

BMBJw – Filters Of Lattice Wajsberg Algebras

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Abstract

In this paper we define the BMBJw - filters of Lattice wajsberg algebras and prove some of the properties of BMBJw - filters. We derive some relations between fuzzy filters, intuitionistic fuzzy filters to BMBJw - filters of Lattice wajsberg algebras. Further we prove that $M_B^{\wedge}J$ – cut sets formed BMBJw – filters. Finally we define the BMBJw - sub algebra and prove that every BMBJw – filter is a BMBJw - sub algebra. Later we give a condition to BMBJw - sub algebra is a BMBJw – filter.

Keywords: Lattice wajsberg algebra, MBJ-neutrosophic sets, BMBJw – filter and BMBJ-Subalgebra.

1. Introduction

Wajsberg presented the concept of wajsberg algebra in 1935. In 1984,[6] Front, Antonio and Torrens led the lattice wajsberg algebra and investigate about filters. Ibrahim and Saravan [1] introduced the strong implicative filters of lattice wajsberg algebras and derived some properties. B.Ahamed introduced [2] the concept of fuzzy implicative filter and obtained some properties of lattice wajsberg algebra. Several researchers [3, 4, 5, 8, 11, 14, 15, 16,] applied the fuzzy theory to different algebras, differential equations and derived some results. Later Gaw derived the vague set as a generalization of fuzzy set. Vague theory applied to several streams by researchers [12,16]. After that Smarandache [7] introduced the concept of neutrosophic sets as a generalization of intuitionistic fuzzy sets. Later Monoranjan and Madhumangal [9] led the neutrosophic sets and define new operations with examples. Then neutrosophic applied to different streams[12,13] Y.B.Jun, R.A.Borzooei and M. Mohseni[10] introduced the MBJ-neutrosophic sets and applied to BCK algebra. T.Anitha, V.Amarendra Babu, G. Bhanu Vinolia[18, 17] introduced the NW- filters and MBJ- filters of lattice wajsberg algebras and proved the some properties.

In this paper we introduce the concepts of $\mathbb{BMBJ}w$ – filter and \mathbb{BMBJ} – Subalgebras of lattice wajsberg algebras and obtain some results on them. For further information of lattice wajsberg algebra refer the wajsberg algebra[6] by Front, Antonio and Torrens and for MBJ-neutrosophic sets refer the [10].

2. Preliminaries

Definition 2.1[6]: Let $(\mathcal{W}, \sim, \prime, 1_m)$ be a wajsberg algebra if it satisfies the following axioms for all $x_m, \eta_m, \delta_m \in \mathcal{W}$

1. $1_m \sim x_m = x_m$

2. $(x_m \sim y_m) \sim ((y_m \sim z_m) \sim (x_m \sim z_m)) = 1_m$
3. $(x_m \sim y_m) \sim y_m = (y_m \sim x_m) \sim x_m$
4. $(x'_m \sim y'_m) \sim (y_m \sim x_m) = 1_m$

Definition 2.2[6]: The wajsberg algebra w is called a lattice wajsberg algebra with the bounds $0_m, 1_m$ if it satisfies the following axioms for all $x_m, y_m \in w$

A partial ordering \leq on w , such that $x_m \leq y_m$ if and only if $x_m \sim y_m = 1_m$, $(x_m \vee y_m) = (x_m \sim y_m) \sim y_m$ and $(x_m \wedge y_m) = ((x'_m \sim y'_m) \sim y'_m)'$

Let \mathbb{I} denote the family of all intervals numbers of $[0, 1]$. If $\mathbb{I}_1 = [a_1, b_1]$, $\mathbb{I}_2 = [a_2, b_2]$ are two elements of \mathbb{I} $[0, 1]$, we call $\mathbb{I}_1 \geq \mathbb{I}_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. We define the term $rmax$ to mean the maximum of two interval as $rmax[\mathbb{I}_1, \mathbb{I}_2] = [\max\{a_1, a_2\}, \max\{b_1, b_2\}]$.

Similarly, we can define the term $rmin$ of any two intervals.

Definition 2.4 [10]: A MJB neutrosophic set ($\mathfrak{M}_{\mathfrak{B}}^{\mathfrak{S}}$ -set) is of the structure $A_m = \{ \langle y_m, M_T^A(y_m), B_I^A(y_m), J_F^A(y_m) \rangle, y_m \in X \}$ where M_T^A is truth membership function, B_I^A is an indeterminate interval –valued membership function and J_F^A is false membership function, on a nonempty set X . The $\mathfrak{M}_{\mathfrak{B}}^{\mathfrak{S}}$ -set is simply denoted by $A_m = (M_T^A, B_I^A, J_F^A)$.

Throughout this paper 'w' denotes the lattice wajsberg algebra and ' $\mathfrak{M}_{\mathfrak{B}}^{\mathfrak{S}}$ -set' denotes the MJB-neutrosophic set.

3. $\mathfrak{BMBJ}w$ - Filters

Definition 3.1: A $\mathfrak{M}_{\mathfrak{B}}^{\mathfrak{S}}$ -set $A_B = (M_T^A, B_I^A, J_F^A)$ of w is called a $\mathfrak{BMBJ}w$ - filter if it satisfies $\forall x_b, y_b \in w$,

(3.1) $M_T^A(x_b) + B_I^{A-}(y_b) \leq 1$ and $B_I^{A+}(y_b) + J_F^A(y_b) \leq 1$

(3.2) $M_T^A(1_b) \geq M_T^A(x_b), B_I^{A-}(1_b) \leq B_I^{A-}(x_b), B_I^{A+}(1_b) \geq B_I^{A+}(x_b)$ and $J_F^A(1_b) \leq J_F^A(x_b)$.

(3.3) $M_T^A(y_b) \geq \min \{ M_T^A(x_b \sim y_b), M_T^A(x_b) \}$, $B_I^{A-}(y_b) \leq \max \{ B_I^{A-}(x_b \sim y_b), B_I^{A-}(x_b) \}$, $B_I^{A+}(y_b) \geq \min \{ B_I^{A+}(x_b \sim y_b), B_I^{A+}(x_b) \}$, $J_F^A(y_b) \leq \max \{ J_F^A(x_b \sim y_b), J_F^A(x_b) \}$.

