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Research Article

j-compactness and j-cocompactness in ditopological texture

spaces

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Abstract

The centre of attention in this paper deals with j-compactness and j-cocompactness in ditopological texture spaces. Also, j-inadequate and j-coinadequate in ditopological texture spaces are discussed.

Keywords: Texture, ditopology, difunction, j-compact, j-cocompact, j-inadequate and j-coinadequate.

1 Introduction

The texture and ditopology texture spaces were placed for first timeby L.M. Brown [1] in point-set setting for the assessment of fuzzy sets. It has been proved convenient as a framework to talk about the complementfree mathematical idea. The ditopological texture space may be considered as a natural combination of texture space, topological space and bitopological space, but ditopology corresponds in a natural way to fuzzy topology. A texture is considered as a generalization of fuzzy lattice. The created to illustrate of ditopology is more general than general topology, fuzzy topology and bitopological concepts to the ditopological texture space. Also concerning textures and ditopologies and some basic notions are recollected and sufficient to present a new idea to the theory and the motivation for its analysis of a subject are acquired from [1,2,3]. In [4], the concept of j-compactness and jcocompactness in ditopological texture spaces, were introduced and also their characterizations are discussed. Also j-inadequate and j-coinadequate in ditopological texture spaces are initiated.

2 Preliminaries

Some basic definitions of textures are listed below.

Definition 2.1. [6] Let S be a set. Then $\psi \subseteq P(S)$ is called a texturing of S and S is

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said to be textured by ψ if

1) (ψ, \subseteq) is a complete lattice containing *S* and ϕ and for any index set *I* and $A_i \in \psi, i \in I$, the meet $\Lambda_{i \in I}$ A_i and the join $\bigvee_{i \in I} A_i$ in ψ are related with the intersection and union in P(S) by the equalities.

$$\Lambda_{i\in I} A_i = \bigcap_{i\in I A_i}$$

for all I, while

$$\bigvee_{i\in I} A_i = \bigcup_{i\in I} A_i$$

for all finite *I*.

2) ψ is completely distributive.

3) ψ separate the point of *S*. That is, given $s_1 \neq s_2$ in *S* we have $L \in \psi$ with $s_1 \in L$, $s_2 \notin L$ If *S* is textured by ψ then (S, ψ) is called a texture space, or simply a texture.

Definition 2.2. [4] A mapping $\sigma: \psi \to \psi$ satisfying $\sigma(\sigma(A)) = A$, $\forall A \in \psi$ and $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A)$, $\forall A, B \in \psi$ is called a complementation on (S, ψ) and (S, ψ, σ) is then said to be a complemented texture. For a texture (S, ψ) most properties are conveniently defined in terms of the sets,

$$p - \text{sets } P_s = \bigcap \{A \in \psi : s \in A\}$$
$$q - \text{sets } Q_s = \bigvee \{A \in \psi : s \notin A\}$$

We recall the following fundamental properties

- 1) For, $A, B \in \psi$, if $A \subseteq B$ then there exists $s \in S$ with $A \not\subseteq Q_s$ and $P_s \not\subseteq B$
- 2) $A = \bigcap \{Q_s \mid P_s \not\subseteq A\}$ for all $A \in \psi$
- 3) $A = \bigvee \{P_s \mid A \not\subseteq Q_s\}$ for all $A \in \psi$

Definition 2.3. [7] A dichotomous topology on a texture (S, ψ) , or ditopology, is a pair (τ, k) of subsets of ψ , where the of open sets τ satisfies

1)
$$S, \emptyset \in \tau$$

2) $G_1, G_2 \in \tau \Longrightarrow G_1 \cap G_2 \in \tau$ and

3) $G_i \in \tau, i \in I \Longrightarrow \bigvee_i G_i \in \tau$

and the set of closed sets k satisfies

1) $S, \emptyset \in k$

2) $K_1, K_2 \in k \Longrightarrow K_1 \bigcup K_2 \in k$ and

3) $K_i \in k, i \in I \Longrightarrow \bigcap K_i \in k$ and

Hence a ditoplogy is essentially a topology for which there is no a priori relation between the open and closed sets.

