Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 5, July, 2021: 4555 - 4565

Research Article

Model predictive control of power converters

Paidipala Surender[1], Rajasekhara Reddy Boggula[2*]

Abstract

In this article, Model predictive control of the dynamical system with fast open-loop dynamics is presented. The success of MPC depends on the model being used to approximate the system's dynamic behavior under consideration. Kautz functions are formulated in a state-space-like representation to develop the system's model, as they are very good at approximating the signals. A two parametric Kautz model is employed to identify the dynamic characteristics of the system, and the same model is utilized as a replica of the system in the design of MPC and MPC design, a suitable objective function is chosen which will minimize the error between the system output and reference input signal. The abilities of the two parametric Kautz model and its capabilities in controlling systems with fast dynamics is demonstrated on DC-DC buck converter.

1 INTRODUCTION

Model predictive control of dynamical systems is successful and well-demonstrated control strategy, both in industry and academic research. Identification of the process model is an integral part of MPC design, traditionally known as system identification. There are several ways of developing the process models in scientific literature, among which system identification is mostly adopted and a successful methodology because of its inherent advantages. The theory of system identification is known as black-box modeling; in this way of system modeling, the model is derived from the input-output data of the system and does not depend on the fundamental physical laws of the system under consideration. It is a way of fitting a model with a degree of accuracy by mapping the input into the output space.

Several physical systems display fast dynamic characteristics, and one can find many such systems in electrical engineering applications. Such a system with fast dynamics requires an efficient and fast responding control design to achieve reasonable tracking control. These fast-dynamic systems can have applications spanning from power electronics applications to robotics. Traditionally and industrially accepted PID controllers, whose design is based on a linear approximation of a system, may be improved by replacing it with an MPC. PID controllers are generally turned on a trial and error basis for the system's best performance under consideration; hence, MPC may be a better alternative to control such a system.

Model predictive control, as the name sounds an explicit process/system model in calculating the optimal control actions required to have excellent and stable control. An appropriate objective function is minimized in the design procedure over a definite period, called the prediction horizon. The first few samples of the control moves are applied to the system, called

[1,2]Department of Electronics and Instrumentation Engineering, VNR Vignana Jyothi Institute of Engineering and Technology, Bachupally, Hyderabad, Telangana, INDIA-500090. [1] surender.paidipala@gmail.com, [2*]rajasekharareddy_b@vnrvjiet.in

the control horizon. The simplest form of representing the dynamic signal is either by finite Impulse Response (FIR) or Infinite Impulse Response (IIR). Still, both the representations are sufferers from the requirement of many shifting filters. In other words, it needs more number filter coefficients to represent the signals, and many times, the optimal solutions resulting from FIR or IIR representation sufferers from lack of stability in the numerical solutions.

Power electronics devices such as power converters need a fast and reliable control strategy because of their meager response time [3, 10]. A detailed review on this topic is presented elsewhere [11]. MPC was introduced by [4] in 1960, the optimal control action was calculated by solving a linear programming problem. PID controller is the popularly used and industrially accepted control strategy that is not demonstrated and expected for the constrained process. Most of the time, PID control parameters are used to turn on the heuristic approach. The controllers designed based on the empirical model of the process are an alternative way of dealing with such systems and are known as model-based control algorithms. Model predictive control (MPC in short) is one of the two crucial model-based control methodologies. MPC is a proven control algorithm in controlling non-linear, constrained, and MIMO systems. The control action is calculated as an optimization problem and process model embedded in the controller in MPC. Model predictive control design based on the orthogonal basis functions (OBF's) is well established and demonstrated [1, 9, 12, 13]. The use of OBF's in system identification will naturally reduce the number of model parameters because of the inherent orthogonal property of these functions. In other words, the resulting models will have parsimonious nature and lead to a simple solution.

The issues referred to above can be addressed by integrating the theory of orthogonal filters makes them very handy and effective in system modeling and/or control design. The orthogonal property of these filters will enable the control system designed to reduce the numerical computational burden to a great extent and reduce the number of model parameters required in modeling the dynamical system.

Laguerre filter and Kautz filters are two famous orthogonal filters that appeared most of the orthogonal filters, which can be parameterized utilizing a single Laguerre parameter (typically a real-valued number). In contrast, the Kautz filter is generally parameterized by a complex conjugate number, and their representation also looks mathematically complex compared to that of Laguerre representation [6, 7, 5, 2]. The complexity of the Kautz filters is because of their complicated mathematical structure. Still, it can be neglected in system identification as it is a one-time job.

