Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue5, May , 3539- 3550

Research Article

j-compactness and j-cocompactness in ditopological texture spaces

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Abstract

The centre of attention in this paper deals with j-compactness and j-cocompactness in ditopological texture spaces. Also, j-inadequate and j-coinadequate in ditopological texture spaces are discussed.

***Keywords**:* Texture,ditopology, difunction, j-compact, j-cocompact, j-inadequate and j-coinadequate.

1 Introduction

The texture and ditopology texture spaces were placed for first timeby L.M. Brown [1] in point-set setting for the assessment of fuzzy sets. It has been proved convenient as a framework to talk about the complementfree mathematical idea. The ditopological texture space may be considered as a natural combination of texture space, topological space and bitopological space, but ditopology corresponds in a natural way to fuzzy topology. A texture is considered as a generalization of fuzzy lattice. The created to illustrate of ditopology is more general than general topology, fuzzy topology and bitopology. So it will be more benefit to generalize some distinct general (fuzzy, bi) topological concepts to the ditopological texture space. Also concerning textures and ditopologies and some basic notions are recollected and sufficient to present a new idea to the theory and the motivation for its analysis of a subject are acquired from [1,2,3]. In [4], the concept of j-compactness and j-cocompactness in ditopological texture spaces, were introduced and also their characterizations are discussed. Also j-inadequateand j-coinadequate in ditopological texture spaces are initiated.

2 Preliminaries

Some basic definitions of textures are listed below.

Definition 2.1. [6] Let S be a set. Then $\psi \subseteq P(S)$ is called a texturing of S and S is

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said to be textured by ψ if

1) $(\psi, \subseteq)$ is a complete lattice containing S and ϕ and for any index set I and $A_i \in \psi,\ i \in I$, the meet $\Lambda_{i\in I} A_i$ and the join $\vee_{i\in I} A_i$ in ψ are related with the intersection and union in $P(S)$ by the equalities.

$$\Lambda_{i\in I} A_i = \bigcap_{i\in I} A_i$$

for all I, while

$$\vee_{i\in I} A_i = \bigcup_{i\in I} A_i$$

for all finite I.

2) ψ is completely distributive.

3) ψ separate the point of S. That is, given $s_1 \neq s_2$ in S we have $L \in \psi$ with $s_1 \in L,\ s_2 \notin L$ If S is textured by ψ then (S, ψ) is called a texture space, or simply a texture.

Definition 2.2. [4] A mapping $\sigma : \psi \to \psi$ satisfying $\sigma(\sigma(A)) = A,\ \forall A \in \psi$ and $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A),\ \ \forall A, B \in \psi$ is called a complementation on (S, ψ) and (S, ψ, σ) is then said to be a complemented texture. For a texture (S, ψ) most properties are conveniently defined in terms of the sets,

$$p-\text{sets } P_s = \bigcap\{A \in \psi : s \in A\}$$

$$q-\text{sets } Q_s = \bigvee\{A \in \psi : s \notin A\}$$

We recall the following fundamental properties

1) For, $A, B \in \psi$, if $A \subseteq B$ then there exists $s \in S$ with $A \not\subseteq Q_s$ and $P_s \not\subseteq B$

2) $A = \bigcap\{Q_s \mid P_s \not\subseteq A\}$ for all $A \in \psi$

3) $A = \bigvee\{P_s \mid A \not\subseteq Q_s\}$ for all $A \in \psi$

Definition 2.3. [7] A dichotomous topology on a texture (S, ψ), or ditopology, is a pair (τ, k) of subsets of ψ, where the of open sets τ satisfies

1) $S, \emptyset \in \tau$

2) $G_1, G_2 \in \tau \Rightarrow G_1 \bigcap G_2 \in \tau$ and

3) $G_i \in \tau,\ i \in I \Rightarrow \bigvee_i G_i \in \tau$

and the set of closed sets k satisfies

1) $S,\ \emptyset \in k$

2) $K_1,\ K_2 \in k \Rightarrow K_1 \bigcup K_2 \in k$ and

3) $K_i \in k,\ i \in I \Rightarrow \bigcap K_i \in k$ and

Hence a ditoplogy is essentially a topology for which there is no a priori relation between the open and closed sets.