For $A \in \psi$ define the closure *clA* and the interior int A of A under (τ, k) by the equalities.

$$clA = \bigcap \{ K \in k : A \subset K \}$$

and

$$int A = \bigvee \{ G \in \tau : G \subseteq A \}$$

We refer to τ as the topology and k as the cotopology of (τ, k) . If (τ, k) is a ditopology on a complemented texture (S, ψ, σ) , then we say that (τ, k) is complemented if the equality $k = \sigma cl(\tau)$ is satisfied. In this study, a complemented ditoplogical texture space is denoted by $(S, \psi, \tau, k, \sigma)$. In this case we have $\sigma(clA) = int \sigma(A)$. We denote by $O(S, \psi, \tau, k)$ or O(S), the set of open sets in ψ . Likewise $C(S, \psi, \tau, k)$ or C(S) will denote the set of closed sets.

Let (S_1, ψ_1) and (S_2, ψ_2) be textures. In the following definition we consider the product texture $P(S_1) \otimes \psi_2$, and denote by $\overline{P}_{s,t}, \overline{Q}_{s,t}$, respectively the *p*-sets and *q*-sets for the product texture $(S_1 X S_2, P(S_1) \otimes \psi_2)$.

Definition 2.4. [7] Let (S_1, ψ_1) and or (S_2, ψ_2) be textures. Then

(1) $r \in P(S_1) \otimes \psi_2$ is called a relation from (S_1, ψ_1) and or (S_2, ψ_2) if it satisfies.

$$R_{1}: r \not\subseteq \overline{Q}_{s,t}, P_{s'} \not\subseteq Q_{s} \Rightarrow r \not\subseteq \overline{Q}_{s',t}$$

$$R_{2}: r \subseteq \overline{Q}_{s,t} \Rightarrow \exists s' \in S_{1} \text{ such that } P_{s} \not\subseteq \overline{Q}_{s'} \text{ and } r \not\subseteq \overline{Q}_{s',t}$$

(2) $R \in P(S_1) \otimes \psi_2$ is called a correlation from (S_1, ψ_1) and (S_2, ψ_2) if it satisfies.

$$CR_1: \overline{P}_{s,t} \nsubseteq R, P_s \nsubseteq Q_{s'} \Longrightarrow \overline{P}_{s',t} \nsubseteq R.$$

$$CR_2: \overline{P}_{s,t} \not\subseteq R \Longrightarrow \exists s' \in S_1$$
 such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{s',t} \not\subseteq R$.

(3) A pair (r, R) where r is a relation and R a correlation from (S_1, ψ_1) to (S_2, ψ_2) is called a direlation from (S_1, ψ_1) to (S_2, ψ_2) . One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.

Definition 2.5. [4] Let (f, F) be a direlation from (S_1, ψ_1) to (S_2, ψ_2) . Then (f, F) is called a difunction from (S_1, ψ_1) to (S_2, ψ_2) if it satisfies the following two conditions:

$$DF_1$$
: For $s, s' \in S_1, P_s \not\subseteq Q_{s'} \Rightarrow \exists t \in S_2$ such that $f \not\subseteq \overline{Q}_{s,t}$ and $\overline{P}_{s',t} \not\subseteq F$.
 DF_2 : For $t, t' \in S_2$ and $s \in S_1, f \not\subseteq \overline{Q}_{s,t}$ and $\overline{P}_{s,t'} \not\subseteq F \Rightarrow P_t' \not\subseteq Q_t$.

Definition 2.6. [4] Let $(f, F): (S_1, \psi_1) \rightarrow (S_2, \psi_2)$ be a difunction.