2 Theoretical developments

The Orthogonal property and the z-domain representation of Kautz functions are summarized in the following [13]:

Theorem 1: The set $\{\Psi_j(z)\}, \forall_j \in [1, 2n]$, defined as following,

$$\Psi_{2n-1}(z) = C_1^n \left(1 - a_1^n z\right) \Gamma^{(n)}(z) \quad (1)$$

$$\Psi_{2n}(z) = C_2^{(n)} \left(1 - a_2^{(n)} z\right) \Gamma^{(n)}(z) \quad (2)$$

for $\forall$ n = 1, 2 … where

$$\Gamma^{(n)}(z) = \frac{\prod_{j=1}^{n-1}\left(1-\beta_j z\right)\left(1-\beta_j^* z\right)}{\prod_{j=1}^{n}\left(z-\beta_j\right)\left(z-\beta_j^*\right)} \quad (3)$$

$$\left(1+a_1^{(n)}a_2^{(n)}\right)\left(1+\beta_n\beta_n^*\right)-\left(a_1^{(n)}+a_2^{(n)}\right)\left(\beta_n+\beta_n^*\right)=0 \quad (4)$$

$$C_1^{(n)} = \left(\frac{\left(1-\beta_n^2\right)\left(1-\beta_n^{*2}\right)\left(1-\beta_n\beta_n^*\right)}{\left(1+\left(a_1^{(n)}\right)^2\right)\left(1+\beta_n\beta_n^*\right)-2a_1^{(n)}\left(\beta_n+\beta_n^*\right)}\right)^{1/2} \quad (5)$$

$$C_{\frac{1}{2}}^{(n)} = \sqrt{\frac{(1-\beta_n^2)(1-\beta_n^{*2})(1-\beta_n\beta_n^*)}{\left\{1+(a_{\frac{1}{2}}^{(n)})^2\right\}\{1+\beta_n\beta_n^*\}-2a_{\frac{1}{2}}^{(n)}\{\beta_n+\beta_n^*\}}} \quad (6)$$

Follow orthogonal property, i.e.,

$$\delta_{mn} = \frac{1}{2\pi i}\oint \Psi_m(z)\Psi_n(z^{-1})\frac{dz}{z} \quad (7)$$

δ_{mn} Is the Kronecker delta function and $\{\beta_n, \beta_n^*\} \in \mathbb{C}, |\beta_n| < 1$. the set of functions $\{\Psi_j(z), \forall j = 1, 2 \ldots$ are refereed as discrete-time Kautz functions.

Condition I

When $\beta_n, \beta_n^* = a_1$, i.e., the parameters are real-valued. The solution of Eq.4 is simplified as

$$a_1^{(n)} = a_1 \text{ and } a_2^{(n)} = \frac{1}{a_1}$$

$$C_1^{(n)} = \sqrt{(1-a_l^2} \text{ and } C_2^{(n)} = \sqrt{1-a_l^2}$$

$$\Psi_j(z) = \sqrt{1-a_l^2}\,\frac{(1-a_1 z)^{j-1}}{(z-a_l)^j} = L_j(z) \quad (8)$$

Kautz functions take a more straightforward version named Laguerre functions.

Condition II

When $\beta_n, \beta_n^* = \beta$, for imaginary and equal values of the parameter, The solution of Eq. 4 is as follows.

$$a_1^{(n)} = \frac{1+\beta\beta^*}{\beta+\beta^*} \text{ and } a_2^{(n)}=0$$

$$\Psi_{2n-1}(z) = \frac{C_1^{(n)}(1-a_1^{(n)}}{z^2+b_k z+c_k}\;{}^{n-1}\sqrt{\frac{c_k z^2+b_k z+1}{Z^2+b_k z+c_k}} \quad (9)$$

$$\Psi_{2n}(z) = \frac{C_1^{(n)}(1-a_1^{(n)}}{z^2+b_k z+c_k}\;{}^{n-1}\sqrt{\frac{c_k z^2+b_k+1}{z^2+b_k z+c_k}} \quad (10)$$

Where $b_k = -(\beta+\beta^*)$ and $c_k = \beta \times \beta^*$

Eq. 9 and Eq. 10 show a simple version of Kautz functions with complex conjugate and constant number with 2n number of Kautz function.

3 Kautz Representation

Any system can be approximated as a weighted sum of orthogonal basis functions if it follows the following,

- Strictly proper, i.e., (G(∞)=0)
- Continuous in |z| ≥ 1.

