

A Hybrid Group Acceptance Sampling Plans For Life Tests Based On Half Logistic Distribution

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Abstract

In this article, when the lifetime of an item follows a half logistic distribution, a hybrid group acceptance sampling method based on shortened lifetimes is developed. The minimum number of testers and acceptance number required for a particular group size are determined for a specified consumer risk and test termination time. The minimum ratios of the true average life to the stipulated life at a certain producer's risk are determined using the values of the operational characteristic function for various quality levels. Examples are used to demonstrate the findings.

***Keywords:** hybrid group acceptance sampling plan, consumer's risk, operating characteristic function, producer's risk, truncated life test, half logistic distribution*

Introduction

Acceptance or rejection of a product is based on its suitability for use. There are various sorts of quality checking processes used in quality control. Acceptance sampling plans are one example of such a procedure. An acceptance sampling strategy is a method for determining the minimal sample size for testing. This is especially essential if a product's quality is determined by how long it lasts. When constructing a sampling plan, it is frequently believed that only one item will be placed in a tester. In practise, however, testers who can handle a large number of items at once are used since testing time and money can be saved by evaluating objects at the same time. A group of objects in a tester can be considered, and the number of items in a group is referred to as the group size. A group acceptance sampling strategy is an acceptance sampling plan based on such groups of items (GASP). The hybrid group acceptance sampling plan is a way of calculating the minimal number of items for a predetermined number of groups (HGASP). When the HGASP is used in conjunction with shortened life tests, it is referred to as an HGASP based on truncated life tests.

Acceptance sampling plans, group acceptance sampling plans, and hybrid group acceptance sampling plans (HGASP) of abbreviated life assessments have all been studied and can be

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found in Epstein(1954) [8], Gupta(1962) [11], Fertig and Mann(1980) [9], Kantam(2001) *et al* [13], Baklizi(2003) [6], Wu and Tsai(2005) [23], Rosaiah and Kantam(2005) [16], Balakrishnan *et al* (2007) [7], Aslam(2007) [1], Srinivasa Rao *et al* (2008) [17], Aslam and Kantam(2008) [2], Aslam *et al* (2009) [3], Srinivasa Rao *et al* (2009) [18], Lio *et al* (2010) [14], Srinivasa Rao *et al* (2010) [20], Aslam *et al* (2011) [4], Aslam *et al* (2011a) [5], Ramaswamy and Anburajan (2012) [15], Gupta and Groll(1961) [10], Kantam and Rosaiah(1998) [12], Tsai and Wu(2006) [22], Srinivasa Rao (2009) [19], Srinivasa Rao(2011) [21].

In section 2, we discuss the proposed hybrid group acceptance sampling strategy (HGASP), which is based on shortened life tests where a product's lifetime follows a half logistic distribution. Section 3 contains the operating characteristic (OC) and producer risk. Section 4 has the results, which are presented with various instances, and section 5 contains the summary and conclusions.

The Hybrid Group Acceptance Sampling Plans (HGASP)

The probability density function (pdf) of a half logistic distribution is given by

$$p(x) = \frac{2e^{-x}}{(1+e^{-x})^2}, \quad x \geq 0 \quad (2.1)$$

Its cumulative distribution function (cdf) is

$$p(x) = \frac{(1-e^{-x})^2}{(1+e^{-x})^2}, \quad x \geq 0 \quad (2.2)$$

An increasing failure rate (IFR) model with a half logistic distribution is most beneficial in reliability investigations. We're interested in studying this distribution because of its IFR character. Assume that a product's lifetime follows a half logistic distribution with σ as the scale parameter. $F(t)$ is the cumulative distribution function of it given by

$$F(t) = \frac{\left(1 - e^{-\frac{t}{\sigma}}\right)^2}{\left(1 + e^{-\frac{t}{\sigma}}\right)^2}, \quad t \geq 0, \sigma \geq 0 \quad (2.3)$$

