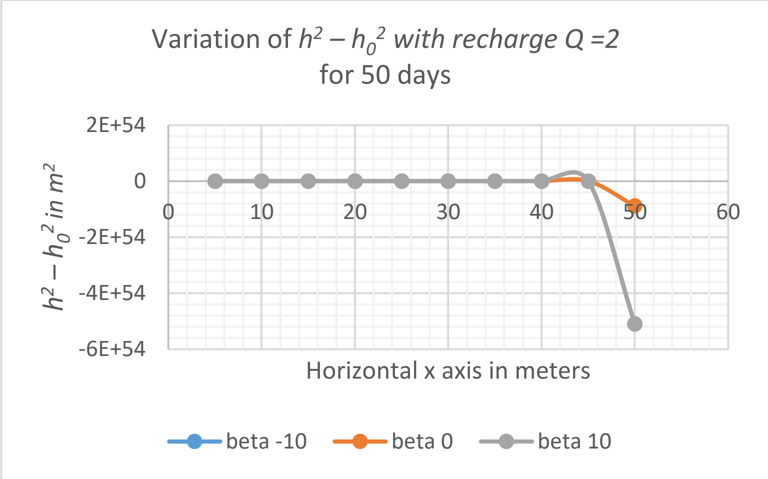

Fig 5(d) Illustrates the comparison of the water table profile for t = 50 days $for\ Q = 2$

In sloping aquifers, the profile remains symmetric during initial stages, however for large values of time, the phreatic surface becomes asymmetric and drifts along the bed slope justifying the extended Dupuit- Forchheimer assumptions.

Initial discharge rate in the drains can be obtained by setting $t = 0$. With initial water head height $h(0,t) = h_1$and $h(L,t) = h_2$ for horizontal aquifer there is no exchange of water at the initial stage.

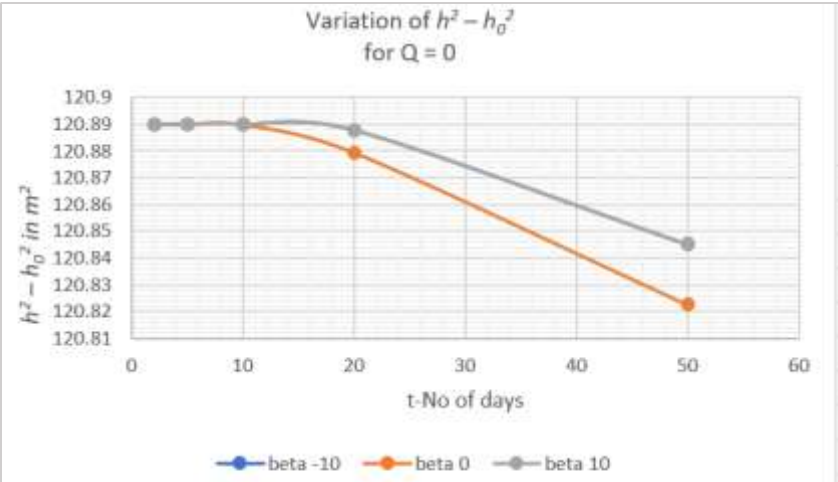

Fig 6(a) Illustrates the comparison of the water table profile for t = 2, 5, 10, 20 and 50 days $for\ Q = 0\ \ for$ analytical solution for Mustafa

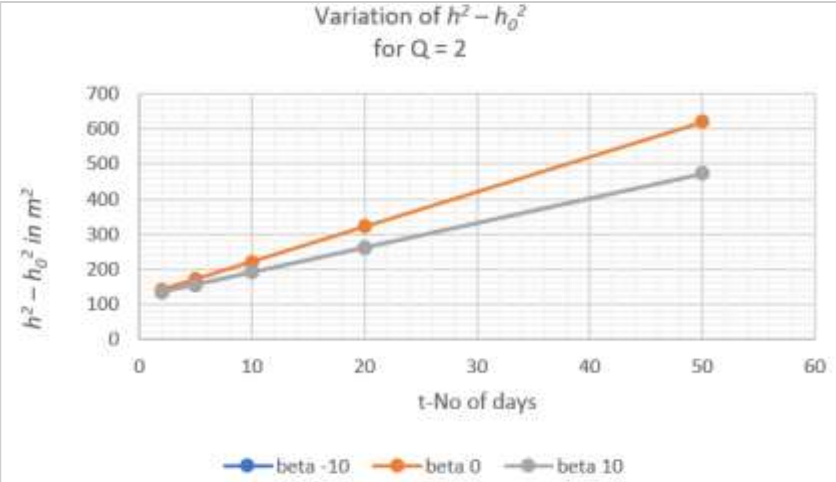

Fig 6(a) Illustrates the comparison of the water table profile for t = 2, 5, 10, 20 and 50 days $for\ Q = 2\ \ for$ analytical solution for Mustafa.

Conclusion

A new analytic solution of the Boussinesq equation was used to characterize water table rise between two canals in a sloping homogeneous, isotropic, unconfined aquifer receiving constant recharge. The new mathematical model presented here is based on Dupuit-Forchheimer assumptions in which spatial coordinates of recharge basins are treated as additional parameters. Unlike existing studies restricted to Laplace transform which leads to solutions in terms of error function with complex argument, the present method includes the substitution with eigen value-eigen function method which produces the analytic solution for water head and flow rate in the form of fast converging infinite sequence. A computational example is considered to simulate the combined effects of bed slopes (upward, zero and downward). The study takes into consideration the following facts:

- Analytical solutions of the linearized Boussinesq equation and the fully implicit finite difference numerical solution of the nonlinear Boussinesq equation were obtained to describe the transient water table rise between two canals.
- The water table rise between two canals computed by the proposed analytical solution for the horizontal aquifer with or without recharge from land surface was closely compared with the values obtained from Mustafa's(1987) solution.
- With constant recharge the water table noticed a rise as compared to the case of no recharge. With increase in the slope of impermeable barrier, the minimum water table rise was found to shift towards the lower canal in the direction of the down slope.
- The comparison of water tables computed from analytical and fully implicit finite difference numerical solutions shows that the difference in the values of water tables in the middle region decreases with the increase in time.

This indicates that linearization of the governing equations results in over-estimation of the water table which is larger initially but decreases with time.

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