In the discrete-time domain, the output of the systems can be expressed as follows,

$$\mathrm{y}(z) = \sum_{j=1}^{2N} \theta_j \Psi_j U(z) \qquad (11)$$

The input to the system can be mapped into the output domain by choosing the suitable parameters, called model parameters as,

$$y(kT_s) = \Theta \Psi_{(q)} u(kT_s) \qquad (12)$$

Where T_s is sampling time, By considering the Kautz function $\Psi_1, \Psi_2, \ldots\ldots, \Psi_{2N-1}, \Psi_{2N}$ as states of the system under consideration and defining delay operator 'q.' It is possible to estimate the model regression parameter Θ using the least-squares approach. A state-space representation can be derived as follows,

$$\Psi_{2n-1}(kT_s) = \begin{cases} \dfrac{1}{q^2 + b_k q + c_k} U(z), & for \quad n = 1 \\ \dfrac{c_k q^2 + b_k q + 1}{q^2 + b_k q + c_k} U(z) & for \quad n > 1 \end{cases} \qquad (13)$$

$$\Psi_{2N}(kT_s) = \Psi_{2N-1}(kT_s - 1) \qquad (14)$$

Where $b_k = -\beta - \beta^* \quad and \quad c_k = \beta \times \beta^*$

For n=1, by defining the following states,

$$\Psi_1(kT_s) = \frac{1}{q^2 + b_k q + c_k} \qquad (15)$$

$$\Psi_1(kT_s) = \Psi_1(kT_s - 1) \qquad (16)$$

Rewriting the above in the time domain leads to the following,

$$\Psi_1(kT_s + 1) = -b_k \Psi_1(kT_s) - c_k \Psi_1(kT_s - 1) + u(kT_s) \qquad (17)$$

Upon replacing $\Psi_1(kT_s - 1)\ with\ \Psi_2(kT_s) in\ Eq. 17$

$\Psi_1(kT_s+1) = -b_k \Psi_1(kT_s) - c_k \Psi_2(kT_s) + u(kT_s)$

$$\Psi_2(kT_s + 1) = \Psi_1(kT_s) \qquad (18)$$

For n=2, by defining the following states,

$$\Psi_3(kT_s) = \frac{c_k q^2 + b_k q + 1}{q^2 + b_k q + c_k} \times \Psi_1(kT_s) \qquad (19)$$

$$\Psi_4(kT_s) = \Psi_3(kT_s - 1) \qquad (20)$$

Applying the inverse transformation leads to,

$$\Psi_3(kT_s + 1) = b_k(1 - c_k)\Psi_1(kT_s) + (1 - c_k^2)\Psi_2(kT_s) - b_k \Psi_3(kT_s) - b_k \Psi_3(kT_s) - c_k \Psi_4(kT_s) + c_k u(kt_s)$$

$$\Psi_4(kT_s) = \Psi_3(kT_s - 1) \qquad (21)$$

From Eq.18 and Eq.21, one can observe that the equations can be written in a matrix notation as follows,

$$\begin{bmatrix}\Psi_1(kT_s+1)\\ \Psi_2(kT_s+1)\\ \Psi_3(kT_s+1)\\ \Psi_4(kT_s+1)\end{bmatrix} = \begin{bmatrix}-b_k & -c_k & 0 & 0\\ 1 & 0 & 0 & 0\\ b_k(1-c_k^2) & 1-c_k^2 & -b_k & -c_k\\ 0 & 1 & 0 & 0\end{bmatrix} \times \begin{bmatrix}\Psi_1(kT_s)\\ \Psi_2(kT_s)\\ \Psi_3(kT_s)\\ \Psi_4(kT_s)\end{bmatrix} + \begin{bmatrix}1\\ 0\\ c_k\\ 0\end{bmatrix} \times u(kT_s) \quad (22)$$

$$y(kT_s = \Theta\Psi(kT_s)$$
(23)

The regression coefficient vector (Θ) is calculated as,

$$\Psi_{2n-1}(kT_s) = C_1^n[\Psi_{2n}(kT_s) - a_1^n\Psi_{2n-1}(kT_s)]$$
$$\Psi_{2n}(kT_s) = c_2^n[\Psi_{2n}(kT_s) - a_2^n\Psi_{2n-1}(kT_s)] \quad (24)$$