Example 3.2: Let $w = \{0_b, x_b, y_b, z_b, v_b, 1_b\}$ with the binary operation \sim as follows:

\sim	0_b	x_b	y_b	z_b	v_b	1_b
0_b	1_b	1_b	1_b	1_b	1_b	1_b
x_b	z_b	1_b	y_b	z_b	y_b	1_b
y_b	v_b	x_b	1_b	y_b	x_b	1_b
z_b	x_b	x_b	1_b	1_b	x_b	1_b
v_b	y_b	1_b	1_b	y_b	1_b	1_b
1_b	0_b	x_b	y_b	z_b	v_b	1_b

The $\mathfrak{M}_{\mathfrak{B}}^{\mathfrak{S}}$ set $A_B = (M_T^A, B_I^A, J_F^A)$ defined on w as follows is a $\mathfrak{BMBJ}w$ - filter of w .

Lemma 3.3: Every $\mathfrak{BMBJ}w$ – filter of w satisfies the following assertion:

β	$M_T^A(\beta)$	$B_T^A(\beta)$	$J_F^A(\beta)$
0_b	0.41	[0.45,0.457]	0.48
x_b	0.48	[0.5,0.51]	0.445
y_b	0.41	[0.45,0.457]	0.48
z_b	0.41	[0.45,0.457]	0.48
v_b	0.41	[0.45,0.457]	0.48
1_b	0.48	[0.5,0.51]	0.445

$\forall x_b, y_b \in w, x_b \leq y_b \Rightarrow M_T^A(y_b) \geq M_T^A(x_b), B_T^{A-}(y_b) \leq B_T^{A-}(x_b), B_T^{A+}(y_b) \geq B_T^{A+}(x_b)$ and $J_F^A(y_b) \leq J_F^A(x_b)$.

Proof: Let $A_B = (M_T^A, B_T^A, J_F^A)$ is a BMBJ_w – filter of w and $x_b, y_b \in w$ such that $x_b \leq y_b$, so $x_b \sim y_b = 1_b$. Then we have

$$M_T^A(y_b) \geq \min \{M_T^A(x_b \sim y_b), M_T^A(x_b)\} = \min \{M_T^A(1_b), M_T^A(x_b)\} = M_T^A(x_b),$$

$$B_T^{A-}(y_b) \leq \max \{B_T^{A-}(x_b \sim y_b), B_T^{A-}(x_b)\} = \max \{B_T^{A-}(1_b), B_T^{A-}(x_b)\} = B_T^{A-}(x_b)$$

$$B_T^{A+}(y_b) \geq \min \{B_T^{A+}(x_b \sim y_b), B_T^{A+}(x_b)\} = \min \{B_T^{A+}(1_b), B_T^{A+}(x_b)\} = B_T^{A+}(x_b)$$

and

$$J_F^A(y_b) \leq \max \{J_F^A(x_b \sim y_b), J_F^A(x_b)\} = \max \{J_F^A(1_b), J_F^A(x_b)\} = J_F^A(x_b)$$

This completes the proof:

Theorem3.4: Let $A_B = (M_T^A, B_T^A, J_F^A)$ is a BMBJ_w – filter of w . Then A_B satisfies the following assertion $\forall x_b, y_b, z_b \in w$

$$(3.5) \quad x_b \leq y_b \sim z_b \Rightarrow M_T^A(z_b) \geq \min \{M_T^A(y_b), M_T^A(x_b)\},$$

$$B_T^{A-}(z_b) \leq \max \{B_T^{A-}(y_b), B_T^{A-}(x_b)\},$$

$$B_T^{A+}(z_b) \geq \min \{B_T^{A+}(y_b), B_T^{A+}(x_b)\}$$

and $J_F^A(z_b) \leq \max \{J_F^A(y_b), J_F^A(x_b)\}.$

Proof: Let $x_b, y_b, z_b \in w$ such that $x_b \leq y_b \sim z_b$. Then

$$M_T^A(y_b \sim z_b) \geq \min \{M_T^A(x_b \sim (y_b \sim z_b)), M_T^A(x_b)\}$$

$$= \min \{M_T^A(1_b), M_T^A(x_b)\}$$

$$= M_T^A(x_b),$$

$$B_T^{A-}(y_b \sim z_b) \leq \max \{B_T^{A-}(x_b \sim (y_b \sim z_b)), B_T^{A-}(x_b)\}$$

$$= \max \{B_T^{A-}(1_b), B_T^{A-}(x_b)\}$$

$$= B_T^{A-}(x_b),$$

$$B_T^{A+}(y_b \sim z_b) \geq \min \{B_T^{A+}(x_b \sim (y_b \sim z_b)), B_T^{A+}(x_b)\}$$

$$= \min \{B_T^{A+}(1_b), B_T^{A+}(x_b)\}$$

$$= B_T^{A+}(x_b)$$

and $J_F^A(y_b \sim z_b) \leq \max \{J_F^A(x_b \sim (y_b \sim z_b)), J_F^A(x_b)\}$

$$= \max \{J_F^A(1_b), J_F^A(x_b)\}$$

$$= J_F^A(x_b).$$

It follows that

$$M_T^A(z_b) \geq \min \{M_T^A(y_b \sim z_b), M_T^A(y_b)\} = \min \{M_T^A(x_b), M_T^A(y_b)\},$$

$$\begin{aligned} \mathbb{B}_I^{A-}(z_b) &\leq \max \{ \mathbb{B}_I^{A-}(\eta_b \sim z_b), \mathbb{B}_I^{A-}(\eta_b) \} = \max \{ \mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A-}(\eta_b) \}, \\ \mathbb{B}_I^{A+}(z_b) &\geq \min \{ \mathbb{B}_I^{A+}(\eta_b \sim z_b), \mathbb{B}_I^{A+}(\eta_b) \} = \min \{ \mathbb{B}_I^{A+}(x_b), \mathbb{B}_I^{A+}(\eta_b) \}, \\ \text{and } \mathbb{J}_F^A(z_b) &\leq \max \{ \mathbb{J}_F^A(\eta_b \sim z_b), \mathbb{J}_F^A(\eta_b) \} = \max \{ \mathbb{J}_F^A(x_b), \mathbb{J}_F^A(\eta_b) \}. \end{aligned}$$