For $A \in \psi$ define the closure clA and the interior $\text{int}\, A$ of A under (τ, k) by the equalities.

$$clA = \bigcap \{K \in k : A \subseteq K\}$$

and

$$\text{int}\, A = \bigvee \{G \in \tau : G \subseteq A\}$$

We refer to τ as the topology and k as the cotopology of $(\tau,\ k)$. If $(\tau,\ k)$ is a ditopology on a complemented texture $(S,\ \psi,\ \sigma)$, then we say that $(\tau,\ k)$ is complemented if the equality $k = \sigma cl(\tau)$ is satisfied. In this study, a complemented ditoplogical texture space is denoted by $(S,\ \psi, \tau, k, \sigma)$. In this case we have $\sigma(clA) = \text{int}\, \sigma(A)$. We denote by $O(S,\ \psi,\ \tau,\ k)$ or $O(S)$, the set of open sets in ψ. Likewise $C(S,\ \psi,\ \tau,\ k)$ or $C(S)$ will denote the set of closed sets.

Let $(S_1,\ \psi_1)$ and $(S_2,\ \psi_2)$ be textures. In the following definition we consider the product texture $P(S_1) \otimes \psi_2$, and denote by $\overline{P}_{s,t},\ \overline{Q}_{s,\,t}$, respectively the $p-$sets and $q-$sets for the product texture $(S_1 \text{X}\, S_2,\ P(S_1) \otimes \psi_2)$.

Definition 2.4. [7] Let $(S_1,\ \psi_1)$ and or $(S_2,\ \psi_2)$ be textures. Then

(1) $r \in P(S_1) \otimes \psi_2$ is called a relation from $(S_1,\ \psi_1)$ and or $(S_2,\ \psi_2)$ if it satisfies.

$$R_1 : r \not\subseteq \overline{Q}_{s,t}, P_{s'} \not\subseteq Q_s \Rightarrow r \not\subseteq \overline{Q}_{s',t}$$

$$R_2 : r \subseteq \overline{Q}_{s,t} \Rightarrow \exists s' \in S_1 \text{ such that } P_s \not\subseteq \overline{Q}_{s'} \text{ and } r \not\subseteq \overline{Q}_{s',t}$$

(2) $R \in P(S_1) \otimes \psi_2$ is called a correlation from $(S_1,\ \psi_1)$ and $(S_2,\ \psi_2)$ if it satisfies.

$$CR_1 : \overline{P}_{s,t} \not\subseteq R,\ P_s \not\subseteq Q_{s'} \Rightarrow \overline{P}_{s',t} \not\subseteq R.$$

$CR_2 : \overline{P}_{s,t} \not\subseteq R \Rightarrow \exists s' \in S_1$ such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{s',t} \not\subseteq R.$

(3) A pair (r, R) where r is a relation and R a correlation from (S_1, ψ_1) to (S_2, ψ_2) is called a direlation from (S_1, ψ_1) to (S_2, ψ_2). One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.

Definition 2.5. [4] Let (f, F) be a direlation from (S_1, ψ_1) to (S_2, ψ_2). Then (f, F) is called a difunction from (S_1, ψ_1) to (S_2, ψ_2) if it satisfies the following two conditions:

DF_1 : For $s, s' \in S_1, P_s \not\subseteq Q_{s'} \Rightarrow \exists t \in S_2$ such that $f \not\subseteq \overline{Q}_{s,t}$ and $\overline{P}_{s',t} \not\subseteq F.$

DF_2 : For $t, t' \in S_2$ and $s \in S_1$, $f \not\subseteq \overline{Q}_{s,t}$ and $\overline{P}_{s,t'} \not\subseteq F \Rightarrow P_{t'} \not\subseteq Q_t.$

Definition 2.6. [4] Let $(f, F) : (S_1, \psi_1) \to (S_2, \psi_2)$ be a difunction.