From the above derivations, it can be generalized (for an N^{th} order Kautz model), and it is possible to develop a mathematical expression as given below

$$\Psi(kT_s+1) = A_k\Psi(kT_s) + b_k(kT_s) \quad (25)$$

Where

$$\Psi(kT_s) = \begin{bmatrix}\Psi_1(kT_s)\\ \Psi_2(kT_s)\\ \Psi_3(kT_s)\\ \Psi_4(kT_s)\\ .\\ .\\ .\\ \Psi_{2N}(kT_s)\end{bmatrix}$$

is the vector that needs to be updated based on the constant-coefficient matrices A_k and vector b_k. The output of the system under consideration can be approximated sum of Kautz states.

$$y(kT_s) = \sum_{j=1}^{2N} \theta_j\,\Psi_j(kT_s) \quad (26)$$

$$y(kT_s) = \Theta\Psi\,(kT_s) \quad (27)$$

And Θ is calculated by applying linear regression.

4 Model predictive control design

A dynamic model of a system is a vital issue in MPC calculations. Generally, a model of the process is utilized as a replica of the process to forecast the process's future dynamic behavior. A moving window, called prediction horizon (N_p), is the time over window called control horizon (N_c). The overall aim of predictive control is to calculate a set of control moves $\{u(kT_s+p-1/kT_s), \forall p\in[1N_c]\}$ by optimizing a suitable defined objective function to obtain maximizing yield and minimizing the control effort required.

The Kautz model developed in section 3 serves as a dynamic model to predict a system's future behavior in the present work.

$$\Psi\left(kT_s + K_i + {1}/{kT_s}\right) = A_k\Psi(kT_s + {k_i}/{kT_s}) + b_k u(kT_s + {k_i}/{kT_s})$$
$$y\left(kT_s + K_i + {1}/{kT_s}\right) = \Theta\Psi\left(kT_s + K_i + {1}/{kT_s}\right) \quad (28)$$

Where k_i is the time window, $k_i \in [1, N_p]$. Derivation between process and model is referred to as process model mismatch ($d\left({kT_s}/{kT_s}\right) = y_p(kT_s) - y_m({kT_s}/{kT_s} - 1)$) and the predicted output is updated as,

$$y_c\left(kT_s + {k_i}/{kT_s}\right) = y_p(kT_s) + d({kT_s}/{kT_s}) \quad (29)$$

In the prediction control calculations, the future control effort is calculated by optimizing the derivation between the know reference trajectory $y_{sp}(kT_s + ({K_i}/{kT_s}))$ and the corrected prediction of the process. A quadratic objective function is optimized over a moving horizon to ensure the global optimal solution,

$$\min_{\Delta u} \sum_{k_i=1}^{N_p} E_c\left(kT_s + {K_i}/{kT_s}\right)^T QE(kT_s + {k_i}/{kT_s})$$

And $E_c\left(kT_s + {K_i}/{kT_s}\right) = y_{sp}\left(kT_s + {K_i}/{kT_s}\right) - y_c(kT_s + {k_i}/{kT_s})$ (30)

5 Results and Discussion

The developed Kautz model is applied to a Dc-Dc buck converter to prove the developed model's capabilities in modeling systems with a fast and underdamped response. The results presented are carried out on MATLAB® (R10a) with Window 10 64-bit PC.

5.1 case study: DC-DC buck converter

The DC-DC buck converter is found everywhere in the power conversion application because of its efficiency in converting a high voltage signal to a low voltage signal. Extended battery life, low heat losses are some of the advantages of power conversion, and these advantages allow for the fabrication of low-power electronic systems. Generally, the circuit components such as capacitors, inductors, and the load resistance decide the mode of operation of buck converter, called continuous mode and discontinuous mode of operation of buck converter, called continuous mode and discontinuous mode. The schematic of a DC-DC buck converter is as shown in Fig 1. IN the present study, continuous conduction mode (CCM)

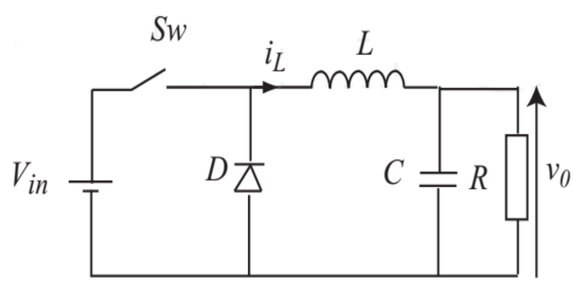

Fig 1: Schematic of DC-DC buck converter

The DC-DC buck converter is considered with a switching period(T). DC-DC buck converter displays linear dynamic characteristics when the switches are closed.