Theorem3.5: Let $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a $\mathfrak{M}_{\mathbb{B}}^{\tilde{S}}$ -set of \mathfrak{w} is a \mathbb{BMBJW} – filter of \mathfrak{w} if and only if A_B satisfying (3.1), (3.2) and

$$\begin{aligned} \Rightarrow \quad & (3.6) \quad M_T^A(x_b \otimes \eta_b) \geq \min \{ M_T^A(\eta_b), M_T^A(x_b) \}, \\ & \mathbb{B}_I^{A-}(x_b \otimes \eta_b) \leq \max \{ \mathbb{B}_I^{A-}(\eta_b), \mathbb{B}_I^{A-}(x_b) \}, \\ & \mathbb{B}_I^{A+}(x_b \otimes \eta_b) \geq \min \{ \mathbb{B}_I^{A+}(\eta_b), \mathbb{B}_I^{A+}(x_b) \} \\ \text{and } & \mathbb{J}_F^A(x_b \otimes \eta_b) \leq \max \{ \mathbb{J}_F^A(\eta_b), \mathbb{J}_F^A(x_b) \} \text{ for all } x_b, \eta_b \in \mathfrak{w}. \end{aligned}$$

Proof: Suppose that $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a \mathbb{BMBJW} – filter of \mathfrak{w} , obviously (3.1) and (3.2) holds. Since $x_b \leq \eta_b \sim (x_b \otimes \eta_b)$, we get $M_T^A(\eta_b \sim (x_b \otimes \eta_b)) \geq M_T^A(x_b)$, $\mathbb{B}_I^{A-}(\eta_b \sim (x_b \otimes \eta_b)) \leq \mathbb{B}_I^{A-}(x_b)$, $\mathbb{B}_I^{A+}(\eta_b \sim (x_b \otimes \eta_b)) \geq \mathbb{B}_I^{A+}(x_b)$ and $\mathbb{J}_F^A(\eta_b \sim (x_b \otimes \eta_b)) \leq \mathbb{J}_F^A(x_b)$.

$$\begin{aligned} \text{By (3.3), it follows that } M_T^A(x_b \otimes \eta_b) &\geq \min \{ M_T^A(\eta_b), M_T^A(\eta_b \sim (x_b \otimes \eta_b)) \} \\ &\geq \min \{ M_T^A(\eta_b), M_T^A(x_b) \}, \\ \mathbb{B}_I^{A-}(x_b \otimes \eta_b) &\leq \max \{ \mathbb{B}_I^{A-}(\eta_b), \mathbb{B}_I^{A-}(\eta_b \sim (x_b \otimes \eta_b)) \} \\ &\leq \max \{ \mathbb{B}_I^{A-}(\eta_b), \mathbb{B}_I^{A-}(x_b) \}, \\ \mathbb{B}_I^{A+}(x_b \otimes \eta_b) &\geq \min \{ \mathbb{B}_I^{A+}(\eta_b), \mathbb{B}_I^{A+}(\eta_b \sim (x_b \otimes \eta_b)) \} \\ &\geq \min \{ \mathbb{B}_I^{A+}(\eta_b), \mathbb{B}_I^{A+}(x_b) \} \\ \text{and } \mathbb{J}_F^A(x_b \otimes \eta_b) &\leq \max \{ \mathbb{J}_F^A(\eta_b), \mathbb{J}_F^A(\eta_b \sim (x_b \otimes \eta_b)) \} \\ &\leq \max \{ \mathbb{J}_F^A(\eta_b), \mathbb{J}_F^A(x_b) \}. \end{aligned}$$

Conversely suppose that Let $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a $\mathfrak{M}_{\mathbb{B}}^{\tilde{S}}$ -set of \mathfrak{w} and satisfies (3.1), (3.2) and (3.6). Clearly $x_b \otimes (x_b \sim \eta_b) \leq \eta_b$, so we have $M_T^A(\eta_b) \geq M_T^A(x_b \otimes (x_b \sim \eta_b))$, $\mathbb{B}_I^{A-}(\eta_b) \leq \mathbb{B}_I^{A-}(x_b \otimes (x_b \sim \eta_b))$, $\mathbb{B}_I^{A+}(\eta_b) \geq \mathbb{B}_I^{A+}(x_b \otimes (x_b \sim \eta_b))$ and $\mathbb{J}_F^A(\eta_b) \leq \mathbb{J}_F^A(x_b \otimes (x_b \sim \eta_b))$. By (3.6), we have

$$\begin{aligned} M_T^A(\eta_b) &\geq \min \{ M_T^A(x_b \sim \eta_b), M_T^A(x_b) \}, \\ \mathbb{B}_I^{A-}(\eta_b) &\leq \max \{ \mathbb{B}_I^{A-}(x_b \sim \eta_b), \mathbb{B}_I^{A-}(x_b) \}, \\ \mathbb{B}_I^{A+}(\eta_b) &\geq \min \{ \mathbb{B}_I^{A+}(x_b \sim \eta_b), \mathbb{B}_I^{A+}(x_b) \} \\ \text{and } \mathbb{J}_F^A(\eta_b) &\leq \max \{ \mathbb{J}_F^A(x_b \sim \eta_b), \mathbb{J}_F^A(x_b) \}. \end{aligned}$$

This completes the proof.

Theorem3.6: Every $\mathfrak{M}_{\mathbb{B}}^{\tilde{S}}$ -set $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ of \mathfrak{w} satisfying (3.1) and (3.4) is a \mathbb{BMBJW} – filter of \mathfrak{w} .

Proof: Let $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a $\mathfrak{M}_{\mathbb{B}}^{\tilde{S}}$ -set of \mathfrak{w} satisfying (3.1) and (3.4).

