

## **Waiting Time Analysis Of A Multiple – Server Queuing System In Covid – 19 Government Hospital In India**

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**Abstract:** This Research Paper Focused On Multiple Lines, Multiple Server Systems Of Patients In Covid - 19 Government Hospital In India. The Underlying Mathematical Concepts Of Queue Models: Arrival And Service Time Distributions, Queue Disciplines, And Queue Behavior. The Operating Characteristic Formulas For The Multiple-Server Queuing Model Meant To Evaluate The Performance Of Practical Queuing Systems Were Also Presented. Patients' Arrival Time And Service Time Spent To Obtain Maintenance Were Gathered And Evaluated. Various Percentages Of Typical Advent Period Combined With A Facility Period  $\frac{\alpha}{\beta}$  Were Found To Assess The Best Assistance Services (Servers) Suitable For A Government Hospital. The Operation Element, Along With The Usual Delaying Period Of The Consumer Appearing In The Structure, Lingered At 0.176 And 0.152, Correspondingly.

**Keywords:** Entrance Level, Facility Ratio, Poison Allocation, Exponential Allocation, Implementation Procedures.

### **1. Introduction**

Queue Or Awaiting Queues Are Rows Of People/Patients Awaiting Their Chance To Obtain Assistance From Maintenance Workers. The Patients At This Point Might Be Humans Or Natural Individuals Expecting Towards Receiving Help Located In Places Like Gas Bunks, Deposit, Mechanic Workshop, Airfield, Automobile Estate, And Supplies. A Row Is Common Knowledge In India, Particularly In A Government Hospital And. Corresponding To Sharma (2013), A Queue Is Created While Any Quantity Of Patients Needing Help Surpasses The Number Of Factories Or Accommodations, Making It Non-Operatable. It Uses Up Extra Time Than Needed To Help A Patient.

Patients' Awaiting Queues Generate Concerns To Administrators Of Facility Suppliers Because Of The Following Effects Of Balking And Reneging Of The Patients. Lengthy Awaiting Queues Create Tension And Provoke Errors And Expense By Equally The Awaiting Patients And Service Workers; See Uche And Ugah (2014). Patients Lose Valuable Time In Other Statements And Help; Workers Also Lose Helpful Patients Because Of Reneging And Balking When A Lengthy Line Is Created.

Usually, Growing The Amount Of Servers By The Worker Would Decrease Patients' Awaiting Period But Add To The Functioning Expense For The Assistance. Yuncheng And Liang (2002) Stated That The Line Question Is A Dilemma About An Equilibrium Amongst The Typical Awaiting Period Of Patients And The Unused Period Of The Helpers In A Help Location. Queuing Theory Is An Efficient Technique Used In Various Circumstances When It Is Impossible To Correctly Calculate The Entrance Percentage /Period Of Patients And Help Percentage (Period) Company Servers. It Is Utilized To Decide The Amount Of Assistance That Will Efficiently Utilize Time To The Patients And Assistance Operator.

Numerous Writers Have Accomplished Valuable Papers In Queueing Schemes In Various Regions Of Social Activities. Example, Ezeliiora Et Al. (2014) Examined Queue Method Administration Of Shoprite Plaza, Enugu, Using Single-Line Multiple Server Evaluation. They Suggested A Reduction In The Amount Of Employees To Lessen The Operational Price Of The Scheme And Lessen The Unused Period Of The Workers. Adamu (2015) Worked On Government Hospitals By Creating A Single Queue- Single Server And Single Queue-Multi Server Method. He Stated That Government Hospitals Should Convert The Multi-Tellers Multi-Server Method To Single Queue Multi-Tellers To Lessen The Total Awaiting Period Of Patients From The Multiple Serving Points And Lessen The Patients ' Jockey Troubles. It Was Told That The Multi-Tellers Should Enhance Their Improvement To Have An Additional Decrease Of Awaiting Patients' Time. Nearly All Of The Workings In The Paper Implemented Single Line Multiple Appearance Ratio Of Patients With Various Appearances And Assistance Percentage Of The Scheme. This Paper Concentrates On Multiple-Line Multiple-Channel Methods Of Government Hospitals With Numerous Typical Appearance Percentages Of Patients And Various Assistance Ratios Of Neutralized Servers.