The dynamics of the converter can be represented as the following equations with closed switches [8].

$$\frac{dx(t)}{dt} = A_1 x(t) + B_1 u(t)$$

$$y(t) = C_1 x(t) + E_1 u(t) \quad (31)$$

With opened switches, the converter dynamics can be represented by the following equations,

$$\frac{dx(t)}{dt} = A_2 x(t) + B_2 u(t)$$

$$y(t) = C_2 x(t) + E_2 u(t) \quad (32)$$

From the above Eq. 31 and Eq. 32, the average model of the converter system in the state-space model as a function of the duty cycle can be written as

$$\frac{d\bar{x}(t)}{dt} = A_{converter}(d)\bar{x} + B_{converter}(d)\bar{u}$$

$$y = C_{converter}(d)\bar{x} + E_{converter}(d)\bar{u} \quad (33)$$

Where $A_{converter}(d), B_{converter}(d), C_{converter}(d)\ and\ E_{converter}(d)$ are defined as

$$A_{converter}(d) = dA_1 + (1-d)A_2$$

$$B_{converter}(d) = dB_1 + (1-d)B_2$$

$$C_{converter}(d) = dC_1 + (1-d)C_2$$

$$E_{converter}(d) = dE_1 + (1-d)E_2 \quad (34)$$

$\overline{x}$(t), $\overline{u}$(t), and $\overline{y}$(t) are the corresponding average value of u(t), x(t), and y(t) during one switching period (T), and d is the duty cycle. The dynamic model of the buck converter is as described below [8].

$$\frac{d\bar{\imath}_L}{dt} = \frac{-1}{L}\bar{v}_o + \frac{d}{L}V_{in}$$

$$\frac{d\bar{v}_o}{dt} = \frac{1}{C}\bar{\imath}_L + \frac{-1}{RC}\bar{v}_o$$

$$\bar{y} = \bar{v}_o \quad (35)$$

The parameter values of DC-DC buck converter in the present study are V_{in} = 15V, R = 10Ω, L = 3, C = 22F

5.2 Identification of DC-DC buck converter

The Dc-Dc buck converter is excited with an appropriate input signal around the operating point, and the data is collected to train, test, and validate the Kautz model.

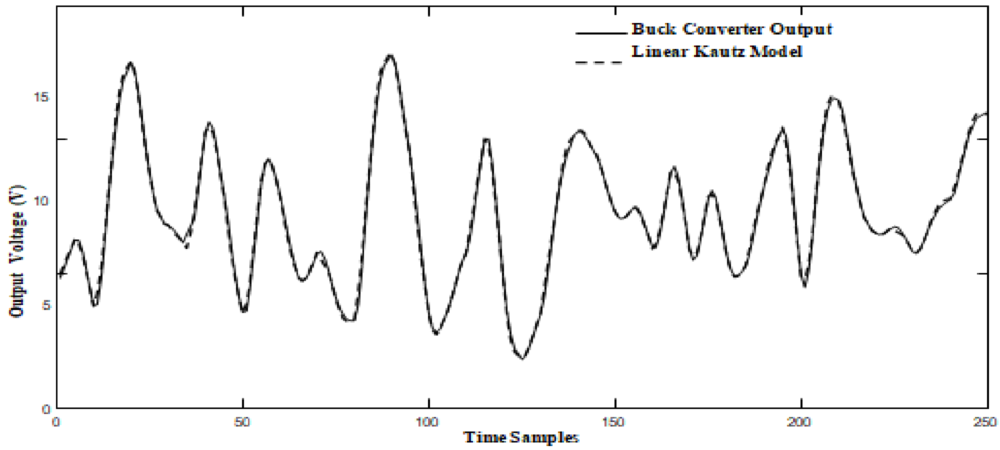

Figure 2: Identification of DC-DC buck converter measured voltage across the load (solid line) and predicted voltage using linear Kautz model (dash-dot line) and predicted voltage using linear Kautz model (dash-dot line).

