Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue6, July , 7094- 7101

Research Article

Scheduling a Semi-Round Robin Tournament using Independent Edge Domination

H.M. Sulthan Ahthar[1], S. Syed Ali Fathima[2], K. Alli[3]

Abstract

This paper discuss about scheduling a semi round robin tournament without equal number of matches and with equal number of matches. In this paper, we introduce a method for scheduling a semi round- robin tournament with the help of independent edge domination.

Keywords: Round-Robin, Domination, independent edge domination.

1. Introduction

Domination in graphs is the one of the most engaging area which is used by many researchers for practical problem. Sports scheduling is one of the important practical problem because many sports schedule involves organizers many, peoples time and lot more. So that, many researchers involve in scheduling sports. Here we discuss about the semi round robin tournament. A **round-robin** is a sports competition in which each player or team plays against every other player or team. The **semi-round robin** is one of the round robin tournaments where uneven divisions are scheduled. This paper discuss about scheduling a semi round robin tournament without equal number of matches and with equal number of matches. In this paper, we introduce a method for scheduling a semi round- robin tournament with the help of independent edge domination.

2. Definition

A **graph** is an ordered pair G = (V, E) where V is the vertex set and E is the edge set. A graph with one component is called connected graph and a graph with more than one component is a **disconnected graph**. A graph may contain loops and multiple edges are called **general graph**. Graphs with no loops or multiple edges are called **simple graphs.** A **complete graph** is a graph in which each pair of graph vertices is connected by an edge. A complete graph with n vertices denoted by 'K_n' is a graph with n vertices in which each vertex is connected to each of the others. The **degree** of a vertex V in a graph G denoted by d(v) is the number of edges incident with v. A graph in which each vertex has the same degree is a **regular graph.** If each vertex has degree r, the graph is **regular of degree r** or **r-regular.** Of special importance are the **cubic graphs,** which are regular of degree 3. A set D of vertices in a graph G = (V, E) is called a **dominating set** of G if every vertex in V – D is adjacent to some vertex in D. A set F of edges in

[1]Research Scholar (Reg.No. 17211072091014), Sadakathullah Appa College affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627012, Tamil Nadu (India). sulthanahthar@gmail.com
[2]Department of Mathematics, Sadakathullah Appa College affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627012, Tamil Nadu (India). syedalifathima2014@gmail.com
[3]Department of Mathematics, The M.D.T. Hindu College affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627012, Tamil Nadu (India). allimdt@gmail.com

a graph G = (V,E) is called an **edge dominating set** of G if every edge in E – F is adjacent to at least one edge in F and is an **independent edge dominating set** if edges are independent. A edge dominating set is called minimal edge dominating set if no proper subset of S is a edge dominating set. A minimal edge dominating set of minimum cardinality is minimum edge dominating set. Every minimum edge dominating set is a minimal edge dominating set, but the converse does not necessarily hold. A minimal edge dominating set of maximum cardinality is **maximal edge dominating set**. An independent edge dominating set of maximal cardinality is **maximal independent edge dominating set**.

3. Semi Round-Robin Tournament

3.1 Scheduling semi round robin tournaments

The semi-round robin is one of the round robin tournaments that solves the problem of uneven divisions. For example, in a cricket tournament with seven entries divided into two divisions, one division would have three entries, and the other would have four. This means that the division with four entries requires each team to compete in one more game than in the division with three entries.