(1) For
$$A \in \psi_1$$
, the image $f \stackrel{\rightarrow}{} A$ and the co-image $F \stackrel{\rightarrow}{} A$ are defined by,
 $f \stackrel{\rightarrow}{} A = \bigcap \{Q_t : \forall s, \ f \not\subseteq \overline{Q}_{s,t} \Rightarrow A \subseteq Q_s\}$
 $F \stackrel{\rightarrow}{} A = \bigvee \{P_t : \forall s, \ \overline{P}_{s,t} \not\subseteq F \Rightarrow P_s \subseteq A\}.$

(2) For $B \in \psi_2$, the inverse image $f^{\leftarrow}B$ and the inverse co-image $F^{\leftarrow}B$ are defined by,

$$f^{\leftarrow}B = \bigvee \{P_s : \forall t, \ f \not\subseteq \overline{Q}_{s,t} \Longrightarrow P_t \subseteq B\}$$
$$F^{\leftarrow}B = \bigcap \{Q_s : \forall t, \ \overline{P}_{s,t} \not\subseteq F \Longrightarrow B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and coimage are usually not.

Definition 2.7. [5] The diffunction (f, F): $(S_1, \psi_1, \tau_1, k_1) \rightarrow (S_2, \psi_2, \tau_2, k_2)$ is called continuous if $B \in \tau_2 \Rightarrow F^{\leftarrow}B \in \tau_1$, cocontinuous if $B \in k_2 \Rightarrow f^{\leftarrow}B \in k_1$, and bicontinuous if it is both continuous and cocontinuous.

Definition 2.8. [4] Let $(f, F): (S_1, \psi_1) \to (S_2, \psi_2)$ be a difunction. Then (f, F) is called surjective if it satisfies the condition.

SUR. For
$$t, t' \in S_2$$
, $P_t \not\subseteq Q_{t'} \Rightarrow \exists s \in S_1$ with $f \subseteq \overline{Q}_{s,t'}$ and $\overline{P}_{s,t'} \not\subseteq F$.

If
$$(f, F)$$
 is surjective then $F^{\rightarrow}(f^{\leftarrow}B) = B = f^{\rightarrow}(F^{\leftarrow}B)$ for all $B \in \psi_2$

Definition 2.9. [8] Let (X, τ) be generalised topological space a set $A \in \tau$ is called

1) j – open if $A \subseteq intpc/A$

2) j – closed if *intpc*/ $A \subseteq A$

Definition 2.10. [9] Let (S, ψ, τ, k) be a ditopological texture space. For $A \in \psi$, we define,

1) The *j*-closure cl_iA of A under (τ, k) by the equality,

$$cl_i A = \bigcap \{B \mid B \in C_i(S) \text{ and } A \subseteq B\}$$

2) The *j*-interior int *A* of A under (τ, k) by the equality,

int $A = \bigvee \{B \mid B \in O_i(S) \text{ and } B \subseteq A\}$

Definition 2.11. [10] Let $(S_i, \psi_i, \tau_i, k_i)$, i = 1, 2 be a ditopological texture spaces and (f, F): $(S_1, \psi_1) \rightarrow (S_2, \psi_2)$ a difunction.

1) If $F^{\leftarrow}(G) \in O_i(S_1)$, for every $G \in O(S_2)$ this called j-continuous.

2) If $f^{\leftarrow}(K) \in C_i(S_1)$, for every $K \in C(S_2)$ this called j-cocontinuous.

3) If it is j – continuous and j – cocontinuous this is called j – bicontinuous.

Definition 2.12. [1] For a direlation $(f, F): (S_1, \psi_1) \to (S_2, \psi_2)$ the following are equivalent.