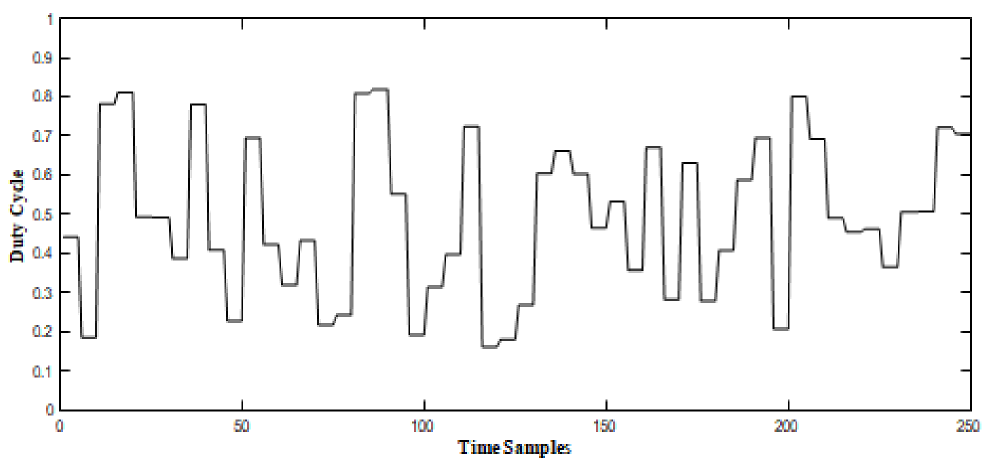

Figure 3: Test signal for DC-DC buck converter identification

Input duty cycle perturbation of mean value 0.47. A total of 5000 data points were collected by solving system describing equations of DC-Dc buck converter(refer Eq. 35) with a sampling rate of $\frac{1}{f_s}sec$. The data is divided into two parts, one to train the model and another one to test the model. 75% of the data is used to train the model and to calculate the model parameters for the best performance, and the remaining 25% of the data is used to validate the best performance. The remaining 25% of the data is used to validate the developed Kautz model is evaluated based on a performance index called root mean square error (RMSE) value between the process/ model mismatch. The identification capabilities of the developed Kautz model are compared graphically and are shown in Fig. 2. It can be easily observed from figure 2 that the Kautz model is very efficient in approximating the resonant zone of the DC-DC buck converter. The RMSE value between process/model mismatch is reported as low as 0.0143.

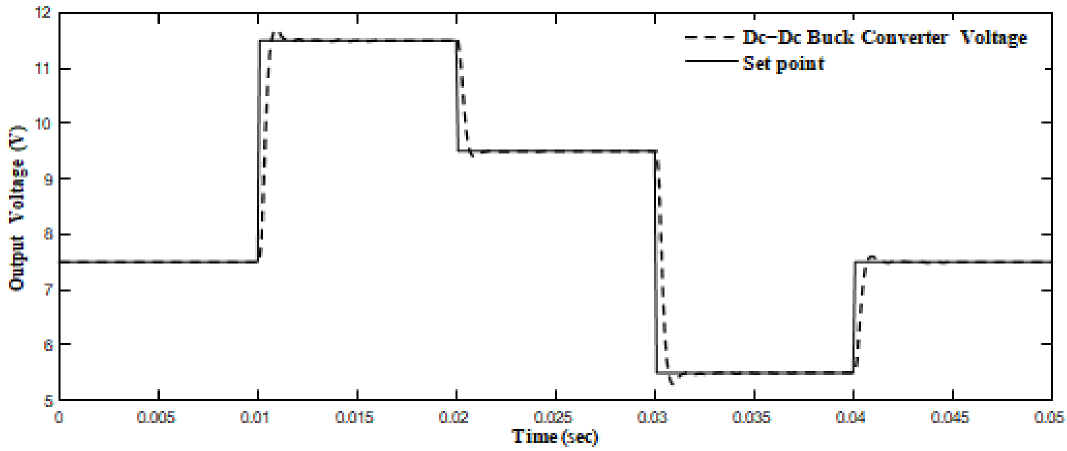

Figure 4: The performance of Kautz MPC in controlling DC-DC buck converter: converter load voltage