Clearly for all $\forall x_b, \eta_b, z_b \in \mathfrak{w}$, $x_b \leq (x_b \sim \eta_b) \sim \eta_b$, by (3.4) we have

$$\begin{aligned} M_T^A(\eta_b) &\geq \min \{ M_T^A(x_b), M_T^A(x_b \sim \eta_b) \}, \\ \mathbb{B}_I^{A-}(\eta_b) &\leq \max \{ \mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A-}(x_b \sim \eta_b) \}, \\ \mathbb{B}_I^{A+}(\eta_b) &\geq \min \{ \mathbb{B}_I^{A+}(x_b), \mathbb{B}_I^{A+}(x_b \sim \eta_b) \} \\ \text{and } \mathbb{J}_F^A(\eta_b) &\leq \max \{ \mathbb{J}_F^A(x_b), \mathbb{J}_F^A(x_b \sim \eta_b) \}. \end{aligned}$$

Theorem 3.7: A $\mathfrak{M}_{\mathbb{B}}^{\tilde{S}}$ -set $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ of \mathfrak{w} is a \mathbb{BMBJW} – filter of \mathfrak{w} if and only if the cut sets $M_T^{A\alpha} = \{ x_b \in \mathfrak{w} / M_T^A(x_b) \geq \alpha \}$,

$$\begin{aligned} \mathbb{B}_I^{A-\beta} &= \{ x_b \in \mathfrak{w} / \mathbb{B}_I^{A-}(x_b) \leq \beta_1 \}, \\ \mathbb{B}_I^{A+\beta} &= \{ x_b \in \mathfrak{w} / \mathbb{B}_I^{A+}(x_b) \geq \beta_2 \} \end{aligned}$$

and $\mathbb{J}_F^{A\gamma} = \{x_b \in \mathbb{w} / \mathbb{J}_F^A(x_m) \leq \gamma\}$, where $\alpha, \gamma, \beta_1, \beta_2 \in [0,1]$

are filters of \mathbb{w} .

Proof: Suppose that $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a BMBJw – filter of \mathbb{w} . Let $\alpha, \beta_1, \beta_2, \gamma \in [0,1]$

such that $\mathbb{M}_T^{A\alpha}, \mathbb{B}_I^{A-\beta}, \mathbb{B}_I^{A+\beta}$ and $\mathbb{J}_F^{A\gamma}$ are nonempty. Obviously $1 \in \mathbb{M}_T^{A\alpha}, \mathbb{B}_I^{A-\beta}, \mathbb{B}_I^{A+\beta}$ and $\mathbb{J}_F^{A\gamma}$. For any $x_{1b}, x_{2b}, \eta_{1b}, \eta_{2b}, u_{1b}, u_{2b}, \beta_{1b}$ and $\beta_{2b} \in \mathbb{w}$ such that $(x_{1b} \rightsquigarrow x_{2b}, x_{1b} \in \mathbb{M}_T^{A\alpha}), (\eta_{1b} \rightsquigarrow \eta_{2b}, \eta_{1b} \in \mathbb{B}_I^{A-\beta}), (u_{1b} \rightsquigarrow u_{2b}, u_{1b} \in \mathbb{B}_I^{A+\beta})$ and $(\beta_{1b} \rightsquigarrow \beta_{2b}, \beta_{1b} \in \mathbb{J}_F^{A\gamma})$.

Then $\mathbb{M}_T^A(x_{2b}) \geq \min \{\mathbb{M}_T^A(x_{1b} \rightsquigarrow x_{2b}), \mathbb{M}_T^A(x_{1b})\} \geq \alpha$ implies $x_{2b} \in \mathbb{M}_T^{A\alpha}$,

$\mathbb{B}_I^{A-}(\eta_{2b}) \leq \max \{\mathbb{B}_I^{A-}(\eta_{1w} \rightsquigarrow \eta_{1w}), \mathbb{B}_I^{A-}(\eta_{1w})\} \leq \beta$ implies $\eta_{2w} \in \mathbb{B}_I^{A-\beta}$,

$\mathbb{B}_I^{A+}(u_{2b}) \geq \min \{\mathbb{B}_I^{A+}(u_{1b} \rightsquigarrow u_{2b}), \mathbb{B}_I^{A+}(u_{1b})\} \geq \beta_2$ implies $u_{2b} \in \mathbb{B}_I^{A+\beta}$

$\mathbb{J}_F^A(\beta_{2b}) \leq \max \{\mathbb{J}_F^A(\beta_{1b} \rightsquigarrow \beta_{2b}), \mathbb{J}_F^A(\beta_{1b})\} \leq \gamma$ implies $\beta_{2b} \in \mathbb{J}_F^{A\gamma}$.

$\mathbb{M}_T^{A\alpha}, \mathbb{B}_I^{A-\beta}, \mathbb{B}_I^{A+\beta}$ and $\mathbb{J}_F^{A\gamma}$ are filters of \mathbb{w} .

Conversely suppose that the nonempty cut sets $\mathbb{M}_T^{A\alpha}, \mathbb{B}_I^{A-\beta}, \mathbb{B}_I^{A+\beta}$ and $\mathbb{J}_F^{A\gamma}$ are implicative filters of \mathbb{w} for all $\alpha, \beta_1, \beta_2, \gamma \in [0,1]$. Assume that $\mathbb{M}_T^A(1_b) < \mathbb{M}_T^A(x_b), \mathbb{B}_I^{A-}(1_b) > \mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A+}(1_b) < \mathbb{B}_I^{A+}(x_b)$ and $\mathbb{J}_F^A(1_b) > \mathbb{J}_F^A(x_b)$ for some $x_b \in \mathbb{w}$.

Then $1_b \notin \mathbb{M}_T^{AM(xb)} \cap \mathbb{B}_I^{A-M(xb)} \cap \mathbb{B}_I^{A+M(xb)} \cap \mathbb{J}_F^{M(xb)}$.

This is a contradiction.

So $\mathbb{M}_T^A(1_b) \geq \mathbb{M}_T^A(x_b), \mathbb{B}_I^{A-}(1_b) \leq \mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A+}(1_b) \geq \mathbb{B}_I^{A+}(x_b)$ and $\mathbb{J}_F^A(1_b) \leq \mathbb{J}_F^A(x_b)$ for all $x_b \in \mathbb{w}$.