- 1) (f, F) is diffunction.
- 2) The following inclusions hold.
- a) $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A)), \forall A \in \psi_1$
- b) $f^{\rightarrow}(F^{\leftarrow}(B)) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}(B)), \forall B \in \psi_2$
- 3) $f \leftarrow B = F \leftarrow B, \forall B \in \psi_2$

Definition 2.13. [4] If (S, \wp) and (T, \Im) are textures $\varphi: S \to T$ an ω - compatible point function, namely one satisfying $P_s \not\subseteq Q_s \Rightarrow P_{\phi(s)} \not\subseteq Q_{\phi(s)}$ then the formulae,

 $f_{\varphi} = \bigvee \{ \overline{P}_{(s,t)} \mid \exists u \in S \text{ with } P_s \not\subseteq Q_u \text{ and } P_{\varphi(u)} \not\subseteq Q_t \},$ $F_{\varphi} = \bigcap \{ \overline{Q}_{(s,t)} \mid \exists v \in S \text{ with } P_v \not\subseteq Q_s \text{ and } P_t \not\subseteq Q_{\varphi(v)} \},$

define a difunction (f_{α}, F_{α}) from (S, \wp) and (T, \mathfrak{I}) .

Definition 2.14. [2] Let (τ, k) be a ditopology on (S, ψ) . Then a subset β and τ is called a base of τ if every set in τ can be written as a join of sets in β , while a subset β of k is a base of k if every set in k can be written as an intersection of sets in β

As usual, a subbase of τ , the set of finite intersections of which is a base of τ while a subbase of k, is a subset of k, the set of finite unions of which is base of k. In the case of a complemented ditopology, will clearly carry a base (subbase) of τ into a base(subbase) of k, and conversely.

Definition 2.15. [2] Let (τ, k) be a ditopology on the texture space (S, ψ) .

1) Let $\beta \subseteq \tau$. Then the following are equivalent.

(i) β is a base of τ

(ii) $G \in \tau, G \not\subseteq Q_s \Longrightarrow \exists B \in \beta$ with $\beta \not\subseteq Q_s$ and $B \subseteq G$

(iii) $G \in \tau$, $G \not\subseteq Q_s \Longrightarrow \exists B \in \beta$ with $P_s \subseteq B \subseteq G$

Definition 2.16. [1] Let (τ, R) be a direlation from (S_1, ψ_1) to (S_2, ψ_2) , J an index set, $A_i \in \psi_1, \forall j \in J$ and $B_j \in \psi_2, \forall j \in J$ then

1)
$$r^{\leftarrow} \left(\bigcap_{j \in J} B_j \right) = \bigcap_{j \in J} r^{\leftarrow} B_j$$
 and $R^{\rightarrow} \left(\bigcap_{j \in J} A_j \right) = \bigcap_{j \in J} R^{\rightarrow} A_j$

2)
$$r \rightarrow (\bigvee_{j \in J} A_j) = \bigvee_{j \in J} r \rightarrow A_j \text{ and } R \leftarrow (\bigvee_{j \in J} B_j) = \bigvee_{j \in J} R \leftarrow B_j$$

Definition 2.17. [1] Let (f, F) be a difunction from (S_1, ψ_1) and (S_2, ψ_2) . Then

- a) For $A \in \psi_1$, $A = \emptyset \Leftrightarrow f^{\rightarrow}A = \emptyset$
- b) For $A \in \psi_1$, $A = S_1 \Leftrightarrow F^{\rightarrow}A = S_2$
- c) $f^{\leftarrow} \emptyset = F^{\leftarrow} \emptyset = \emptyset$ and $f^{\leftarrow} S_2 = F^{\leftarrow} S_2 = S_1$.

3 j-compactness and j-cocompactness

Definition 3.1. Let (τ, k) be a ditopology on the texture space (S, ψ) and take $A \in \psi$. The family $\{G_i | i \in I\}$ is said to be a j-open cover of A if $G_i \in \tau$ for all $i \in I$ and $A \subseteq \bigvee_{i \in I} G_i$ **Definition 3.2.** Let (τ, k) be a ditopology on the texture space (S, ψ) and take $A \in \psi$. The family $\{F_i | i \in I\}$ is said to be a j-closed cover of A if $F_i \in K$ for all $i \in I$ and $\bigcap_{i \in I} F_i \subseteq A_i$ **Definition 3.3.** Let (τ, k) be a ditopology on the texture space (S, ψ) and take $A \in \psi$.