Suppose 18 teams are divided into 4 Groups where 3 teams in group A, 4 teams in group B, 5 teams in group C and 6 teams in group D. In group A, there are 3 teams; each team (1, 2 and 3) will play 2 matches—one match against each of the other 2 teams. In group B, there are 4 teams; each team (4, 5, 6 and 7) will play 3 matches—one match against each of the other 3 teams. In group C, there are 5 teams; each team (8, 9, 10, 11 and 12) will play 4 matches—one match against each of the other 4 teams. In group D, there are 6 teams; each team (13, 14, 15, 16, 17 and 18) will play 5 matches—one match against each of the other 5 teams. Every group winner move to knock out matches. In this case every group has different number of matches; it is unfair to other teams who play more matches. To make it fair we should make every team to play equal matches or nearly equal matches.
On this process of making equal matches or nearly equal, some teams play same team twice or some team play with less matches than other team in that group or some group team will not play with all teams in that group. So it makes confusion to schedule the matches. To make it easy we use iteratively eliminating maximal independent edge domination method.

3.2 Scheduling Semi round robin tournaments without equal matches using iteratively eliminating maximal independent edge domination method.

Scheduling Semi round robin tournaments without equal number of matches and Scheduling Semi round robin tournaments with equal number of matches have same algorithm as shown below

Algorithm

Step 1 : Construct a disconnected graph whose vertices are teams and two vertices are joined by an edge if there is a match between them.

Step 2 : Find the maximal independent edge domination set S_1 in the graph.

Step 3 : Consider the maximal independent edge domination set S_1 as Round 1.

Step 4 : Now eliminate the maximal independent edge domination set S_1 edges from the graph.

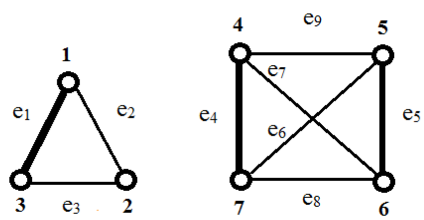
Step 5 : Consider the resulted sub graph as a new graph.
Step 6 : Find the maximal independent edge domination set S_2 in the new graph.
Step 7 : Consider the maximal independent edge domination set S_2 as next Round 2.
Step 8 : Now eliminate the maximal independent edge domination set S_2 edges from the graph
Step 8 : Repeat this process until no edges remains.

3.2.1 Scheduling semi round robin method without equal number of matches

For example : In a cricket tournament there are seven teams which is divided into two divisions that is three teams in 1st division and four teams in 2nd Division. Each division play round robin which means that, Teams in 2nd Division play one more match when it compare to 1st division teams. Here we schedule the tournament

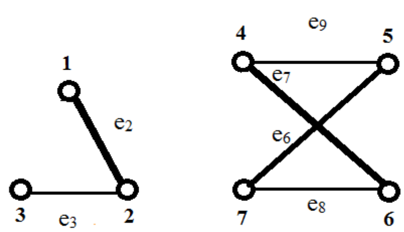
Consider 1, 2, 3, 4, 5, 6 and 7 are teams, divided into two divisions. Team 1, 2 and 3 are in 1st Division **and** 4, 5, 6 and 7 are in 2nd Division.

First, we construct a graph G whose vertices are teams and there is an edge between them if they have matches between teams. In 1st division there are 3 teams where each team plays against every other team in that division. And in 2nd Division there are 4 teams where each team plays against every other team in that division. So that there is k3 and k4 in graph G. Here e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 and e_9 are edges represent matches between



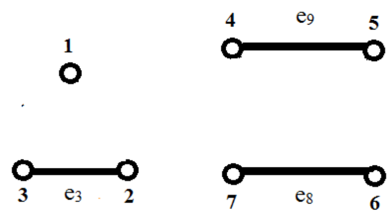
G

Now the maximal independent edge domination is founded in this graph G. Let the maximal independent edge dominating set S_1 be $\{e_1, e_4, e_5\}$. The maximal independent edge dominating set vertex is considered as Round 1. The maximal independent edge dominating set S_1 vertex is eliminated from the graph.