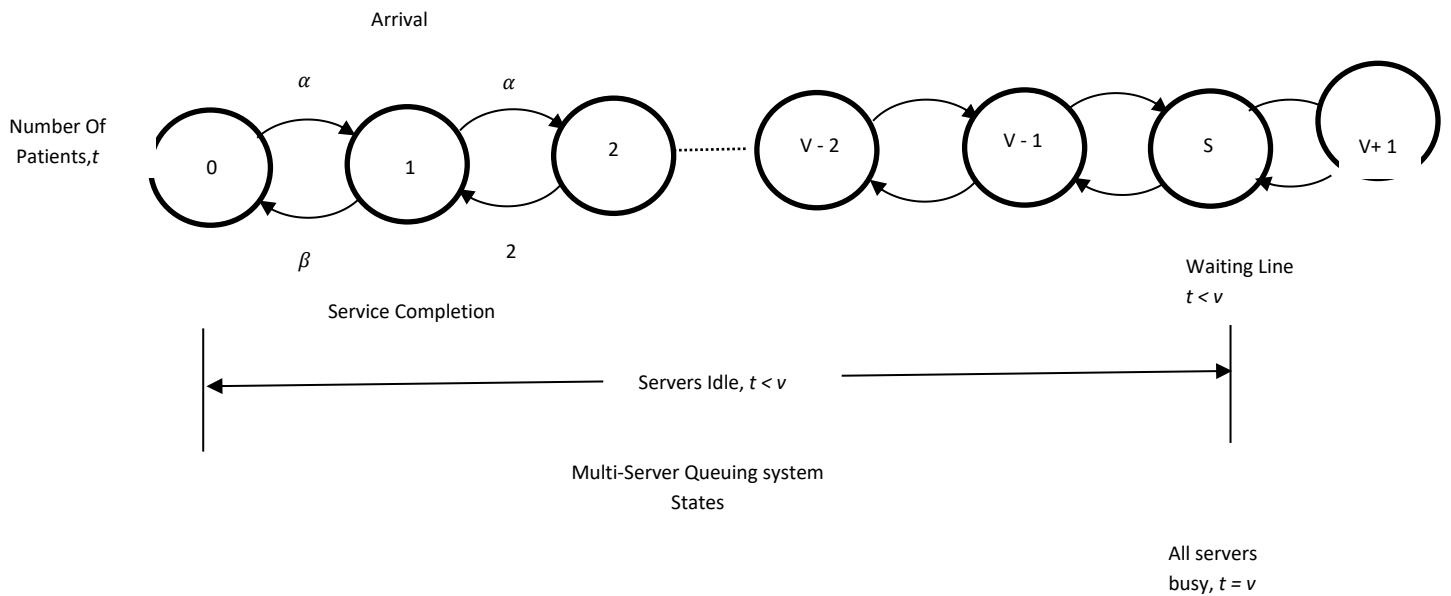
## **2 The P/P/V Model**

The Version Explained Here Is P/P/V Of Unlimited Phoning People With A First-Come, First-Served Multiple Server Queue System ( $\infty$ /FCFS). There Are Numerous Than The Same System In Similar Lines Which Offer Similar Assistance To Patients In This Situation. The Customer At The Entrance Enters Any Line, Then Stays In His Line Till He Gets Assistance. The Line Attributes Are As Follows:

- Entrances Of Consumers Adhere To Poisson Possibility Division With An Appearance Ratio, Consumers Period.  $\alpha$