- A is called *j*-compact if whenever {G_i | *i* ∈ I} is a *j*-open cover of A then there is a finite subset J of I with A ⊆ ⋃_{j∈J}G_j. In particular the ditopological texture space (S, ψ, τ, k) is called *j*-compact if S is *j*-compact.
- A is called *j*-cocompact if whenever {F_i | i ∈ I} is a *j*-closed cover of A then there is a finite subset J of I with ∩_{j∈J} F_j ⊆ A. In particular the ditopological texture space (S, ψ, τ, k) is called *j*-cocompact if Ø is *j*-cocompact.

Theorem 3.4. Let (τ, k) be a complemented ditopology on (S, ψ, σ) . Then $(S, \psi, \sigma, \tau, k)$ is *j*-compact if and only if it is *j*-cocompact.

Proof: Let (τ, k) be *j*-compact and suppose $\mathcal{F} = \{F_i | i \in I\}$ be a family of *j*-closed sets with $\bigcap \mathcal{F} = \emptyset$. Consider the family $\mathcal{G} = \{\sigma(F_i | i \in I)\}$ of *j*-open sets. Then

$$\bigvee g = \bigvee \{ \sigma(F_i) \mid i \in I \} = \sigma(\bigcap \{ (F_i) \mid i \in I \})$$
$$= \sigma(\emptyset)$$
$$= S$$

and so $J \subseteq I$ finite with $\bigvee \{\sigma(F_i) | i \in I\} = S$ when $\bigcap \{(F_i) | i \in J\} = \emptyset$. Therefore we get (τ, k) is *j*-compact. If (τ, k) is *j*-compact then it is obvious.

Theorem 3.5. Let $(f, F): (S_1, \psi_1, \tau_1, k_1) \to (S_2, \psi_2, \tau_2, k_2)$ be a *j*-continuous diffunction. If $A \in \psi$ is *j*-compact then $f^{\to}A \in \psi_2$ is *j*-compact.

Proof: Take $f \rightarrow A \subseteq \bigvee_{i \in J} G_i$ where $G_i \in O_i(S_2), j \in J$

Then from the inclusion,

 $f^{\leftarrow}(F^{\rightarrow}A) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}A), \,\forall A \in \psi_1$

and also $r^{\rightarrow}(\bigvee_{j\in J}A_j) = \bigvee_{j\in J}(r^{\rightarrow}A_j)$ and

 $R^{\leftarrow}(\bigvee_{j\in J}B_j) = \bigvee_{j\in J}(R^{\leftarrow}B_j), A_j \in \psi_1, \forall j \in J \text{ and } B_j \in \psi_2, \forall j \in J$

We have $A \subseteq F^{\leftarrow}(f^{\rightarrow}A) \subseteq F^{\leftarrow}(\bigvee_{j \in J}G_j) = \bigvee_{j \in J}F^{\leftarrow}G_j$

Since (f, F) is j-continuous, $F \leftarrow G_j \in O_j(s_1)$ and also by the j-compacteness of A there exists $J' \subseteq J$ finite such that $A \subseteq \bigcup_{i \in J'} F \leftarrow G_j$.

Also from $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}B), \forall B \in \psi_2$

 $r^{\rightarrow}(\bigvee_{j\in J}A_j) = \bigvee_{j\in J}(r^{\rightarrow}A_j)$ and

 $R^{\leftarrow}(\bigvee_{j\in J}B_j) = \bigvee_{j\in J}(R^{\leftarrow}B_j), A_j \in \psi_1, \forall j \in J \text{ and } B_j \in \psi_2, \forall j \in J$

We have $f \to A \subseteq f \to (\bigcup_{j \in J'} F \leftarrow G_j = \bigcup_{j \in J'} f \to (F \leftarrow G_j) \subseteq \bigcup_{j \in J'} G_j$

that is $f \stackrel{\rightarrow}{\to} A \subseteq \bigcup_{i \in J'} G_i$. Therefore $f \stackrel{\rightarrow}{\to} A$ is *j*-compact.