(1) For $A \in \psi_1$, the image $f^{\to}A$ and the co-image $F^{\to}A$ are defined by,

$$f^{\to}A = \bigcap\{Q_t : \forall s,\ f \not\subseteq \overline{Q}_{s,t} \Rightarrow A \subseteq Q_s\}$$

$$F^{\to}A = \bigvee\{P_t : \forall s,\ \overline{P}_{s,t} \not\subseteq F \Rightarrow P_s \subseteq A\}.$$

(2) For $B \in \psi_2$, the inverse image $f^{\leftarrow}B$ and the inverse co-image $F^{\leftarrow}B$ are defined by,

$$f^{\leftarrow}B = \bigvee\{P_s : \forall t,\ f \not\subseteq \overline{Q}_{s,t} \Rightarrow P_t \subseteq B\}$$

$$F^{\leftarrow}B = \bigcap\{Q_s : \forall t,\ \overline{P}_{s,t} \not\subseteq F \Rightarrow B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.

Definition 2.7. [5] The difunction (f, F): $(S_1, \psi_1, \tau_1, k_1) \to (S_2, \psi_2, \tau_2, k_2)$ is called continuous if $B \in \tau_2 \Rightarrow F^{\leftarrow}B \in \tau_1$, cocontinuous if $B \in k_2 \Rightarrow f^{\leftarrow}B \in k_1$, and bicontinuous if it is both continuous and cocontinuous.

Definition 2.8. [4] Let $(f, F) : (S_1, \psi_1) \to (S_2, \psi_2)$ be a difunction. Then (f, F) is called surjective if it satisfies the condtion.

$SUR.$ For $t, t' \in S_2$, $P_t \not\subseteq Q_{t'} \Rightarrow \exists s \in S_1$ with $f \subseteq \overline{Q}_{s,t'}$ and $\overline{P}_{s,t'} \not\subseteq F.$

If (f, F) is surjective then $F^{\to}(f^{\leftarrow}B) = B = f^{\to}(F^{\leftarrow}B)$ for all $B \in \psi_2$

Definition 2.9. [8] Let (X, τ) be generalised topological space a set $A \in \tau$ is called

1) $j-$open if $A \subseteq intpcl\ A$

2) $j-$closed if $intpcl A \subseteq A$

Definition 2.10. [9] Let (S, ψ, τ, k) be a ditopological texture space. For $A \in \psi,$ we define,

1) The $j-$closure $cl_j A$ of A under (τ, k) by the equality,

$$cl_j A = \bigcap\{B \mid B \in C_j(S) \text{ and } A \subseteq B\}$$

2) The $j-$interior $\text{int}_j A$ of A under (τ, k) by the equality,

$$\text{int}_j A = \bigvee\{B \mid B \in O_j(S) \text{ and } B \subseteq A\}$$

Definition 2.11. [10] Let $(S_i, \psi_i, \tau_i, k_i),\ i = 1,2$ be a ditopological texture spaces and $(f, F):(S_1, \psi_1) \to (S_2, \psi_2)$ a difunction.

1) If $F^{\leftarrow}(G) \in O_j(S_1),$ for every $G \in O(S_2)$ this called $j-$continuous.

2) If $f^{\leftarrow}(K) \in C_j(S_1),$ for every $K \in C(S_2)$ this called $j-$cocontinuous.

3) If it is $j-$continuous and $j-$cocontinuous this is called $j-$bicontinuous.

Definition 2.12. [1] For a direlation $(f, F):(S_1, \psi_1) \to (S_2, \psi_2)$ the following are equivalent.

1) (f, F) is difunction.

2) The following inclusions hold.

$a)$ $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A)), \forall A \in \psi_1$

$b)$ $f^{\rightarrow}(F^{\leftarrow}(B)) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}(B)), \forall B \in \psi_2$

3) $f^{\leftarrow}B = F^{\leftarrow}B, \forall B \in \psi_2$

Definition 2.13. [4] If $(S, \wp)$ and $(T, \Im)$ are textures $\varphi: S \to T$ an $\omega-$ compatible point function, namely one satisfying $P_s \not\subseteq Q_s \Rightarrow P_{\phi(s)} \not\subseteq Q_{\phi(s)}$ then the formulae,

$$f_\varphi = \bigvee\{\overline{P}_{(s,t)} \mid \exists\ u \in S \text{ with } P_s \not\subseteq Q_u \text{ and } P_{\varphi(u)} \not\subseteq Q_t\},$$

$$F_\varphi = \bigcap\{\overline{Q}_{(s,t)} \mid \exists\ v \in S \text{ with } P_v \not\subseteq Q_s \text{ and } P_t \not\subseteq Q_{\varphi(v)}\},$$

define a difunction (f_φ, F_φ) from $(S, \wp)$ and $(T, \Im)$.