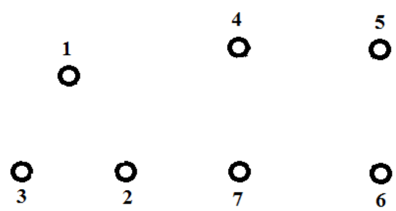
H_1

This process is repeated in such way that the maximal independent edge dominating set $S_2 = \{e_2, e_6, e_7\}$ is found in the resulting Sub graph H_1 which is $G - S_1$. This maximal independent edge dominating set containing edges e_2, e_6 and e_7 is considered as round 2. The edges e_2, e_6 and e_7 of second maximal independent edge domination set are eliminated from the graph H_1.



H2

This process is again repeated. Let the third **maximal independent edge dominating** set S_3= $\{e_3, e_8, e_9\}$ which is considered as round 3. The edges of **maximal independent edge domination** set are eliminated from the resulting sub graph H_2.



H3

The resultant sub graph H_3 is null graph so the process is terminated.

This process shows schedule:

Rounds	set	Edges	Edge representing Matches			Number of matches
1	S_1	e_1, e_4, e_5	1-3	4-7	5-6	3
2	S_2	e_2, e_6, e_7	1-2	4-6	5-7	3
3	S_3	e_3, e_8, e_9	2-3	4-5	6-7	3
Total						9

3.2.2 Scheduling semi round robin method with equal number of matches

To make equal number of matches for every team in uneven distribution groups results in increase in matches in some group or decrease in matches in some group or both can happen in various groups. For example: Suppose in a Team Penalty Kick Soccer Shoot-Out Tournament with 18 entries divided into four uneven divisions because of number of entries are different in every area. In every area topper is selected for knock out. So we need to have equal number of matches for every team. 1st division would have three entries, 2nd division would have four entries, 3rd division would have five entries, and 4th division would have six entries.

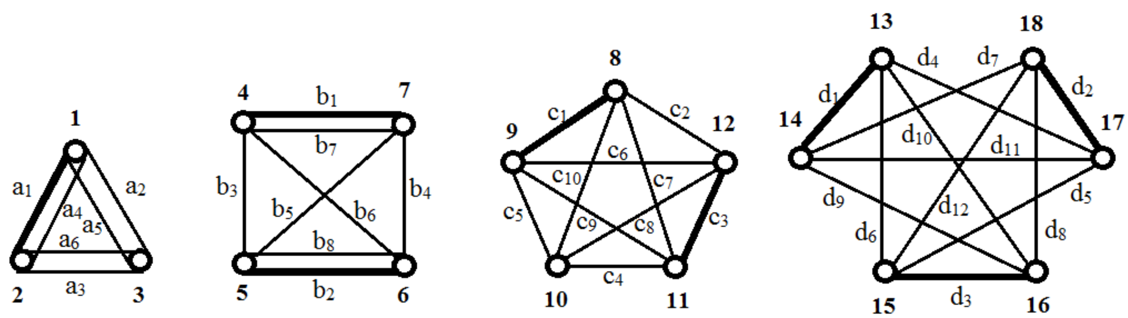
Here every team represents as vertex. Every match represents as edges. Number of Matches can be represented as degree in graph. To get equal number of matches in every vertex, degrees should be same.

	3 vertices	4 vertices	5 vertices	6 vertices
2- regular graph	✓	✓	✓	✓

3- regular graph	×	✓	×	✓
4- regular graph	✓	✓	✓	✓
5- regular graph	×	✓	×	✓
6- regular graph	✓	✓	✓	✓