- Maintenance Periods Are Exponentially Distributed With Mean Percentage, Consumers For Each Division Of Time. The Assistance Period Differs After A Single Consumer To The New Consumer However Similar For Every System.  $\beta$
- There Is No Threshold For The Amount In The Line, Therefore Endless Calling People.
- The Line Restraint Is First Come, First Serve, And Balking Or Reneging Is Not Permitted.
- If  $t < v$ , The Amount Of Consumers Is Fewer Than The Amount Of Systems So That No Line Will Be Generated, Implying That  $v - t$ , The Amount Of Systems, Will Be Empty. They Combine The Assistance Ratio Is.  $\beta_t = t\beta$
- If  $T \geq V$ , All Systems Are Functional, And The Highest Number Of Consumers In The Line Is  $t - v$ . The Merged Assistance Ratio Is  $\beta_t = V\beta$  See Akinnuli And Olugbade (2014).
- Overall Assistance Rate Should Be More Significant Than The Entry Ratio.  $V\beta > \alpha$  Then The Awaiting Line Will Be Considerably Extensive, Giving An Unwanted Chart (Adamu, 2015).

As In The Model, P/P/V Fig.1 Shows The Movement Of Patients Among Several States.



Thus, To Derive The Result For This Model, We Have  $\alpha_t > \alpha \forall t \geq 0$

$$\beta_t = \begin{cases} t\beta; & t < v \\ v\beta; & t \geq v \end{cases}$$

The Method Of Determining Probability,  $S_t$  Of  $t$  Patients In The Queuing System At Time  $W$  And Value Of Performance Measures Is Summarized Below:

### 2.1 Obtain The System Of Differential-Difference Equations.

Using The Same Logic As In Model P/P/1, We Have

$$\begin{aligned} S_t(w + \nabla w) &= S_t(w)\{1 - \alpha\nabla w\}\{1 - t\beta\nabla w\} + S_{t+1}(w)\{1 - \alpha\nabla w\}\{(t + 1)\{1 - \alpha\nabla w\}\} \\ &\quad + S_{t+1}(w)\{\alpha\nabla w\}\{1 - (n - 1)\beta\nabla w\} \\ &= -(\alpha + w\beta)S_t(w) + (t + 1)\beta S_{t+1}(w)\nabla w + \alpha S_{t-1}(w)\nabla w + S_t(w) \\ &\quad + \text{terms involving } (\nabla w)^2; \quad 1 \leq t < v \end{aligned}$$

$$\begin{aligned} S_t(w + \nabla w) &= S_t(w)\{1 - \alpha\nabla w\}\{1 - \beta\nabla w\} + S_{t+1}(w)\{1 - \alpha\nabla w\}\{v\beta\nabla w\} \\ &\quad + S_{t-1}(w)\{\alpha\nabla w\}\{1 - v\beta\nabla w\} \\ &= -(\alpha + v\beta)S_t(w)\nabla w + v\beta S_{t+1}(w)\nabla w + \alpha S_{t-1}(w)\nabla w + S_t(w) \\ &\quad + \text{terms involving } (\nabla w)^2; \quad t \geq v \end{aligned}$$

And  $S_0(w + \nabla w) = S_0(w)\{1 - \alpha\nabla w\} + S_1(w)B\nabla w; t = 0$

By Dividing These Equations By  $\nabla w$  And Then By Taking Limit As  $\nabla w \rightarrow 0$ , we get

$$S_t^1(w) = -(\alpha + t\beta)S_t(w) + (t + 1)\beta S_{t+1}(w) + \alpha S_{t-1}(w) \quad ; 1 \leq t < v$$

$$S_t^1(w) = -(\alpha + v\beta)S_t(w) + v\beta S_{t+1}(w) + \alpha S_{t-1}(w) \quad ; t \geq v$$

$$S_0^1(w) = -\alpha S_0(w) + \beta S_1(w) \quad ; t = 0$$

### 2.2 Obtain The System Of Steady-State Equations.

In This Steady – State Conditions, The Differential – Difference Equations Obtained From The Above Equations As  $W \rightarrow \infty$ , are :

$$-\alpha S_0 + \beta S_1 = 0 \quad ; t = 0$$

$$-(\alpha + t\beta)S_t + (t + 1)\beta S_{t+1} + \alpha S_{t-1} = 0 \quad ; 0 < t < v$$

$$-(\alpha + v\beta)S_t + v\beta S_{t+1} + \alpha S_{t-1} = 0 \quad ; t \geq v$$

### 2.3 Solve The System Of Difference Equations.

Employing The Iterative Technique, The Possibility Of T Patients In The Scheme Is Given By.