Definition 2.14. [2] Let (τ, k) be a ditopology on (S, ψ). Then a subset β and τ is called a base of τ if every set in τ can be written as a join of sets in β, while a subset β of k is a base of k if every set in k can be written as an intersection of sets in β

As usual, a subbase of τ, the set of finite intersections of which is a base of τ while a subbase of k, is a subset of k, the set of finite unions of which is base of k. In the case of a complemented ditopology, will clearly carry a base (subbase) of τ into a base(subbase) of k, and conversely.

Definition 2.15. [2] Let (τ, k) be a ditopology on the texture space (S, ψ).

1) Let $\beta \subseteq \tau$. Then the following are equivalent.

(i) β is a base of τ

(ii) $G \in \tau, G \not\subseteq Q_s \Rightarrow \exists B \in \beta$ with $\beta \not\subseteq Q_s$ and $B \subseteq G$

(iii) $G \in \tau, G \not\subseteq Q_s \Rightarrow \exists B \in \beta$ with $P_s \subseteq B \subseteq G$

Definition 2.16. [1] Let (τ, R) be a direlation from (S_1, ψ_1) to (S_2, ψ_2), J an index set, $A_j \in \psi_1, \forall j \in J$ and $B_j \in \psi_2, \forall j \in J$ then

1) $r^{\leftarrow}\left(\bigcap_{j\in J} B_j\right) = \bigcap_{j\in J} r^{\leftarrow} B_j$ and $R^{\rightarrow}\left(\bigcap_{j\in J} A_j\right) = \bigcap_{j\in J} R^{\rightarrow} A_j$

2) $r^{\rightarrow}\left(\bigvee_{j\in J} A_j\right) = \bigvee_{j\in J} r^{\rightarrow} A_j$ and $R^{\leftarrow}\left(\bigvee_{j\in J} B_j\right) = \bigvee_{j\in J} R^{\leftarrow} B_j$

Definition 2.17. [1] Let (f, F) be a difunction from (S_1, ψ_1) and (S_2, ψ_2). Then

a) For $A \in \psi_1$, $A = \emptyset \Leftrightarrow f^{\rightarrow} A = \emptyset$

b) For $A \in \psi_1$, $A = S_1 \Leftrightarrow F^{\rightarrow} A = S_2$

c) $f^{\leftarrow}\emptyset = \mathrm{F}^{\leftarrow}\emptyset = \emptyset$ and $f^{\leftarrow} S_2 = \mathrm{F}^{\leftarrow} S_2 = S_1$.

3 j-compactness and j-cocompactness

Definition 3.1. Let (τ, k) be a ditopology on the texture space (S, ψ) and take $A \in \psi$. The family $\{G_i \mid i \in I\}$ is said to be a $j-$open cover of A if $G_i \in \tau$ for all $i \in I$ and $A \subseteq \bigvee_{i\in I} G_i$
Definition 3.2. Let (τ, k) be a ditopology on the texture space (S, ψ) and take $A \in \psi$. The family $\{F_i \mid i \in I\}$ is said to be a $j-$closed cover of A if $F_i \in K$ for all $i \in I$ and $\bigcap_{i\in I} F_i \subseteq A_i$

Definition 3.3. Let (τ, k) be a ditopology on the texture space (S, ψ) and take $A \in \psi$.

- A is called $j-compact$ if whenever $\{G_i \mid i \in I\}$ is a $j-open$ cover of A then there is a finite subset J of I with $A \subseteq \bigcup_{j \in J} G_j$. In particular the ditopological texture space (S, ψ, τ, k) is called $j-compact$ if S is $j-compact$.
- A is called $j-cocompact$ if whenever $\{F_i \mid i \in I\}$ is a $j-closed$ cover of A then there is a finite subset J of I with $\bigcap_{j \in J} F_j \subseteq A$. In particular the ditopological texture space (S, ψ, τ, k) is called $j-cocompact$ if $\emptyset$ is $j-cocompact$.