If $\mathbb{M}_T^A(a_{2b}) < \min \{\mathbb{M}_T^A(a_{1b} \rightsquigarrow a_{2b}), \mathbb{M}_T^A(a_{1b})\}$ for some $a_{1b}, a_{2b} \in \mathbb{w}$ then $a_{1b} \rightsquigarrow a_{2b}, a_{1b} \in \mathbb{M}_T^{A\alpha_0}$

but $a_{2b} \notin \mathbb{M}_T^{A\alpha_0}$ for some $\alpha_0 = \min \{\mathbb{M}_T^A(a_{1b} \rightsquigarrow a_{2b}), \mathbb{M}_T^A(a_{1b})\}$. This is a contradiction, so

$\mathbb{M}_T^A(x_{2b}) \geq \min \{\mathbb{M}_T^A(x_{1b} \rightsquigarrow x_{2b}), \mathbb{M}_T^A(x_{1b})\}$ for all $x_{1b}, x_{2b} \in \mathbb{w}$. Similarly we can prove that

$\mathbb{J}_F^A(\beta_{2b}) \leq \max \{\mathbb{J}_F^A(\beta_{1b} \rightsquigarrow \beta_{2b}), \mathbb{J}_F^A(\beta_{1b})\}$ for all $x_{1b}, x_{2b} \in \mathbb{w}$. If $\mathbb{B}_I^{A-}(a_{2b}) > \max \{\mathbb{B}_I^{A-}(a_{1w} \rightsquigarrow a_{1w}), \mathbb{B}_I^{A-}(a_{1w})\}$ for some $a_{1b}, a_{2b} \in \mathbb{w}$ then $a_{1b} \rightsquigarrow a_{2b}, a_{1b} \in \mathbb{B}_I^{A-\beta_0}$ but $a_{2b} \notin \mathbb{B}_I^{A-\beta_0}$

For some $\beta_0 = \max \{\mathbb{B}_I^{A-}(a_{1w} \rightsquigarrow a_{2w}), \mathbb{B}_I^{A-}(a_{1w})\}$ This is a contradiction, so $\mathbb{B}_I^{A-}(\eta_{2b}) \leq \max \{\mathbb{B}_I^{A-}(\eta_b \rightsquigarrow \eta_b), \mathbb{B}_I^{A-}(\eta_{1b}), \mathbb{B}_I^{A-}(\eta_{1b})\}$ for all $\eta_{1b}, \eta_{2b} \in \mathbb{w}$. Similarly we can prove that $\mathbb{B}_I^{A+}(u_{2b}) \geq \min \{\mathbb{B}_I^{A+}(u_{1b} \rightsquigarrow u_{2b}), \mathbb{B}_I^{A+}(u_{1b})\}$ for all $u_{1b}, u_{2b} \in \mathbb{w}$. Therefore $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a BMBJw – filter of \mathbb{w} .

Theorem 3.8: A $\mathfrak{M}_{\mathbb{B}}^{\tilde{\Sigma}}$ -set $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ of \mathbb{w} is a BMBJw – filter of \mathbb{w} if and only if the $(\mathbb{M}_T^A, \mathbb{B}_I^{A-})$ and $(\mathbb{B}_I^{A+}, \mathbb{J}_F^A)$ are intuitionistic fuzzy filters of \mathbb{w} .

Theorem 3.9: For any nonempty subset A of \mathbb{w} , let $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a $\mathfrak{M}_{\mathbb{B}}^{\tilde{\Sigma}}$ -set in \mathbb{w} defined by $\mathbb{M}_T^A(x_w) = \alpha$ if $x_w \in A$ and other wise 0,

$\mathbb{B}_I^{A-}(x_w) = \beta_1$ if $x_w \in A$ and other wise 1,

$\mathbb{B}_I^{A+}(x_w) = \beta_2$ if $x_w \in A$ and other wise 0,

$\mathbb{J}_F^A(x_w) = \gamma$ if $x_w \in A$ and other wise 1, where $\alpha, \beta_2 \in (0,1], \beta_1, \gamma \in \{0,1\}$.

Then A is a filter of \mathbb{w} if and only if $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a BMBJw – filter of \mathbb{w} .

Proof: Suppose that A is an filter of \mathbb{w} and $x_b, \eta_b \in \mathbb{w}$.

It is obvious that $\mathbb{M}_T^A(1_b) \geq \mathbb{M}_T^A(x_b), \mathbb{B}_I^{A-}(1_b) \leq \mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A+}(1_b) \geq \mathbb{B}_I^{A+}(x_b)$ and $\mathbb{J}_F^A(1_b) \leq \mathbb{J}_F^A(x_b)$ for all $x_b \in \mathbb{w}$.

Case(i): If $x_b \sim y_b, x_b \in A$ then $y_b \in A$ and so

$$\begin{aligned} M_T^A(y_b) &= \alpha = \min \{M_T^A(x_b \sim y_b), M_T^A(x_b)\}, \\ \mathbb{B}_I^{A-}(y_b) = \beta_1 &= \max \{\mathbb{B}_I^{A-}(x_b \sim y_b), \mathbb{B}_I^{A-}(x_b)\}, \\ \mathbb{B}_I^{A+}(y_b) = \beta_2 &= \min \{\mathbb{B}_I^{A+}(x_b \sim y_b), \mathbb{B}_I^{A+}(x_b)\}, \\ \mathbb{J}_F^A(y_b) = \gamma &= \max \{\mathbb{J}_F^A(x_b \sim y_b), \mathbb{J}_F^A(x_b)\} \end{aligned}$$

Case(ii): If any one of $x_b \sim y_b, x_b$ is contained in A, say $x_b \sim y_b \in A$, then

$M_T^A(x_b \sim y_b) = \alpha, M_T^A(x_b) = 0, \mathbb{B}_I^{A-}(x_b \sim y_b) = \beta_1, \mathbb{B}_I^{A-}(x_b) = 1, \mathbb{B}_I^{A+}(x_b \sim y_b) = \beta_2, \mathbb{B}_I^{A+}(x_b) = 0, \mathbb{J}_F^A(x_b \sim y_b) = \gamma, \mathbb{J}_F^A(x_b) = 1$. Therefore

$$\begin{aligned} M_T^A(y_b) &\geq 0 = \min \{\alpha, 0\} = \min \{M_T^A(x_b \sim y_b), M_T^A(x_b)\}, \\ \mathbb{B}_I^{A-}(y_b) \leq 1 &= \max \{\mathbb{B}_I^{A-}(x_b \sim y_b), \mathbb{B}_I^{A-}(x_b)\}, \\ \mathbb{B}_I^{A+}(y_b) \geq 0 &= \min \{\mathbb{B}_I^{A+}(x_b \sim y_b), \mathbb{B}_I^{A+}(x_b)\}, \\ \mathbb{J}_F^A(y_b) \leq 1 &= \max \{\mathbb{J}_F^A(x_b \sim y_b), \mathbb{J}_F^A(x_b)\}. \end{aligned}$$

Case(iii): If $x_b \sim y_b$ and x_b are not contained in A.