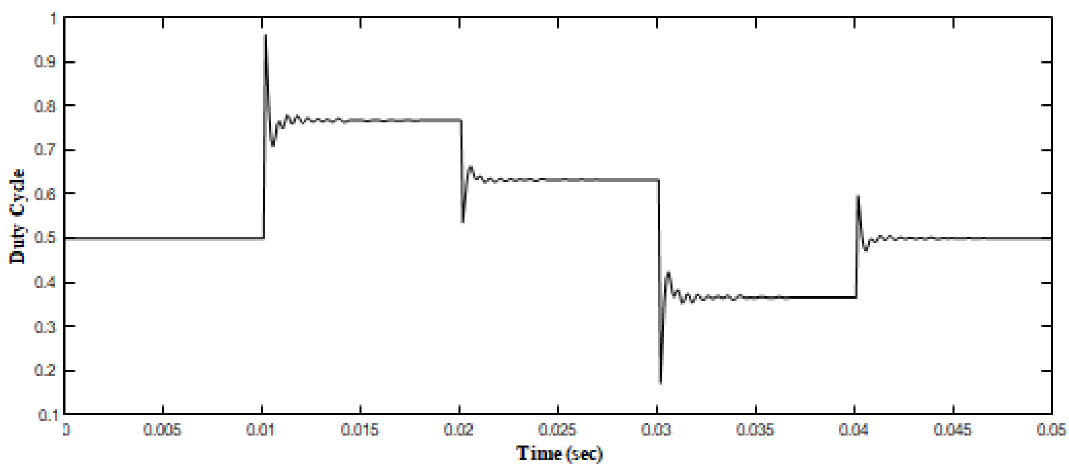

Figure 5: The performance of Kautz MPC in controlling DC-DC converter: Input duty cycle profile

5.3 MPC of DC-DC buck converter

Both unconstrained and constrained model-based predictive control strategies are implemented on the DC-DC buck converter system. In the design, the prediction and control horizons are chosen as 10 and 1, respectively. The Kautz model-based predictive control performance is shown in Fig. 4, Fig. 5, Fig. 6, and Fig. 7. In both cases, the Kautz model is based on tracking the reference voltage even with a fast switching in the reference voltage values. From fig. 6 and Fig. 7, one can easily observe the reference tracking capabilities of Kautz MPC with a constant of duty cycle(0.25 ≤ Duty cycle ≤ 0.75).

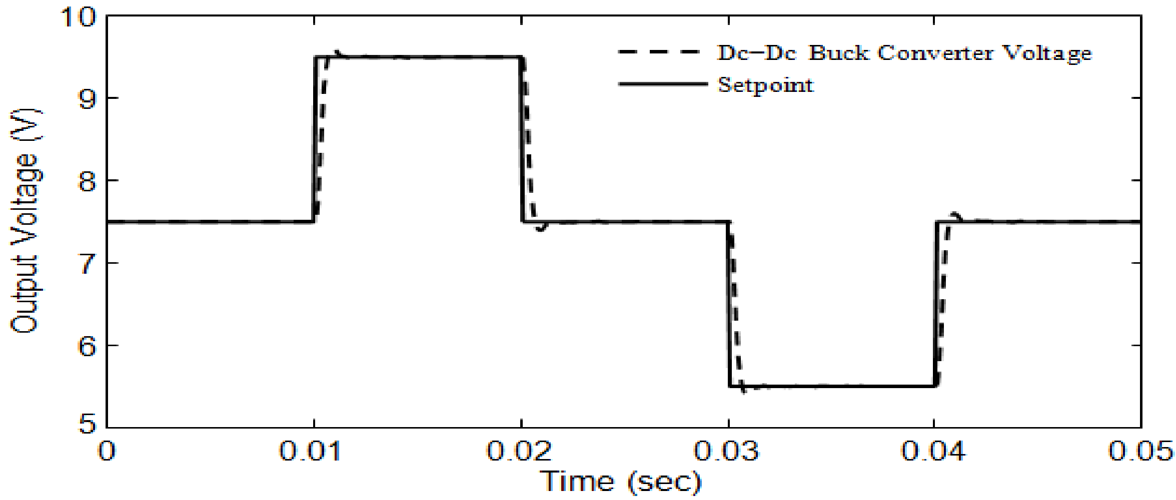

Figure 6: the performance of Kautz MPC in controlling DC-DC buck converter with constraints on duty cycle: converter load voltage

6 Conclusions

This work introduces a methodology for model identification and predictive control of underdamped systems using the Kautz model. Kautz's efficiency in modeling and controlling the resonating system is demonstrated through a case study. It has been observed that the process-model mismatch is significantly more minor in approximating the dynamics of DC-DC buck converters, whose dynamics are high-speed. An MPC has been designed based on the model developed, the capabilities of Kautz model-based MPC id presented. From simulation studies, it is evident that the Kautz filters are good at dealing with resonating systems.