Theorem 3.4. Let (τ, k) be a complemented ditopology on (S, ψ, σ). Then $(S, \psi, \sigma, \tau, k)$ is $j-compact$ if and only if it is $j-cocompact$.

Proof: Let (τ, k) be $j-compact$ and suppose $\mathscr{F} = \{F_i \mid i \in I\}$ be a family of $j-closed$ sets with $\bigcap \mathscr{F} = \emptyset$. Consider the family $\mathscr{G} = \{\sigma(F_i \mid i \in I)\}$ of $j-open$ sets. Then

$$\begin{aligned} \bigvee \mathscr{G} = \bigvee\{\sigma(F_i) \mid i \in I\} &= \sigma(\bigcap\{(F_i) \mid i \in I\}) \\ &= \sigma(\emptyset) \\ &= S \end{aligned}$$

and so $J \subseteq I$ finite with $\bigvee\{\sigma(F_i) \mid i \in I\} = S$ when $\bigcap\{(F_i) \mid i \in J\} = \emptyset$. Therefore we get (τ, k) is $j-compact$. If (τ, k) is $j-cocompact$ then it is obvious.

Theorem 3.5. Let $(f, F) : (S_1, \psi_1, \tau_1, k_1) \to (S_2, \psi_2, \tau_2, k_2)$ be a $j-continuous$ difunction. If $A \in \psi$ is $j-compact$ then $f^{\to} A \in \psi_2$ is $j-compact$.

Proof: Take $f^{\to} A \subseteq \vee_{j \in J} G_j$ where $G_j \in O_j(S_2),\ j \in J$

Then from the inclusion,

$f^{\leftarrow}(F^{\to} A) \subseteq A \subseteq F^{\leftarrow}(f^{\to} A),\ \forall A \in \psi_1$

and also $r^{\to}(\bigvee_{j \in J} A_j) = \bigvee_{j \in J} (r^{\to} A_j)$ and

$R^{\leftarrow}(\bigvee_{j \in J} B_j) = \bigvee_{j \in J} (R^{\leftarrow} B_j),\ A_j \in \psi_1,\ \forall j \in J$ and $B_j \in \psi_2,\ \forall j \in J$

We have $A \subseteq F^{\leftarrow}(f^{\to} A) \subseteq F^{\leftarrow}(\bigvee_{j \in J} G_j) = \bigvee_{j \in J} F^{\leftarrow} G_j$

Since (f, F) is $j-continuous$, $F^{\leftarrow} G_j \in O_j(s_1)$ and also by the $j-compacteness$ of A there exists $J' \subseteq J$ finite such that $A \subseteq \bigcup_{j \in J'} F^{\leftarrow} G_j$.

Also from $f^{\to}(F^{\leftarrow} B) \subseteq B \subseteq F^{\to}(f^{\leftarrow} B),\ \forall B \in \psi_2$

$r^{\to}(\bigvee_{j\in J} A_j) = \bigvee_{j\in J}(r^{\to}A_j)$ and

$R^{\leftarrow}(\bigvee_{j\in J} B_j) = \bigvee_{j\in J}(R^{\leftarrow}B_j),\ A_j \in \psi_1, \forall j \in J$ and $B_j \in \psi_2, \forall j \in J$

We have $f^{\to}A \subseteq f^{\to}(\bigcup_{j\in J'} F^{\leftarrow}G_j = \bigcup_{j\in J'} f^{\to}(F^{\leftarrow}G_j) \subseteq \bigcup_{j\in J'} G_j$

that is $f^{\to}A \subseteq \bigcup_{j\in J'} G_j$. Therefore $f^{\to}A$ is $j-compact$.

Theorem 3.6. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $(f,F): S_1 \to S_2$ a surjective $j-continuous$ difunction. Then if $(S_1, \psi_1, \tau_1, k_1)$ is $j-compact$, $(S_2, \psi_2, \tau_2, k_2)$ is also $j-compact$.