$$S_t = \begin{cases} \frac{\gamma^t}{t!} S_0 & ; t \leq v \\ \frac{\gamma^t}{v!v^{t-v}} S_0 & ; t > v; \gamma = \frac{\alpha}{v\beta} \end{cases}$$

Using The Following Conditions To Find Value Of  $S_0$

$$\sum_{t=0}^{\infty} S_t = 1$$

$$\sum_{t=0}^{v-1} S_t + \sum_{t=v}^{\infty} S_t = 1$$

$$\sum_{t=0}^{v-1} \frac{1}{t!} \left(\frac{\alpha}{\beta}\right)^t S_0 + \sum_{t=v}^{\infty} \frac{1}{v!v^{t-v}} \left(\frac{\alpha}{v\beta}\right)^t S_0 = 1$$

$$S_0 \left[ \sum_{t=0}^{v-1} \frac{v^t}{t!} \left(\frac{\alpha}{v\beta}\right)^t + \sum_{t=v}^{\infty} \frac{v^t}{v!v^{t-v}} \left(\frac{\alpha}{v\beta}\right)^t \right] = 1$$

$$S_0 \left[ \sum_{t=0}^{v-1} \frac{(v\gamma)^t}{t!} + \frac{v^v}{v!} \sum_{t=v}^{\infty} \gamma^t \right] = 1$$

$$S_0 \left[ \sum_{t=0}^{v-1} \frac{(v\gamma)^t}{t!} + \frac{v^v}{v!} \frac{\gamma^v}{1-\gamma} \right] = 1$$

Thus, The Possibility That The Structure Shall Be Unused Is.

$$S_0 = \left[ \sum_{t=0}^{v-1} \frac{(v\gamma)^t}{t!} + \frac{1}{v!} \frac{(v\gamma)^v}{1-\gamma} \right]^{-1}$$

$$S_0 = \left[ \sum_{t=0}^{v-1} \frac{1}{t!} \left(\frac{\alpha}{\beta}\right)^t + \frac{1}{v!} \left(\frac{\alpha}{\beta}\right)^v \frac{v\alpha}{v\alpha-\beta} \right]^{-1}$$

### 2.4 Performance Measures For Model

- The Anticipated Number Of Patients Awaiting In The Queue.  $T_r = \left[ \frac{1}{(v-1)!} \left(\frac{\alpha}{\beta}\right)^v \frac{\alpha\beta}{(v\beta-\alpha)^2} \right] S_0$
- The Anticipated Delaying Period Of A Patient In The row  $T_v = T_r + \frac{\alpha}{\beta}$
- The Required Pending Period Of A Patient In The Queue  $W_r = \left[ \frac{1}{(v-1)!} \left(\frac{\alpha}{\beta}\right)^v \frac{\beta}{(v\beta-\alpha)^2} \right] S_0 = \frac{T_r}{\alpha}$
- The Estimated Awaiting Period That A Patient Spends In The System.  $W_v = W_r + \frac{1}{\beta} = \frac{T_r}{\alpha} + \frac{1}{\mu}$
- The Likelihood That All Servers Are Concurrently Active  $S(t \geq v) = \frac{1}{v!} \left(\frac{\alpha}{\beta}\right)^v \frac{v\beta}{v\beta-\alpha} S_0$