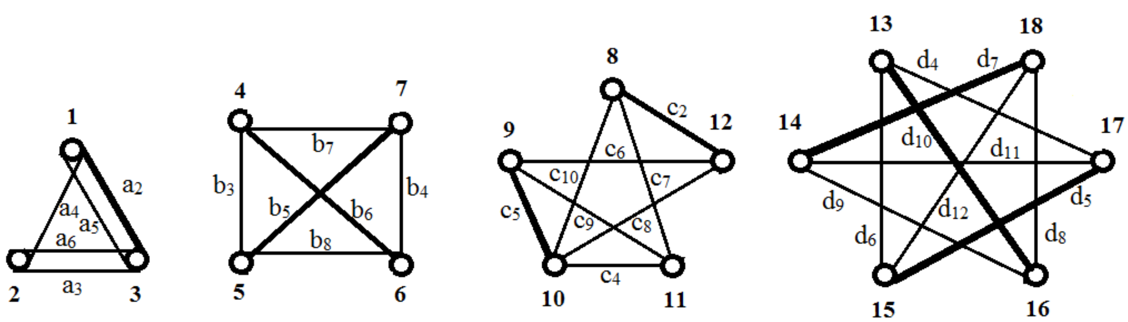
In simple words, odd number of vertices cannot have odd number – regular graph. So here we consider 4- regular graph in every division. 1st division have 6 matches, every team play with other team twice in that division , 2nd division have 8 matches, every team have one additional match in that division, 3rd division have 10 matches, every team play with other teams once in that division, and 4th division have 12 matches, every team play only four matches in that division. Every team in all division play four matches.

Consider a disconnected 4- regular graph whose vertices are 1 to 18 which represents teams and whose edges are a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11} and d_{12} representing matches between their end vertices.



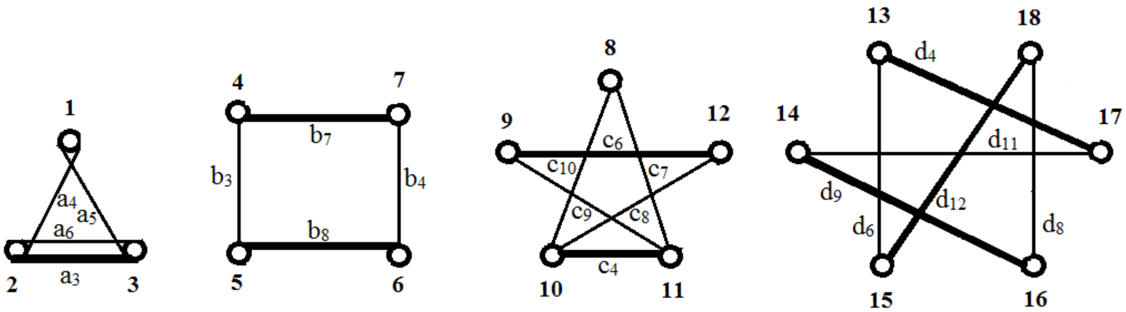
G

By using the algorithm, Now the maximal independent edge domination is founded in this graph G. Let the maximal independent edge domination S_1 be {a_1, b_1, b_2, c_1, c_3, d_1, d_2, d_3}. The maximal independent edge dominating S_1 set vertex is considered as Round 1. The maximal independent edge dominating S_1 set vertex is eliminated from the graph.



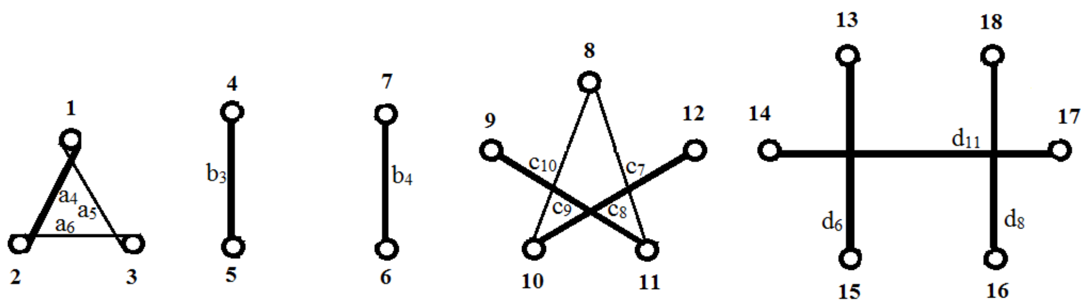
H_1

This process is repeated in such way that the maximal independent edge domination set S_2 is found as {a_2, b_5, b_6, c_2, c_5, d_5, d_7, d_{10}} in the resulting Sub graph H_1 which is $G - S_1$. This maximal independent edge domination S_2 set containing edges a_2, b_5, b_6, c_2, c_5, d_5, d_7 and d_{10} is considered as round 2. The edges a_2, b_5, b_6, c_2, c_5, d_5, d_7 and d_{10} of second maximal independent edge domination set S_2 are eliminated from the graph H_1.