$M_T^A(x_b \sim y_b) = 0 = M_T^A(x_b), \mathbb{B}_I^{A-}(x_b \sim y_b) = 1 = \mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A+}(x_b \sim y_b) = 0 = \mathbb{B}_I^{A+}(x_b), \mathbb{J}_F^A(x_b \sim y_b) = 1 = \mathbb{J}_F^A(x_b)$. Therefore

$$\begin{aligned} M_T^A(y_b) &\geq 0 = \min \{M_T^A(x_b \sim y_b), M_T^A(x_b)\}, \\ \mathbb{B}_I^{A-}(y_b) \leq 1 &= \max \{\mathbb{B}_I^{A-}(x_b \sim y_b), \mathbb{B}_I^{A-}(x_b)\}, \\ \mathbb{B}_I^{A+}(y_b) \geq 0 &= \min \{\mathbb{B}_I^{A+}(x_b \sim y_b), \mathbb{B}_I^{A+}(x_b)\}, \\ \mathbb{J}_F^A(y_b) \leq 1 &= \max \{\mathbb{J}_F^A(x_b \sim y_b), \mathbb{J}_F^A(x_b)\}. \end{aligned}$$

Therefore $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a \mathbb{BMBJw} – filter of w .

Conversely suppose that $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is a \mathbb{BMBJw} – filter of w . Obviously $1 \in A$. Let $x_b \sim y_b, x_b \in A$.

Then $M_T^A(x_b \sim y_b) = \alpha = M_T^A(x_b), \mathbb{B}_I^{A-}(x_b \sim y_b) = \beta_1 = \mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A+}(x_b \sim y_b) = \beta_2 = \mathbb{B}_I^{A+}(x_b), \mathbb{J}_F^A(x_b \sim y_b) = \gamma = \mathbb{J}_F^A(x_b)$. Then

$$\begin{aligned} M_T^A(y_b) &\geq \min \{M_T^A(x_b \sim y_b), M_T^A(x_b)\} = \alpha, \\ \mathbb{B}_I^{A-}(y_b) \leq \max \{\mathbb{B}_I^{A-}(x_b \sim y_b), \mathbb{B}_I^{A-}(x_b)\} &= \beta_1, \\ \mathbb{B}_I^{A+}(y_b) \geq \min \{\mathbb{B}_I^{A+}(x_b \sim y_b), \mathbb{B}_I^{A+}(x_b)\} &= \beta_2, \\ \mathbb{J}_F^A(y_b) \leq \max \{\mathbb{J}_F^A(x_b \sim y_b), \mathbb{J}_F^A(x_b)\} &= \gamma, \end{aligned}$$

and so $x_b \in A$. Hence A is a filter of w .

Theorem 3.10: A $\mathfrak{M}_{\mathbb{B}}^{\mathfrak{S}}$ set $A_B = (M_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is \mathbb{BMBJw} – filter of w if and only if it holds (3.1), (3.2) and

$$\begin{aligned} (3.5) \quad M_T^A(x_b \sim z_b) &\geq \min \{M_T^A(y_b \sim (x_b \sim z_b)), M_T^A(y_b)\}, \\ \mathbb{B}_I^{A-}(x_b \sim z_b) &\leq \max \{\mathbb{B}_I^{A-}(y_b \sim (x_b \sim z_b)), \mathbb{B}_I^{A-}(y_b)\}, \\ \mathbb{B}_I^{A+}(x_b \sim z_b) &\geq \min \{\mathbb{B}_I^{A+}(y_b \sim (x_b \sim z_b)), \mathbb{B}_I^{A+}(y_b)\}, \\ \mathbb{J}_F^A(x_b \sim z_b) &\leq \max \{\mathbb{J}_F^A(y_b \sim (x_b \sim z_b)), \mathbb{J}_F^A(y_b)\} \forall x_b, y_b, z_b \in w. \end{aligned}$$

Proof: Let A_b is a \mathbb{BMBJw} – filter of w , perceptibly it hold (3.1), (3.2) and (3.5).

Conversely suppose that A_m is a $\mathfrak{M}_{\mathbb{B}}^{\mathfrak{S}}$ - set with(2.1) and (2.3).

Taking $x_b = 1_b$ in (3.5), we get

$$\begin{aligned} M_T^A(1_b \sim z_b) &\geq \min \{M_T^A(y_b \sim (1_b \sim z_b)), M_T^A(y_b)\} \\ M_T^A(z_b) &\geq \min \{M_T^A(y_b \sim z_b), M_T^A(y_b)\}, \\ \mathbb{B}_I^{A-}(1_b \sim z_b) &\leq \max \{\mathbb{B}_I^{A-}(y_b \sim (1_b \sim z_b)), \mathbb{B}_I^{A-}(y_b)\}, \end{aligned}$$

$$\begin{aligned} \mathbb{B}_I^{A-}(\mathfrak{z}_b) &\leq \max\{\mathbb{B}_I^{A-}(\mathfrak{v}_b \sim \mathfrak{z}_b), \mathbb{B}_I^{A-}(\mathfrak{v}_b)\}, \\ \mathbb{B}_I^{A+}(1_b \sim \mathfrak{z}_b) &\geq \min\{\mathbb{B}_I^{A+}(\mathfrak{v}_b \sim (1_b \sim \mathfrak{z}_b)), \mathbb{B}_I^{A+}(\mathfrak{v}_b)\} \\ \mathbb{B}_I^{A+}(\mathfrak{z}_b) &\geq \min\{\mathbb{B}_I^{A+}(\mathfrak{v}_b \sim \mathfrak{z}_b), \mathbb{B}_I^{A+}(\mathfrak{v}_b)\}, \end{aligned}$$

and

$$\begin{aligned} \mathbb{J}_F^A(1_b \sim \mathfrak{z}_b) &\leq \max\{\mathbb{J}_F^A(\mathfrak{v}_b \sim (1_b \sim \mathfrak{z}_b)), \mathbb{J}_F^A(\mathfrak{v}_b)\}, \\ \mathbb{J}_F^A(\mathfrak{z}_b) &\leq \max\{\mathbb{J}_F^A(\mathfrak{v}_b \sim \mathfrak{z}_b), \mathbb{J}_F^A(\mathfrak{v}_b)\}. \end{aligned}$$