**3 Demonstration Of Information****Table 1. Appearance And Assistance Information Of Government Hospital**

S.No	Arrival Time	Inter Arrival Time(Min)	Assistance Starts	Assistance Concludes	Assistance Period (Min)
1	8.10	-	8.15	8.19	4
2	8.12	2	8.20	8.25	5
3	8.12	0	8.25	8.27	2
4	8.15	3	8.28	8.45	17
5	8.17	2	8.45	9.00	15
6	8.25	8	9.00	9.03	3
7	8.40	15	9.03	9.08	5
8	9.01	21	9.08	9.14	6
9	9.02	1	9.14	9.20	6
10	9.03	1	9.20	9.29	9
11	9.05	2	9.29	9.32	3
12	9.10	5	9.32	9.35	3
13	9.16	6	9.35	9.40	5
14	9.20	4	9.41	9.52	11
15	9.24	4	9.52	10.03	11
16	9.27	3	10.03	10.07	5
17	9.30	3	10.07	10.15	8
18	9.30	0	10.15	10.19	4
19	9.40	10	10.19	10.22	3
20	9.44	4	10.22	10.30	8
21	9.45	1	10.31	10.39	8
22	9.52	7	10.40	10.46	6
23	9.55	3	10.46	10.53	7
24	9.58	3	10.53	10.58	5
25	9.59	1	10.58	11.01	3
26	10.00	1	11.01	11.03	2
27	10.02	2	11.04	11.06	2
28	10.03	1	11.06	11.15	9
29	10.05	2	11.15	11.19	4
30	10.13	8	11.19	11.23	4
31	10.18	5	11.23	11.30	7
32	10.25	7	11.30	11.38	8
33	10.25	0	11.38	11.40	2
		135			200

The Arrival And Assistance From The 2<sup>nd</sup>,3<sup>rd</sup>, And 4<sup>th</sup> Selected Are Reviewing Below.

**Table2. Regular Arrival And Assistance Time Of Patients For 4 Days Amongst 4 Servers**

Day		Server1		Server2		Server3		Server4	
		$\alpha_1$	$\beta_1$	$\alpha_2$	$\beta_2$	$\alpha_3$	$\beta_3$	$\alpha_4$	$\beta_4$
	Overall	135	200	165	221	201	208	157	177
Mon	Mean	4.218	6.060	4.459	7.892	4.902	5.778	5.413	6.321
Tues	Mean	5.905	7.095	3.963	5.667	4.831	6.983	4.912	6.592
Wed	Mean	5.714	7.905	5.668	7.505	3.167	6.593	4.913	6.146
Thurs	Mean	3.857	5.592	4.329	7.715	3.671	5.112	4.257	6.322
Overall		19.694	26.650	18.419	28.779	16.571	24.466	19.495	25.381
Mean ( $\alpha_1, \beta_1$ )		4.924	6.663	4.605	7.195	4.143	6.117	4.874	6.345

**Table3. The System Features Of This System.**

Amount Of Servers	4
Appearance Ratio ( $\alpha$ )	4.635 Patients / Minute
Assistance Rates ( $\beta$ )Each Hour	6.580 Patients / Minute
Typical Time Among Arrivals	0.216
Typical Assistance Period	0.152
The Likelihood That The Scheme Is Vacant	$S_0 = 0.494$
A Typical Amount Of Patients Awaiting In The Line	$T_r = 0.001$ Patients
The Typical Amount Of Patients Awaiting In The Scheme	$T_v = 0.7059$ Patients
The Standard Period A Patient Consumes Awaiting In The Queue	$U_r = 0.003$ Minutes
The Typical Period A Patient Spends Awaiting In The System	$U_v = 0.152$ Minutes
The Possibility That An Appearance Patient Should Await Assistance	$S_v = 0.006$
The Usual Traffic Force Is	$\gamma_0 = 0.176$