Proof: In this, by taking $A = S_1$ in theorem 3.5 and noting that $f^{\to}S_1 = f^{\to}(F^{\leftarrow}S_2) = S_2$, by using $f^{\leftarrow}\emptyset = F^{\leftarrow}\emptyset = \emptyset$ and $f^{\leftarrow}T = F^{\leftarrow}T = S$ and if (f,F) is surjective then $F^{\to}(f^{\leftarrow}B) = B = f^{\to}(F^{\leftarrow}B)$

Theorem 3.7. Let $(f,F):(S_1, \psi_1, \tau_1, k_1) \to (S_2, \psi_2, \tau_2, k_2)$ be a $j-cocontinuous$ difunction. If $A \in \psi_1$ is $j-cocompact$ then $F^{\to}A$ is $j-cocompact$

Proof: Take $F^{\to}A \supseteq \bigcap_{j\in J} F_j$, where $F_j \in \tau_2,\ j \in J$

Then form the inclusion

$f^{\leftarrow}(F^{\to}A) \subseteq A \subseteq F^{\leftarrow}(f^{\to}A), \forall A \in \psi_1$ and also $r^{\leftarrow}(\bigcap_{j\in J} B_j) = \bigcap_{j\in J} r^{\leftarrow}B_j$

and $R^{\to}(\bigcap_{j\in J} A_j) = \bigcap_{j\in J} R^{\to}A_j,\ A_j \in \psi_1, \forall j \in J, B_j \in \psi_2, \forall_j \in J$

We have $A \supseteq f^{\leftarrow}(F^{\to}A) \supseteq f^{\leftarrow}(\bigcap_{j\in J} F_j) = \bigcap_{j\in J} f^{\leftarrow}F_j$

$= \bigcap_{j\in J} f^{\leftarrow}F_j \subseteq A$

Also $f^{\leftarrow}F_j \in \tau_1$ because (f,F) is $j-cocontinuous$, then by the $j-cocompactness$, of A, there exists $J' \subseteq J$ finite we have $A \supseteq \bigcap_{j\in J'} f^{\leftarrow}F_j$. Then $F^{\leftarrow}A \supseteq F^{\leftarrow}(\bigcap_{j\in J'} f^{\leftarrow}F_j) = \bigcap_{j\in J'} F^{\to}(f^{\leftarrow}F_j)$

$F^{\leftarrow}A \supseteq \bigcap_{j\in J'} F_j$

That is $\bigcap_{j\in J'} F_j \subseteq F^{\leftarrow}A$

Therefore $F^{\leftarrow}A$ is $j-cocompact$.

Theorem 3.8. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $(f, F): S_1 \to S_2$ a surjective $j-cocontinuous$ difunction. Then if $(S_1, \psi_1, \tau_1, k_1)$ is $j-cocompact$, $(S_2, \psi_2, \tau_2, k_2)$ is also $j-cocompact$.

Proof: In this, by taking $A = S_1$ in theorem 3.6 and noting that $F^{\to}S_1 = F^{\to}(f^{\leftarrow}S_2) = S_2$ by using $f^{\leftarrow}\emptyset = F^{\leftarrow}\emptyset = \emptyset$ and $f^{\leftarrow}T = F^{\leftarrow}T = S$ and if (f, F) is surjective then $F^{\to}(f^{\leftarrow}B) = B = f^{\to}(F^{\leftarrow}B)$

Theorem 3.9. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $\varphi: S_1 \to S_2$ a $j-continuous$ surjective $w-preserving$ point function. If $(S_1, \psi_1, \tau_1, k_1)$ is $j-compact$, then $(S_2, \psi_2, \tau_2, k_2)$ is $j-compact$.

Proof: The result will prove from the theorem 3.6 if we have to prove that the associated difunction (f_φ, F_φ) is surjective. Thus, put $t, t' \in \psi_2$ with $P_t \subseteq Q_{t'}$. Let $w \in S_2$ with $P_t \subseteq Q_w$ and $P_w \not\subset Q_{t'}$. Because φ is surjective there exists $s \in S_1$ with $w = \varphi(s)$.