Theorem 3.6. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $(f, F): S_1 \rightarrow S_2$ a surjective *j*-continuous diffunction. Then if $(S_1, \psi_1, \tau_1, k_1)$ is *j*-compact, $(S_2, \psi_2, \tau_2, k_2)$ is also *j*-compact.

Proof: In this, by taking $A = S_1$ in theorem 3.5 and noting that $f \to S_1 = f \to (F \to S_2) = S_2$, by using $f \to \emptyset = F \to \emptyset = \emptyset$ and $f \to T = F \to T = S$ and if (f, F) is surjective then $F \to (f \to B) = B = f \to (F \to B)$

Theorem 3.7. Let $(f, F) : (S_1, \psi_1, \tau_1, k_1) \to (S_2, \psi_2, \tau_2, k_2)$ be a *j*-cocontinuous diffunction. If $A \in \psi_1$ is *j*-cocompact then $F^{\to}A$ is *j*-cocompact

Proof: Take $F^{\rightarrow}A \supseteq \bigcap_{i \in J} F_i$, where $F_i \in \tau_2$, $j \in J$

Then form the inclusion

$$f^{\leftarrow}(F^{\rightarrow}A) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}A), \forall A \in \psi_1 \text{ and also } r^{\leftarrow}(\bigcap_{j \in J} B_j) = \bigcap_{j \in J} r^{\leftarrow} B_j$$

and $R^{\rightarrow}(\bigcap_{j\in J}A_j) = \bigcap_{j\in J}R^{\rightarrow}A_j, A_j \in \psi_1, \forall j \in J, B_j \in \psi_2, \forall_j \in J$

We have $A \supseteq f^{\leftarrow}(F^{\rightarrow}A) \supseteq f^{\leftarrow}(\bigcap_{j \in J} F_j) = \bigcap_{j \in J} f^{\leftarrow}F_j$

$$= \bigcap_{j \in J} f^{\leftarrow} F_j \subseteq A$$

Also $f^{\leftarrow}F_j \in \tau_1$ because (f, F) is j-cocontinuous, then by the j-cocompactness, of A, there exists $J' \subseteq J$ finite we have $A \supseteq \bigcap_{j \in J'} f^{\leftarrow}F_j$. Then $F^{\leftarrow}A \supseteq F^{\leftarrow}(\bigcap_{j \in J'} f^{\leftarrow}F_j) = \bigcap_{j \in J'} F^{\rightarrow}(f^{\leftarrow}F_j)$

$$F \leftarrow A \supseteq \bigcap_{j \in J'} F_j$$

That is $\bigcap_{j \in J'} F_j \subseteq F^{\leftarrow} A$

Therefore $F^{\leftarrow}A$ is *j*-cocompact.

Theorem 3.8. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $(f, F): S_1 \to S_2$ a surjective *j*-cocontinuous difunction. Then if $(S_1, \psi_1, \tau_1, k_1)$ is *j*-cocompact, $(S_2, \psi_2, \tau_2, k_2)$ is also *j*-cocompact.

Proof: In this, by taking $A = S_1$ in theorem 3.6 and noting that $F^{\rightarrow}S_1 = F^{\rightarrow}(f^{\leftarrow}S_2) = S_2$ by using $f^{\leftarrow} \emptyset = F^{\leftarrow} \emptyset = \emptyset$ and $f^{\leftarrow}T = F^{\leftarrow}T = S$ and if (f, F) is surjective then $F^{\rightarrow}(f^{\leftarrow}B) = B = f^{\rightarrow}(F^{\leftarrow}B)$

Theorem 3.9. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $\varphi: S_1 \to S_2$ a *j*-continuous surjective *w*-preserving point function. If $(S_1, \psi_1, \tau_1, k_1)$ is *j*-compact, then $(S_2, \psi_2, \tau_2, k_2)$ is *j*-compact.