**Table 4. The Performing Characteristic Of Multiple – Lines Servers , While 4 Servers Are Implicated.**

S.No	$\frac{\alpha}{\beta}$	$S_0$	$T_v$	$T_r$	$U_r$	$U_v$	$S_v$	Utilization Factor
1	0.100	0.905	0.100	0.000	0.000	0.100	0.000	0.025
2	0.200	0.819	0.200	0.000	0.000	0.100	0.000	0.050
3	0.300	0.741	0.300	0.000	0.000	0.100	0.000	0.075
4	0.400	0.670	0.400	0.000	0.000	0.100	0.000	0.100
5	0.500	0.607	0.500	0.001	0.000	0.100	0.001	0.125
6	0.600	0.549	0.600	0.005	0.000	0.100	0.003	0.150
7	0.700	0.497	0.701	0.001	0.000	0.100	0.006	0.175

8	0.705	0.494	0.706	0.001	0.000	0.152	0.006	0.176
9	0.800	0.449	0.802	0.002	0.000	0.100	0.001	0.200
10	0.900	0.406	0.904	0.004	0.001	0.101	0.014	0.225
11	1.000	0.367	1.007	0.006	0.001	0.101	0.020	0.250
12	1.100	0.332	1.111	0.010	0.001	0.101	0.028	0.275
13	1.200	0.300	1.216	0.015	0.001	0.101	0.037	0.300
14	1.300	0.271	1.323	0.023	0.002	0.101	0.047	0.325
Sum								2.451
Mean								0.175

From Table 4 Above, The Mean Of The Whole Consumption Element Is 0.1750, Thus The Standards For The Perfect Functioning Of The Structure. This Implies That Any Value Lower Than The Standard Efficiency Element Will Always Enhance The Unused Time Of The Servers, Lowers The Manufacturing Period, And Improve The Price Of The Assistance. Centered On This, Some Can Tell That The Structure Operates Efficiently As The Standard Service Is Higher Than The Computed Service Element 0.176 Achieved From The Information Gathered.

In The Introduction Of This Document, We Notified That The Extra The Amount Of Systems, The Extra The Operational Price Of The Structure, And Consistently The Fewer The Awaiting Queue In The Scheme. Once More, As The Workers Will Not Enjoy Suffering The Loss Of Patients Through Balking And Reneging Because Of The Duration Of The Line In The Design, They Would Also Like To Utilize The Lowest Amount Of Systems That Would Lessen The Structure Duration Of The Fee. Therefore, The Amount Of Systems Should Be Improved.

**Table 5. Review Of The Execution Attributes Of The Government Hospital With A Various Number Of Servers.**

No Of Servers	$S_0$	$T_v$	$T_r$	$U_r$	$U_v$	Utility Factor	% Of The System Idle	%Of The System Busy
2	0.479	0.805	0.099	0.022	0.173	0.352	47.9	35.2
3	0.493	0.716	0.011	0.003	0.155	0.235	49.3	23.5
4	0.494	0.706	0.001	0.000	0.152	0.176	49.4	17.6
5	0.496	0.705	0.000	0.000	0.152	0.141	49.6	14.0
Sum								90.4
Mean								22.6

From Table 5 Above, The Mean Awaiting Period Of Customers In The Structure Are 0.173, 0.155, 0.152, And 0.152 Hours For The 2nd, 3<sup>rd</sup>, 4<sup>th</sup>, And 5<sup>th</sup> Servers, Individually. This Indicates That As The Number Of Systems Rises, The Typical Patients Awaiting The Structure Drops And Vice Versa.

Once More, The Different Systems In The Structure, The Additional Unused Period Of The Structure, And Fewer Efficiency Elements.



Ultimately, The Correct Number Of Servers That Would Enhance The Structure Is About Three Servers, Hence The Standard Above.

#### **4.1 Conclusion**

The Amount Of 4 Systems Remaining In Usage By Government Hospitals Is Doing Efficiently, As Demonstrated In Table 4. Though, The Optimum Amount Of Systems For By Government Hospital Is 3 To Lessen The Unused Period And Price Of The Procedure.

#### **Recommendation**

Queuing Theory Is Exceptionally Efficient In Overseeing Awaiting Queues And Expenditure In A Government Hospital; We Advise Using This Method In Other Parts Of The Government Hospital.

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