Thus $P_{\varphi(s)} \not\subseteq Q'_t$ because $\bar{P}_{(s,\varphi,(s))} \not\subset \bar{Q}_{(s,t')}$ and then we have $f \not\subseteq \bar{Q}_{(s,t')}$ because $f = \bigvee\{\bar{P}_{(s,\varphi(s))} \mid s \in S_1\}$. Also in the same sense $\bar{P}_{(s,t)} \not\subseteq F$ and we have proved that (f_φ, F_φ) is surjecive.

Theorem 3.10. Let $(S_1, \psi_1, \tau_1, k_1)$ and $(S_2, \psi_2, \tau_2, k_2)$ be a ditopological texture spaces and $\varphi: S_1 \to S_2$ a $j-cocontinuous$ surjective $w-preserving$ point function. If $(S_1, \psi_1, \tau_1, k_1)$ is $j-cocompact$, then $(S_2, \psi_2, \tau_2, k_2)$ is $j-cocompact$.

Proof: The result will prove from the theorem 3.8 if we have to prove that the associated difunction (f_φ, F_φ) is surjective. The dual result of $j-cocompactness$, the proof is clear.

Definition 3.11. Let (S, ψ) be a texture

1) $\mathcal{A} \subseteq \psi$ is $j-inadequate$ provided $\bigvee \mathcal{A} \neq S$. It is finitely $j-inadequate$ provided no finite sub collection covers S.

2) $\mathcal{A} \subseteq \psi$ is $j-coinadequate$ provided $\bigcap \mathcal{A} \neq \emptyset$. It is finitely $j-coinadequate$ provided no finite sub collection cocovers $\emptyset$.

Clearly, finite $j-coinadequancy$ is same as the finite intersection property.

Theorem 3.12. Let (τ, k) be a ditopology on the texture space (S, ψ) and γ a subbase for τ. Then (τ, k) is $j-compact$ if and only if every $j-open$ cover of S by elements of γ has a finite $j-$subcover.

Proof:

Suppose that (τ,k) is $j-compact$. Because $\gamma \subseteq \tau$, every $j-open$ cover of S by elements of γ has a finite $j-$subcover.

Let every cover of S by members of γ has a finite $j-$subcover. we have to prove that (τ,k) is $j-compact$. So we have to first prove that every finitely $j-inadequate$ collection of $j-open$ sets is $j-inadequate$. Let B be finitely $j-inadequate$ collection of $j-open$ sets and P the set of all finitely $j-inadequate$ collection of G of $j-open$ sets such that B $\subseteq$ G. The $(P,\subseteq)$ is a poset. Suppose C be a chain in P and also suppose G^* be the union of all members of C. Then G^* is a collection of $j-open$ sets and thus B $\subseteq$ G^*. Also for each G$\in C$, G $\subseteq G^*$. Let there is finite subcollection of $U_1,U_2,.....U_n$ of G^* that covers S. For each $i=1,2,....,n$ there exists $G_i \in C$ such that $U_i \in G_i$. Because C is a chain there exists k with $1 \le k \le n$ such that G_k contains all the sets G_i , $i=1,2,....,n$ and also for each $i=1,2,....,n$ we get $U_i \in G_k$. But G_k is finitely $j-inadequate$, thus we get a contradiction. Hence we get G^* is finitely $j-inadequate$. Also thus $G^* \in$P and is G^* upperbound of C.

By Zorns Lemma P has a maximal element $\mathcal{A}$. We have to prove that B is $j-inadequate$. Because B $\subseteq \mathcal{A}$, it is sufficient to prove that $\mathcal{A}$, is $j-inadequate$. Then $\mathcal{A}$ has the following properties.

1) If $U \in \tau$ and $U \notin \mathcal{A}$ there exists a finite subcollection $U_1,U_2,.....U_n$ of $\mathcal{A}$. such that $S = U \bigcup \left(\bigcup_{i=1}^{n} U_i \right)$.

2) If $U_1,U_2,.....U_n$ is a finite collection of $j-open$ sets none of which belong to $\mathcal{A}$, $(\bigcap_{i=1}^{n} U_i) \notin A$.