Given $0 < q < 1$, the 100th percentile is given by

$$t_q = \sigma \log\left(\frac{1+q}{1-q}\right) \quad (2.4)$$

Substituting σ in the equation 2.3 in the scaled form we get

$$F(t) = \frac{1 - e^{-\left(\frac{t}{t_q}\right) \log\left(\frac{1+q}{1-q}\right)}}{1 + e^{-\left(\frac{t}{t_q}\right) \log\left(\frac{1+q}{1-q}\right)}} \quad (2.5)$$

$$F(t) = \frac{1 - e^{-\delta \log\left(\frac{1+q}{1-q}\right)}}{1 + e^{-\delta \log\left(\frac{1+q}{1-q}\right)}} \quad (2.6)$$

where $\delta = \frac{t}{t_q}$.

It is obvious that the mean and median are the same when the distribution is symmetric. When the distribution is skewed, that is, one side of the tail is longer than the other, the mean is expected to tend toward that side of the distribution. We can make the mean considerably greater and bigger by increasing the amount of skewness, in which case the fraction of the population below the mean can be made excessively enormous. That is what it means when it

is said that the mean would not represent the distribution's centre because more than 80% of the population could be below it. However, if the median is used, there is always 50% of the population less than the median. As a result, we may conclude that sampling plans based on population median are more cost-effective than sampling plans based on population mean in terms of sample size. For our current skewed population, the median is a more approximate average for decision-making concerning the quality of life than the population mean, especially if we take $q=0.50$.

Let μ be the true value of the median of a product's lifespan distribution, and 0 be the prescribed median, assuming that an item's lifetime follows a half logistic distribution. We want to test the hypothesis $H_0: \mu \geq \mu_0$ against $H_1: \mu < \mu_0$ based on the failure data. If the sample information supports the hypothesis, a lot is regarded good and accepted for consumer usage; on the other hand, if $\mu < \mu_0$, the lot of the product is rejected. This hypothesis is tested in acceptance sampling plans based on the number of failures from a sample in a pre-determined time. We reject the lot if the number of failures exceeds the action limit c .

If there is sufficient evidence that 0 at a given threshold of consumer risk, we will accept the entire package. Otherwise, we would reject the entire batch. Based on the truncated life test, let us suggest the following HGASP:

1. Determine the number of testers r and assign the r items to each predefined group g , the required sample for a lot is $n = g.r$.
2. Pre-fix the acceptance c for each group and the experiment time t_0 .
3. Accept the lot if at most c failures occur in each of all groups.
4. Terminate the experiment if more than c failures occur in any group and reject the lot.

We want to know how many testers r are needed for a half logistic distribution and what values of acceptance number c are acceptable, assuming that the group size g and the termination time t_0 are known. We will choose $t_0 = \delta\mu_0$ for a particular constant δ since it is more convenient to establish the termination time as a multiple of the provided value μ_0 of the median (termination ratio). The producer's risk is of probability (α) of rejecting a good lot, whereas the consumer's risk is the probability (β) of accepting a bad lot. The recommended sampling plan's parameter value g is determined to ensure the consumer's risk. The consumer's risk is frequently indicated by the consumer's level of confidence. The consumer's risk is equal to $1 - p^*$ if the confidence level is p^* . We'll figure out how many groups g to include in the recommended sampling plan such that the consumer's risk doesn't reach a certain threshold β . We can use the binomial distribution to create the HGASP if the lot size is large enough. According to the HGASP, a lot of items is only accepted if each of the g groups has at least c failures.

As a result, the probability of a lot being accepted is given by

$$\left(\sum_{i=0}^c \binom{r}{i} P_0^i (1 - P_0)^{r-i}\right)^g \leq \beta \tag{2.7}$$

where $p_0 = F_t(\delta_0)$ is the probability of a failure during the time $t = \delta t_q^0$. To save space, only the results of small sample sizes for $\beta=0.25,0.10,0.05,0.01$; $g=2(1)10$; $c=0(1)8$; $\delta=0.7,0.8,1.0,1.2,1.5,2.0$ are displayed in table 1 .