Therefore A_b is a \mathbb{BMBJW} – filter of w

Theorem 3.11: A $\mathfrak{M}_{\mathbb{B}}^{\mathfrak{S}}$ set $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is \mathbb{BMBJW} – filter of w if and only if it holds (3.1) , (3.2) and

$$\begin{aligned} (3.6) \quad \mathbb{M}_T^A(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b &\geq \min\{\mathbb{M}_T^A(\mathfrak{x}_b), \mathbb{M}_T^A(\mathfrak{v}_b)\}, \\ \mathbb{B}_I^{A-}(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b &\leq \max\{\mathbb{B}_I^{A-}(\mathfrak{x}_b), \mathbb{B}_I^{A-}(\mathfrak{v}_b)\}, \\ \mathbb{B}_I^{A+}(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b &\geq \min\{\mathbb{B}_I^{A+}(\mathfrak{x}_b), \mathbb{B}_I^{A+}(\mathfrak{v}_b)\}, \\ \mathbb{J}_F^A(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b &\leq \max\{\mathbb{J}_F^A(\mathfrak{x}_b), \mathbb{J}_F^A(\mathfrak{v}_b)\} \forall \mathfrak{x}_b, \mathfrak{v}_b, \mathfrak{z}_b \in w \end{aligned}$$

Proof: Suppose that $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is \mathbb{BMBJW} – filter of w and $\mathfrak{x}_b, \mathfrak{v}_b, \mathfrak{z}_b \in w$.

Clearly $\mathbb{M}_T^A((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b) \geq \min\{\mathbb{M}_T^A((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)), \mathbb{M}_T^A(\mathfrak{v}_b)\}$ ----- (i)

and $((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) = \mathfrak{x}_b \vee (\mathfrak{v}_b \sim \mathfrak{z}_b) \geq \mathfrak{x}_b$.

So $\mathbb{M}_T^A(((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b))) \geq \mathbb{M}_T^A(\mathfrak{x}_b)$ ----- (ii)

From (i) and (ii), $\mathbb{M}_T^A(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b \geq \min\{\mathbb{M}_T^A(\mathfrak{x}_b), \mathbb{M}_T^A(\mathfrak{v}_b)\}$, .

Clearly $\mathbb{B}_I^{A-}((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b) \leq \max\{\mathbb{B}_I^{A-}((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)), \mathbb{B}_I^{A-}(\mathfrak{v}_b)\}$ ----- (iii)

So $\mathbb{B}_I^{A-}(((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b))) \leq \mathbb{B}_I^{A-}(\mathfrak{x}_b)$ ----- (iv)

From (iii) and (iv), $\mathbb{B}_I^{A-}(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b \leq \max\{\mathbb{B}_I^{A-}(\mathfrak{x}_b), \mathbb{B}_I^{A-}(\mathfrak{v}_b)\}$,

Clearly $\mathbb{B}_I^{A+}((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b) \geq \min\{\mathbb{B}_I^{A+}((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)), \mathbb{B}_I^{A+}(\mathfrak{v}_b)\}$ ----- (v)

So $\mathbb{B}_I^{A+}(((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b))) \geq \mathbb{B}_I^{A+}(\mathfrak{x}_b)$ ----- (vi)

From (v) and (vi), $\mathbb{B}_I^{A+}(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b \geq \min\{\mathbb{B}_I^{A+}(\mathfrak{x}_b), \mathbb{B}_I^{A+}(\mathfrak{v}_b)\}$,

and $\mathbb{J}_F^A((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b) \leq \max\{\mathbb{J}_F^A((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)), \mathbb{J}_F^A(\mathfrak{v}_b)\}$ ----- (vii)

So $\mathbb{J}_F^A(((\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim (\mathfrak{v}_b \sim \mathfrak{z}_b))) \leq \mathbb{J}_F^A(\mathfrak{x}_b)$ ----- (ii)

From (i) and (ii), $\mathbb{J}_F^A(\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b)) \sim \mathfrak{z}_b \leq \max\{\mathbb{J}_F^A(\mathfrak{x}_b), \mathbb{J}_F^A(\mathfrak{v}_b)\}$

Conversely suppose that A_b is a $\mathfrak{M}_{\mathbb{B}}^{\mathfrak{S}}$ -set with (3.1) , (3.2) and (3.6).

$\mathbb{M}_T^A(\mathfrak{v}_b) = \mathbb{M}_T^A(1_b \sim \mathfrak{v}_b) = \mathbb{M}_T^A(((\mathfrak{x}_b \sim \mathfrak{v}_b) \sim (\mathfrak{x}_b \sim \mathfrak{v}_b)) \sim \mathfrak{v}_b) \geq \min\{\mathbb{M}_T^A(\mathfrak{x}_b \sim \mathfrak{v}_b), \mathbb{M}_T^A(\mathfrak{x}_b)\}$,

$\mathbb{B}_I^{A-}(\mathfrak{v}_b) = \mathbb{B}_I^{A-}(1_b \sim \mathfrak{v}_b) = \mathbb{B}_I^{A-}(((\mathfrak{x}_b \sim \mathfrak{v}_b) \sim (\mathfrak{x}_b \sim \mathfrak{v}_b)) \sim \mathfrak{v}_b) \leq \max\{\mathbb{B}_I^{A-}(\mathfrak{x}_b \sim \mathfrak{v}_b), \mathbb{B}_I^{A-}(\mathfrak{x}_b)\}$,

$\mathbb{M}_T^A(\mathfrak{v}_b) = \mathbb{M}_T^A(1_b \sim \mathfrak{v}_b) = \mathbb{M}_T^A(((\mathfrak{x}_b \sim \mathfrak{v}_b) \sim (\mathfrak{x}_b \sim \mathfrak{v}_b)) \sim \mathfrak{v}_b) \geq \min\{\mathbb{M}_T^A(\mathfrak{x}_b \sim \mathfrak{v}_b), \mathbb{M}_T^A(\mathfrak{x}_b)\}$.,

$\mathbb{J}_F^A(\mathfrak{v}_b) = \mathbb{J}_F^A(1_b \sim \mathfrak{v}_b) = \mathbb{J}_F^A(((\mathfrak{x}_b \sim \mathfrak{v}_b) \sim (\mathfrak{x}_b \sim \mathfrak{v}_b)) \sim \mathfrak{v}_b) \leq \max\{\mathbb{J}_F^A(\mathfrak{x}_b \sim \mathfrak{v}_b), \mathbb{J}_F^A(\mathfrak{x}_b)\}$

So $A_B = (\mathbb{M}_T^A, \mathbb{B}_I^A, \mathbb{J}_F^A)$ is \mathbb{BMBJW} – filter of w

Theorem3. :

Theorem 3.11: Let A and B are two \mathbb{BMBJW} – filters of w , then $A \cap B$ is a \mathbb{MBJW} – filter of w .