Proof: The result will prove from the theorem 3.6 if we have to prove that the associated difunction $(f_{\varphi}, F_{\varphi})$ is surjective. Thus, put $t, t' \in \psi_2$ with $P_t \subseteq Q_{t'}$. Let $w \in S_2$ with $P_t \subseteq Q_w$ and $P_w \not\subset Q_{t'}$. Because φ is surjective there exists $s \in S_1$ with $w = \varphi(s)$.

Thus $P_{\varphi(s)} \not\subseteq Q'_t$ because $\overline{P}_{(s,\varphi,(s))} \not\subseteq \overline{Q}_{(s,t')}$ and then we have $f \not\subseteq \overline{Q}_{(s,t')}$ because $f = \bigvee \{\overline{P}_{(s,\varphi(s))} \mid s \in S_1\}$. Also in the same sense $\overline{P}_{(s,t)} \not\subseteq F$ and we have proved that $(f_{\varphi}, F_{\varphi})$ is surjecive.

Theorem 3.10. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $\varphi: S_1 \to S_2$ a *j*-cocontinuous surjective *w*-preserving point function. If $(S_1, \psi_1, \tau_1, k_1)$ is *j*-cocompact, then $(S_2, \psi_2, \tau_2, k_2)$ is *j*-cocompact.

Proof: The result will prove from the theorem 3.8 if we have to prove that the associated difunction $(f_{\varphi}, F_{\varphi})$ is surjective. The dual result of *j*-cocompactness, the proof is clear.

Definition 3.11. Let (S, ψ) be a texture

1) $\mathcal{A} \subseteq \psi$ is *j*-inadequate provided $\bigvee \mathcal{A} \neq S$. It is finitely *j*-inadequate provided no finite sub collection covers *S*.

2) $\mathcal{A} \subseteq \psi$ is *j*-coinadequate provided $\bigcap \mathcal{A} \neq \emptyset$. It is finitely *j*-coinadequate provided no finite sub collection cocovers \emptyset .

Clearly, finite j-coinadequancy is same as the finite intersection property.

Theorem 3.12. Let (τ, k) be a ditopology on the texture space (S, ψ) and γ a subbase for τ . Then (τ, k) is *j*-compact if and only if every *j*-open cover of *S* by elements of γ has a finite *j*-subcover.

Proof:

Suppose that (τ, k) is *j*-compact. Because $\gamma \subseteq \tau$, every *j*-open cover of *S* by elements of γ has a finite *j*-subcover.

Let every cover of *S* by members of γ has a finite *j*-subcover. we have to prove that (τ,k) is *j*-compact. So we have to first prove that every finitely *j*-inadequate collection of *j*-open sets is *j*-inadequate. Let B be finitely *j*-inadequate collection of *j*-open sets and P the set of all finitely *j*-inadequate collection of G of *j*-open sets such that B \subseteq G. The (P, \subseteq) is a poset. Suppose C be a chain in P and also suppose G* be the union of all members of C. Then G* is a collection of U_1, U_2, \dots, U_n of G* that covers S. For each $i = 1, 2, \dots, n$ there exists $G_i \in C$ such that $U_i \in G_i$. Because C is a chain there exists k with $1 \le k \le n$ such that G_k contains all the sets G_i , $i = 1, 2, \dots, n$ and also for each $i = 1, 2, \dots, n$ there G_k . But G_k is finitely *j*-inadequate, thus we get a contradiction. Hence we get G* is finitely *j*-inadequate. Also thus $G^* \in P$ and is G* upperbound of C.

By Zorns Lemma P has a maximal element \mathcal{A} . We have to prove that B is j-inadequate. Because $B \subseteq \mathcal{A}$, it is sufficient to prove that \mathcal{A} , is j-inadequate. Then \mathcal{A} has the following properties.

1) If $U \in \tau$ and $U \notin \mathcal{A}$ there exists a finite subcollection U_1, U_2, \dots, U_n of \mathcal{A} . such that $S = U \bigcup \left(\bigcup_{i=1}^n U_i \right).$

2) If U_1, U_2, \dots, U_n is a finite collection of j-open sets none of which belong to \mathcal{A} , $(\bigcap_{i=1}^n U_i) \notin A$.