Table.1. Minimum no.of testers(r) required for the proposed plan in the case of HLD

β	g	c	δ					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	1	1	1	1	1	1
0.25	3	1	3	3	3	1	1	1
0.25	4	2	5	5	5	5	4	4
0.25	5	3	7	6	6	6	4	4
0.25	6	4	9	8	8	6	4	4
0.25	7	5	11	8	8	6	4	4
0.25	8	6	13	11	11	10	4	4
0.25	9	7	15	13	11	10	4	4
0.25	10	8	17	15	11	10	10	4
0.10	2	0	2	2	1	1	1	1
0.10	3	1	4	3	3	1	1	1
0.10	4	2	6	5	3	1	1	1
0.10	5	3	8	7	6	4	1	1
0.10	6	4	10	9	6	4	1	1
0.10	7	5	12	11	9	8	8	6
0.10	8	6	14	11	9	8	8	6
0.10	9	7	16	14	12	8	8	6
0.10	10	8	18	16	12	12	8	6
0.05	2	0	3	2	2	1	1	1
0.05	3	1	5	4	2	2	1	1
0.05	4	2	7	6	5	3	3	3
0.05	5	3	9	8	5	3	3	3
0.05	6	4	11	8	8	7	6	3
0.05	7	5	13	11	8	7	6	3
0.05	8	6	15	13	11	7	6	3
0.05	9	7	17	15	11	11	6	3
0.05	10	8	19	17	14	11	11	3
0.01	2	0	5	4	3	2	2	1
0.01	3	1	5	4	3	2	2	1
0.01	4	2	8	7	6	5	2	1
0.01	5	3	10	9	6	5	2	1
0.01	6	4	12	11	9	5	2	1
0.01	7	5	14	13	9	9	2	1
0.01	8	6	16	15	12	9	9	7
0.01	9	7	19	15	14	12	9	7
0.01	10	8	21	18	14	12	9	7

Operating characteristic of the sampling plan

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the median from its true value μ . This function is called operating characteristic (OC) function of the sampling plan. Once the minimum number of testers r is obtained, one may be interested to find the probability of acceptance of a lot when the quality is considered to be good if $\mu \geq \mu_0$ or $\frac{\mu}{\mu_0}$.

The OC is given by

$$L(p) = \left(\sum_{i=0}^c \binom{r}{i} p^i (1-p)^{r-i}\right)^g \tag{3.1}$$

Using equation 3.1 the OC values can be obtained for any sampling plan. To save space we present the OC values for sampling plans with $\mu/\mu_0=2,4,6,8,10,12$; $\beta=0.25,0.10,0.05,0.01$; $\delta=0.7,0.8,1.0,1.2,1.5,2.0$ are given in table 2.

Producer’s Risk

The producer may be interested in enhancing the quality level of the product so that the acceptance probability should be larger than a specified level. For a given value of the producer’s risk, say α , the minimum ratio can be obtained by satisfying the following inequality

$$\left(\sum_{i=0}^c \binom{r}{i} p^i (1-p)^{r-i}\right)^g \geq 1-\alpha \tag{3.2}$$

To save space, the minimum values of the ratio $\mu/\mu_0 = 2$ in case of half logistic distribution based on the values given in table 1 for the acceptability of a lot at the producer’s risk of $\alpha = 0.05$ are presented in table 3.