3) If $U_1,U_2,.....U_n$ is a finite collection of $j-open$ sets and $V \in \mathcal{A}$ satisfies $(\bigcap_{i=1}^{n} U_i) \subseteq V$, then there exists j with $1 \le j \le n$ such that $U_j \in \mathcal{A}$.

Proof of (1):

The proof is clear because $\mathcal{A} \bigcup \{U\}$ cannot finitely $j-inadequate$.

Proof of (2):

It is sufficient of prove (2) we take 2 sets $U_1,U_2 \notin \mathcal{A}$. Using (1) there are finite subcollection $V_1,V_2,.....V_n$ and $W_1,W_2,....,W_m$ of $\mathcal{A}$ such that $S = U_1 \bigcup \bigcup_{i=1}^{n} V_i = U_2 \bigcup_{j=1}^{m} W_j$. Thus

$S=(U_1\bigcap U_2)\bigcup\bigcup_{i=1}^{n}V_i\bigcup\bigcup_{j=1}^{m}W_j.$

$U_1\bigcap U_2\notin\mathcal{A},$ because $\mathcal{A}$ is finitely $j-adequate.$ Thus we get (2) is proved.

Proof of (3):

Let $U_1,U_2,.....U_n$ are $j-open$ sets and $V\in\mathcal{A}$ satisfies $\bigcap_{i=1}^{n}U_i\subseteq V.$ If none of the sets $U_i, i=1,2,.....,n$ belong to $\mathcal{A}$ then by using (2), $\bigcap_{i=1}^{n}U_i\notin\mathcal{A}.$ Using (1) there is a finite subcollection $V_1,V_2,.....V_m$ of $\mathcal{A}$ such that $S=(\bigcap_{i=1}^{n}U_i)\bigcup\bigcup_{j=1}^{m}V_j,$ then $S=V\bigcup(\bigcup_{j=1}^{m}V_j),$ because $\mathcal{A}$ is finitely $j-inadequate,$ there is a contradiction, therefore we get (3).

The collection $\gamma\bigcap\mathcal{A}$ is finitely $j-inadequate,$ since $\gamma\bigcap\mathcal{A}\subseteq\mathcal{A}$ and $\mathcal{A}$ is finitely $j-inadequate.$ We have to show that $\bigvee\mathcal{A}\subseteq\bigvee(\mathcal{A}\bigcap\gamma).$ Put $s\in S$ with $\bigvee\mathcal{A}\not\subseteq Q_s.$ Also $A\in\mathcal{A}$ with $A\not\subseteq Q_s,$ because $A\in\tau$ and γ is a subbase for $\tau,$ by using $G\in\tau, G\not\subseteq Q_s\Rightarrow\exists B\in\beta$ with $B\not\subseteq Q_s$ and $B\subseteq G,$ there exists $G_1,G_2,.....G_n\in\gamma,$ then $\bigcap_{i=1}^{n}G_i\subseteq A$ and $\bigcap_{i=1}^{n}G_i\not\subseteq Q_s.$

By (3) there exists $k, 1\le k\le n,$ then $G_k\in\mathcal{A}\bigcap\gamma$ and $G_k\not\subseteq Q_s,$ because $\bigvee(\mathcal{A}\bigcap\gamma)\not\subseteq Q_s.$ This prove that $\bigvee\mathcal{A}\subseteq\bigvee(\mathcal{A}\bigcap\gamma).$ If $\bigvee\mathcal{A}=S$, then $S=\bigvee(\mathcal{A}\bigcap\gamma)$ and since by every cover of S by sets in γ has a finite $j-sub\text{cov}er$ thus we get a contradiction that the family $(\mathcal{A}\bigcap\gamma)$ is finitely $j-inadequate.$ So $\mathcal{A}$ and B is $j-inadequate$ and we get that (τ,k) is $j-compact.$

Theorem 3.13. Let (τ,k) be a ditopology on the texture space (S,ψ) and γ a subbase for the closed set $k.$ Then (τ,k) is $j-cocompact$ if and only if every $j-closed$ cocover of $\emptyset$ by member of γ has a finite $j-sub\text{cov}er.$

Proof: The dual property of the above theorem, the proof is clear.

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