3) If U_1, U_2, \dots, U_n is a finite collection of j-open sets and $V \in \mathcal{A}$ satisfies $(\bigcap_{i=1}^n U_i) \subseteq V$, then there exists j with $1 \le j \le n$ such that $U_j \in \mathcal{A}$.

Proof of (1):

The proof is clear because $\mathcal{A} \cup \{U\}$ cannot finitely *j*-*inadequate*.

Proof of (2):

It is sufficient of prove (2) we take 2 sets $U_1, U_2 \notin \mathcal{A}$. Using (1) there are finite subcollection V_1, V_2, \dots, V_n and W_1, W_2, \dots, W_m of \mathcal{A} such that $S = U_1 \bigcup_{i=1}^n V_i = U_2 \bigcup_{i=1}^m W_i$. Thus

$$S = (U_1 \cap U_2) \bigcup \bigcup_{i=1}^n V_i \bigcup \bigcup_{j=1}^m W_j.$$

 $U_1 \cap U_2 \notin \mathcal{A}$, because \mathcal{A} is finitely *j*-adequate. Thus we get (2) is proved.

Proof of (3):

Let U_1, U_2, \dots, U_n are j-open sets and $V \in \mathcal{A}$ satisfies $\bigcap_{i=1}^n U_i \subseteq V$. If none of the sets $U_i, i = 1, 2, \dots, n$ belong to \mathcal{A} then by using (2), $\bigcap_{i=1}^n U_i \notin \mathcal{A}$. Using (1) there is a finite subcollection V_1, V_2, \dots, V_m of \mathcal{A} such that $S = (\bigcap_{i=1}^n U_i) \bigcup_{j=1}^m V_j$, then $S = V \bigcup_{i=1}^m V_j$, because \mathcal{A} is finitely j-inadequate, there is a contradiction, therefore we get (3).

The collection $\gamma \cap \mathcal{A}$ is finitely j-inadequate, since $\gamma \cap \mathcal{A} \subseteq \mathcal{A}$ and \mathcal{A} is finitely j-inadequate. We have to show that $\bigvee \mathcal{A} \subseteq \bigvee (\mathcal{A} \cap \gamma)$. Put $s \in S$ with $\bigvee \mathcal{A} \not\subseteq Q_s$. Also $A \in \mathcal{A}$ with $A \not\subseteq Q_s$, because $A \in \tau$ and γ is a subbase for τ , by using $G \in \tau, G \not\subseteq Q_s \Rightarrow \exists B \in \beta$ with $B \not\subseteq Q_s$ and $B \subseteq G$, there exists $G_1, G_2, \dots, G_n \in \gamma$, then $\bigcap_{i=1}^n G_i \subseteq A$ and $\bigcap_{i=1}^n G_i \not\subseteq Q_s$.

By (3) there exists $k, 1 \le k \le n$, then $G_k \in \mathcal{A} \cap \gamma$ and $G_k \not\subseteq Q_s$, because $\bigvee (\mathcal{A} \cap \gamma) \not\subseteq Q_s$. This prove that $\bigvee \mathcal{A} \subseteq \bigvee (\mathcal{A} \cap \gamma)$. If $\bigvee \mathcal{A} = S$, then $S = \bigvee (\mathcal{A} \cap \gamma)$ and since by every cover of S by sets in γ has a finite $j - sub \operatorname{cov} er$ thus we get a contradiction that the family $(\mathcal{A} \cap \gamma)$ is finitely j - inadequate. So \mathcal{A} and B is j - inadequate and we get that (τ, k) is j - compact.

Theorem 3.13. Let (τ, k) be a ditopology on the texture space (S, ψ) and γ a subbase for the closed set k. Then (τ, k) is *j*-cocompact if and only if every *j*-closed cocover of \emptyset by member of γ has a finite *j*-subcover.

Proof: The dual property of the above theorem, the proof is clear.

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