Table.2. OC values of the hybrid group acceptance sampling plan for HLD with $g=4$ and $c=2$

	r	δ	$\frac{\mu}{\mu_0}$					
			2	4	6	8	10	12
0.25	5	0.7	0.3297	0.7306	0.8637	0.9189	0.9464	0.9621
0.25	5	0.8	0.2452	0.6690	0.8281	0.8965	0.9313	0.9511
0.25	5	1.0	0.1253	0.5459	0.7508	0.8462	0.8965	0.9259
0.25	5	1.2	0.0582	0.4309	0.6690	0.7903	0.8568	0.8965
0.25	4	1.5	0.0587	0.4399	0.6784	0.7979	0.8627	0.9011
0.25	4	2.0	0.0095	0.2505	0.5160	0.6784	0.7748	0.8348
0.10	6	0.7	0.2140	0.6383	0.8088	0.8838	0.9224	0.9447
0.10	5	0.8	0.2452	0.6690	0.8281	0.8965	0.9313	0.9511
0.10	3	1.0	0.4590	0.8118	0.9093	0.9473	0.9656	0.9759
0.10	1	1.2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1	1.5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1	2.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.05	7	0.7	0.1336	0.5484	0.7502	0.8450	0.8953	0.9248
0.05	6	0.8	0.1435	0.5643	0.7617	0.8530	0.9011	0.9291
0.05	5	1.0	0.1253	0.5459	0.7508	0.8642	0.8965	0.9259
0.05	3	1.2	0.3349	0.7439	0.8733	0.9255	0.9512	0.9656
0.05	3	1.5	0.1922	0.6359	0.8118	0.8873	0.9255	0.9473
0.05	3	2.0	0.0625	0.4590	0.6964	0.8118	0.8733	0.9093

0.01	8	0.7	0.0808	0.4642	0.6899	0.8035	0.8656	0.9027
0.01	7	0.8	0.0804	0.4666	0.6926	0.8057	0.8674	0.9041
0.01	6	1.0	0.0585	0.4258	0.6632	0.7855	0.8530	0.8935
0.01	5	1.2	0.0582	0.4309	0.6690	0.7903	0.8568	0.8965
0.01	2	1.5	0.5166	0.8449	0.9276	0.9585	0.9732	0.9813
0.01	1	2.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table.3. Minimum ratio of the values of true median and the specified median for the producer’s risk of $\alpha = 0.05$ in the case of HLD

β	g	c	δ					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	15.1828	17.3517	21.6896	26.0275	32.5343	43.3790
0.25	3	1	4.9720	5.6823	7.1028	1.0001	1.0001	1.0001
0.25	4	2	3.3256	3.8007	4.7508	5.7009	5.3344	7.1125
0.25	5	3	2.6692	2.4996	3.1244	3.7492	2.5026	3.3369
0.25	6	4	2.3125	2.2712	2.8389	2.2543	1.0001	1.0001
0.25	7	5	2.0861	1.5480	1.9350	1.3944	1.0001	1.0001
0.25	8	6	1.9280	1.7724	2.2154	2.3283	1.0001	1.0001
0.25	9	7	1.8105	1.7190	1.6992	1.7587	1.0001	1.0001
0.25	10	8	1.7194	1.6721	1.3241	1.3349	1.6698	1.0001
0.10	2	0	30.1767	34.4876	21.6896	26.0275	32.5343	43.3790
0.10	3	1	6.9615	5.6823	7.1028	1.0001	1.0001	1.0001
0.10	4	2	4.1488	3.8007	2.8035	1.0001	1.0001	1.0001
0.10	5	3	3.1455	3.0505	3.1244	2.0022	1.0001	1.0001
0.10	6	4	2.6346	2.6429	1.8786	1.0001	1.0001	1.0001
0.10	7	5	2.3240	2.3841	2.2898	2.3219	2.9024	2.3238
0.10	8	6	2.1141	1.7724	1.6581	1.6352	2.0439	1.0001
0.10	9	7	1.9621	1.8949	1.9262	1.1331	1.4163	1.0001
0.10	10	8	1.8462	1.8191	1.5231	1.8277	1.0001	1.0001
0.05	2	0	45.1698	34.4876	43.1095	26.0275	32.5343	43.3790
0.05	3	1	8.9363	7.9560	4.1951	5.0341	1.0001	1.0001
0.05	4	2	4.9658	4.7414	4.7508	2.7703	3.4628	4.6170
0.05	5	3	3.6185	3.5948	2.4189	1.0001	1.0001	1.0001
0.05	6	4	2.9546	2.2712	2.8389	2.8397	2.8178	1.0001
0.05	7	5	2.5605	2.3841	1.9350	1.8792	1.1429	1.0001
0.05	8	6	2.2992	2.2034	2.1254	1.2424	1.0001	1.0001
0.05	9	7	2.1127	2.0692	1.6992	2.0390	1.0001	1.0001
0.05	10	8	1.9725	1.9651	1.9044	1.5888	1.9860	1.0001
0.01	2	0	75.1557	68.7575	64.5283	51.7314	64.6642	43.3790
0.01	3	1	8.9363	7.9560	7.1028	5.0341	6.2925	1.0001
0.01	4	2	5.7793	5.6752	5.9268	5.7009	1.0001	1.0001
0.01	5	3	4.0893	4.1354	3.1244	2.9027	1.0001	1.0001
0.01	6	4	3.2732	3.3767	3.3036	1.6218	1.0001	1.0001