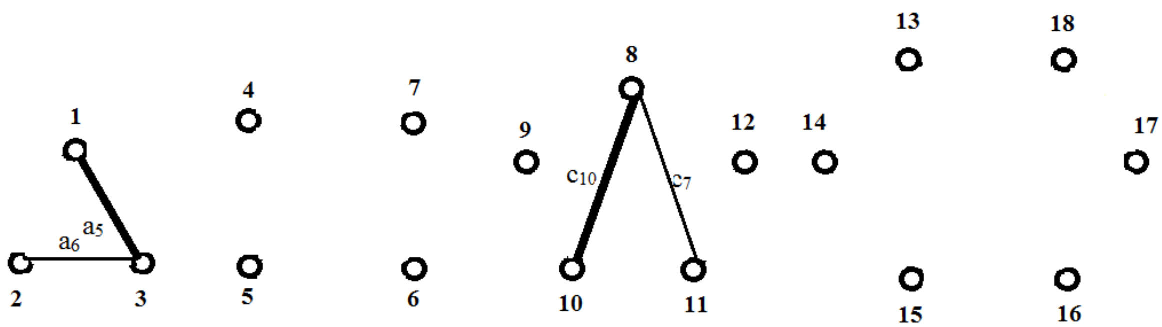
H_2

This process is again repeated. Let the third maximal independent edge domination set $S_3 = \{a_3, b_7, b_8, c_4, c_6, d_4, d_9, d_{12}\}$ which is considered as round 3. The edges of maximal independent edge domination set S_3 are eliminated from the resulting sub graph H_2.



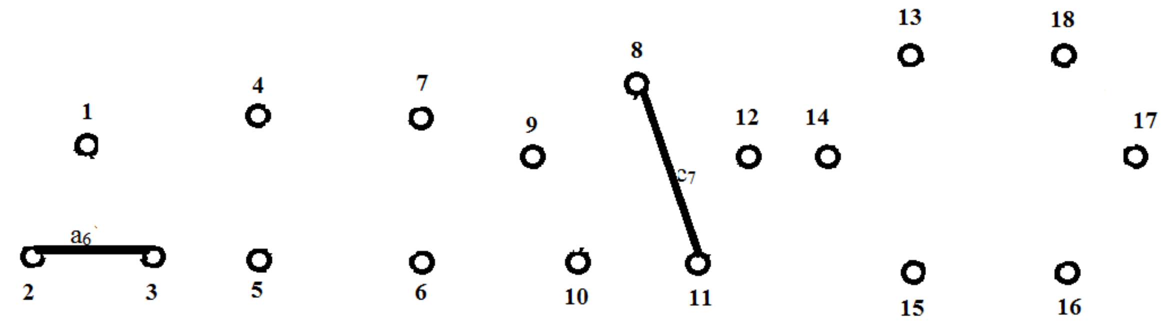
H_3

This process is again repeated. Let the fourth maximal independent edge domination set $S_4 = \{a_4, b_3, b_4, c_8, c_9, d_6, d_8, d_{11}\}$ which is considered as round 4. The edges of maximal independent edge domination set S_4 are eliminated from the resulting sub graph H_3.