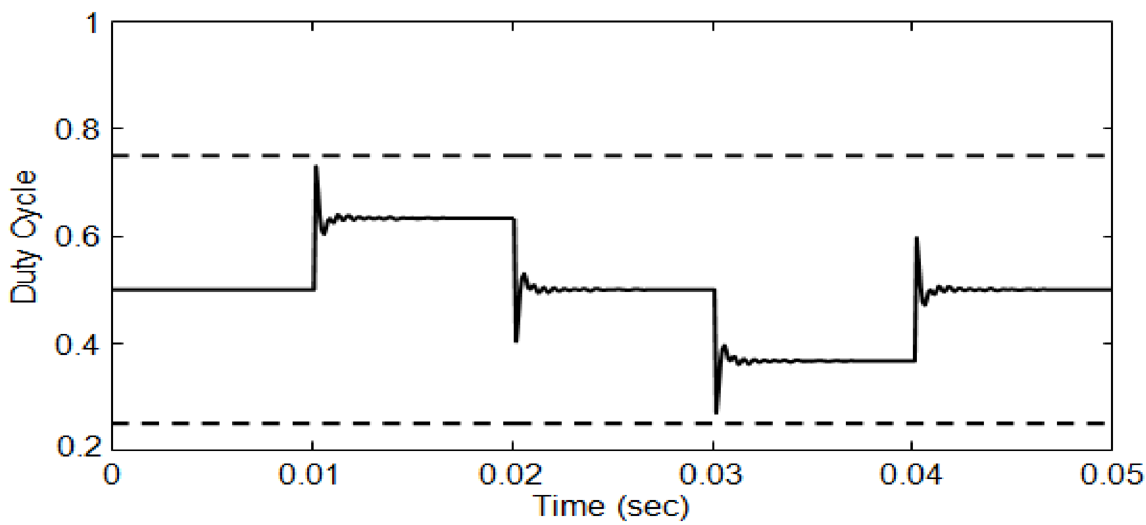

Figure 7: The performance of Kautz MPC in controlling DC-DC converter with constraints on duty cycle: Input duty cycle profile.

REFERENCES

[1] W. Kautz. Brief synthesis in the time domain. Transactions of the IRE Professional Group on circuit theory 1954; CT-1 (3): 29-39. Doi:10.1109/TCT.1954.1083588
[2] Samik M, Reddy R, Saha P. Model predictive control of resonant systems using Kautz model. International journal of automation & computing 2016; 13(5): 501-515. Doi: 10.1007/s11633-016-0954-x
[3] Perez MA, Bernet S, Rodriguez J, Kouro S, Lizana R et al. circuit Topologies, Modeling, Control Schemes, and applications of Modular Multilevel Converters. IEEE Transactions on Power Electronics 2015; 30(1): 4-17. Doi:10.1109/TPEL.2014.2310127
[4] Andrew IP. Use of linear programming methods for synthesizing sampled data automatic systems. Automation & Remote control 1963; 24(7): 837-844
[5] Reddy R, Saha p.Kautz filters based model predictive control for resonating systems. International journal of dynamics & control 2017; 5(3): 477-495.DOI: 10.1007/s40435-016-0242-1
[6] Reddy R, Saha P. Modeling and control of non-linear resonating processes: part I-system identification using orthogonal basis function. International journal of dynamics & control 2017; 5(4): 1222-1236. Doi: 10.1007/s40435-016-0277-3
[7] Reddy R, Saha P. Modeling and control of non-linear resonating processes: Part II-model based control using orthogonal basis function-based wiener models. International Journal of dynamics and control 2017; 5(4): 1237-1251. DOI: 10.1007/s40435-016-0276-4
[8] Kamel BS, sahbani, Abdelaziz, Mohamed B. sliding Mode control. Poland: InTech,2011
[9] Saha P, Krishnan SH, Rao VSR, Patwardhan SC. Modeling and predictive control of MIMO non-linear systems using Winer-Laguerre models. Chemical Engineering Communications 2004: 191(8): 1083-1119.doi:10.1080/0098644049276452
[10] Tarakanth K, Patwardhan S, Agarwal V. internal model control f DC-DC boost converter Exhibiting non-minimum phase behavior. In: IEEE 2014 international conference Mumbai, India; 2014. Pp.1-7.
[11] Vaquez S Rodriguez J, Rivera M, Franquelo LG, Norambuena M et al. Model predictive control for power converters and drives: Advances and Trends. IEEE Transactions on Industrial Electronics 2017: 64(2): 935-947.doi: 10.1109/TIE.2016.2625238
[12] Wahlberg B. Identification of resonant system using Kautz filters, In: IEEE 1991 conference on Decision and control; Brighton, England; 1991 pp.2005-2010
[13] Wahlberg B system identification using lauguerre models. IEEE transactions on Automatic control 1991 36(5): 551-562. Doi: 1.1109/9.76361.