0.01	7	5	2.7960	2.9263	2.2898	2.7477	1.0001	1.0001
0.01	8	6	2.4835	2.6276	2.4863	1.9897	2.4871	2.0706
0.01	9	7	2.4122	2.0692	2.3686	2.3131	1.8293	1.0001
0.01	10	8	2.2234	2.1100	1.9044	1.8277	1.3129	1.0001

Tables and Examples

Table 1 shows the HGASP design parameters for various values of the consumer risk and the test termination time multiplier. It should be noted that $n = r \times g$. can be used to get the minimal sample size. Table 1 shows that when the test termination time multiplier grows, the number of testers required r decreases, implying that fewer testers are required. if for a constant group size, the test termination time multiplier increases For instance, if $\beta=0.10$, $g=4$, $c=2$, and δ changes from 0.7 to 0.8, the needed values of the HGASP design parameters change from $r=6$ to $r=5$. This tendency, however, is not constant because it is influenced by the acceptance rate. Table 2 shows the probability of acceptance for the lot at the median ratio that corresponds to the producer's risk. Finally, for certain parameter values, table 3 shows the minimum ratios of true median to defined median for the acceptance of a lot with producer's risk $\alpha=0.05$.

If a product's lifetime follows a half logistic distribution, an HGASP should be designed to see if the median is more than 1,000 hours, with a testing time of 700 hours and four groups. The values $c = 2$ and $\beta = 0.10$ are assumed. As a result, the termination multiplier is equal to $\delta = 0.700$. Table 1 shows that the minimal number of testers necessary is $r =6$. As a result, we'll select a random sample of $n=24$ items and assign 6 items to each of the four groups to test for 700 hours. This means that a total of 24 products are required, with 6 items assigned to each of the four groups. We shall accept the lot if no more than two failures occur in any of the four groups before 700 hours. When the experiment reaches its third failure before the 700th hour, we call it a day. When the true value of the median is $\mu = 4,000$ hours, the probability of acceptance for this proposed sample plan is $p = 0.6383$. The producer's risk is equal to $\alpha =0.3617$ if the true value of the median is 4 times the required value $\mu_0= 1000$ hours.

If we need the ratio to assure a producer's risk of $\alpha = 0.05$, we can obtain it from table 3. For example, when $\beta = 0.10$, $r = 6$, $g =4$, $c = 2$ and $\delta = 0.700$, the required ratio is $\mu/\mu_0 =4.1488$.

Summary and Conclusions

In the case of half logistic distribution, this work proposes a hybrid group acceptance sampling plan based on a truncated life test. When the consumer's risk (β) and other plan characteristics are given, the number of groups and acceptance number are determined. As the test termination time multiplier grows, it is seen that the minimal number of groups required lowers. When a large number of objects are being examined at the same time, this HGASP can be employed. Clearly, such a tester would save time and money during the testing process.

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