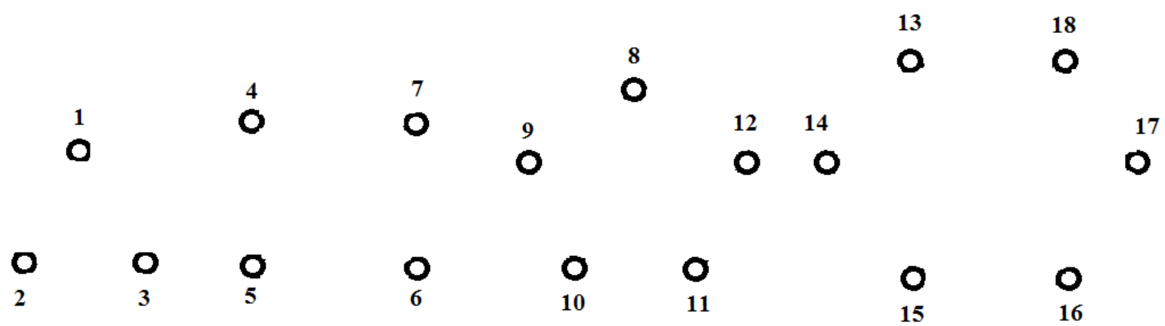
H_4

This process is again repeated. Let the fifth maximal independent edge domination set $S_5 = \{a_5, c_{10}\}$ which is considered as round 5. The edges of maximal independent edge domination set S_5 are eliminated from the resulting sub graph H_4



H_5

This process is again repeated. Let the fifth maximal independent edge domination set S_6 = $\{a_6, c_7\}$ which is considered as round 6. The edges of maximal independent edge domination set S_6 are eliminated from the resulting sub graph H_5



H_6

The resultant sub graph H_3 is null graph so the process is terminated.

Rounds	sets	Edges	Edge representing Matches								Number of matches
1	S_1	a_1, b_1, b_2, c_1, c_3, d_1, d_2, d_3	1-2	4-7	5-6	9-8	11-12	13-14	15-16	17-18	8
2	S_2	a_2, b_5, b_6, c_2, c_5, d_5, d_7, d_{10}	1-3	5-7	4-6	8-12	9-10	15-17	14-18	13-16	8
3	S_3	a_3, b_7, b_8, c_4, c_6, d_4, d_9, d_{12}	2-3	4-7	5-6	10-11	9-12	13-17	14-16	15-18	8
4	S_4	a_4, b_3, b_4, c_8, c_9, d_6, d_8, d_{11}	1-2	4-5	6-7	10-12	9-11	13-15	16-18	14-17	8
5	S_5	a_5, c_{10}	1-3			10-8					2
6	S_6	a_6, c_7	2-3			8- 11					2
			Total								36

Conclusion

This paper discuss about scheduling a semi round robin tournament without equal number of matches and with equal number of matches. This paper deals with a method for scheduling a semi round- robin tournament with the help of independent edge domination. This method makes easy access to schedule the semi round robin with or without equal number of matches. This method also used in other scheduling purpose which has same structure.

Reference

1. Gary Chatrand and Ping Zhag, Introduction to graph theory**,** Tata McGraw-Hill (edition 2006).
2. Richard Hoshino and Ken-ichi Kawarabayashi, Graph Theory and Sports Scheduling, Pg.no:726, Volume 60, number 6, AMS journals (2013).
3. Kulli V.R, Theory of domination in graphs, Vishwa international publication.
4. Teresa W. Haynes, Stephen T.Hedetniemi and Peter J.Slater, Fundamentals of domination in graphs Publication Marcel dekker Inc, New York.
5. Berdeward OK and Deo SD**,** Application of graph theory in scheduling tournament**,** international journal of science engineering and applied science (USEAS) - volume1, issue-5, August (2015)
6. H.M. Sulthan Ahthar, S. Syed Ali Fathima and K. Alli, Scheduling a Round-Robin Tournament using Independent Edge Domination **,** ICCSPAM 19, Pg. no: 115 – 118, AIJRSTEM (2019), IASIR Journal.