Proof: Let $\mathfrak{x}_b, \mathfrak{v}_b, \mathfrak{z}_b \in w$ such that $\mathfrak{x}_b \leq (\mathfrak{v}_b \sim \mathfrak{z}_b)$, then $\mathfrak{x}_b \sim (\mathfrak{v}_b \sim \mathfrak{z}_b) = 1_b$.

Since A and B are two \mathbb{BMBJW} – filters of w , then we have

$$\begin{aligned} M_T^A(\mathfrak{z}_b) &\geq \min \{M_T^A(\eta_b), M_T^A(x_b)\}, \\ \mathbb{B}_I^{A-}(\mathfrak{z}_b) &\leq \max \{\mathbb{B}_I^{A-}(\eta_b), \mathbb{B}_I^{A-}(x_b)\}, \\ \mathbb{B}_I^{A+}(\mathfrak{z}_b) &\geq \min \{\mathbb{B}_I^{A+}(\eta_b), \mathbb{B}_I^{A+}(x_b)\} \\ \text{and } J_F^A(\mathfrak{z}_b) &\leq \max \{J_F^A(\eta_b), J_F^A(x_b)\} \end{aligned}$$

$$\begin{aligned} M_T^B(\mathfrak{z}_b) &\geq \min \{M_T^B(\eta_b), M_T^B(x_b)\}, \\ \mathbb{B}_I^{B-}(\mathfrak{z}_b) &\leq \max \{\mathbb{B}_I^{B-}(\eta_b), \mathbb{B}_I^{B-}(x_b)\}, \\ \mathbb{B}_I^{B+}(\mathfrak{z}_b) &\geq \min \{\mathbb{B}_I^{B+}(\eta_b), \mathbb{B}_I^{B+}(x_b)\} \\ \text{and } J_F^B(\mathfrak{z}_b) &\leq \max \{J_F^B(\eta_b), J_F^B(x_b)\} \end{aligned}$$

$$\begin{aligned} M_T^{A \cap B}(\mathfrak{z}_b) &= \min \{M_T^A(\mathfrak{z}_b), M_T^B(\mathfrak{z}_b)\} \\ &= \min \{\min \{M_T^A(x_b), M_T^A(\eta_b)\}, \min \{M_T^B(x_b), M_T^B(\eta_b)\}\} \\ &= \min \{\min \{M_T^A(x_b), M_T^B(x_b)\}, \min \{M_T^A(\eta_b), M_T^B(\eta_b)\}\} \\ &= \min \{M_T^{A \cap B}(x_b), M_T^{A \cap B}(\eta_b)\}. \end{aligned}$$

Similarly we can prove that

$$\mathbb{B}_I^{A \cap B}(\mathfrak{z}_b) = \min \{\mathbb{B}_I^{A \cap B}(x_b), \mathbb{B}_I^{A \cap B}(\eta_b)\}, J_F^{A \cap B}(\mathfrak{z}_b) = \max \{J_F^{A \cap B}(x_b), J_F^{A \cap B}(\eta_b)\}.$$

So $A \cap B$ is a $\mathbb{BMBJ}\mathcal{W}$ – filter of \mathcal{W} .

Definition 3.12 : A $\mathfrak{S}_{\mathbb{B}}$ -set $A_B = (M_T^A, \mathbb{B}_I^A, J_F^A)$ on \mathcal{W} is called a \mathbb{BMBJ} - s algebra of \mathcal{W} if it satisfies $\forall x_b, \eta_b \in \mathcal{W}$,

$$M_T^A(x_b) + \mathbb{B}_I^{A-}(\eta_b) \leq 1 \text{ and } \mathbb{B}_I^{A+}(\eta_b) + J_F^A(\eta_b) \leq 1$$

and

$$\begin{aligned} M_T^A(x_b \sim \eta_b) &\geq \min \{M_T^A(\eta_b), M_T^A(x_b)\}, \\ \mathbb{B}_I^{A-}(x_b \sim \eta_b) &\leq \max \{\mathbb{B}_I^{A-}(\eta_b), \mathbb{B}_I^{A-}(x_b)\}, \\ \mathbb{B}_I^{A+}(x_b \sim \eta_b) &\geq \min \{\mathbb{B}_I^{A+}(\eta_b), \mathbb{B}_I^{A+}(x_b)\}, \\ J_F^A(x_b \sim \eta_b) &\leq \max \{J_F^A(\eta_b), J_F^A(x_b)\} \end{aligned}$$

Theorem 3.13: Every $\mathbb{BMBJ}\mathcal{W}$ – filter of \mathcal{W} is a \mathbb{BMBJ} - s algebra of \mathcal{W} .

Proof: Let $A_B = (M_T^A, \mathbb{B}_I^A, J_F^A)$ be a \mathbb{BMBJ} - filter of \mathcal{W} .

Since $\eta_b \leq x_b \sim (\eta_b \sim x_b)$ for all $x_b, \eta_b \in \mathcal{W}$, it follows from lemma 3.1 that

$$\begin{aligned} M_T^A(x_b \sim \eta_b) &\geq \min \{M_T^A(\eta_b), M_T^A(x_b)\}, \\ \mathbb{B}_I^{A-}(x_b \sim \eta_b) &\leq \max \{\mathbb{B}_I^{A-}(\eta_b), \mathbb{B}_I^{A-}(x_b)\}, \\ \mathbb{B}_I^{A+}(x_b \sim \eta_b) &\geq \min \{\mathbb{B}_I^{A+}(\eta_b), \mathbb{B}_I^{A+}(x_b)\}, \\ J_F^A(x_b \sim \eta_b) &\leq \max \{J_F^A(\eta_b), J_F^A(x_b)\} \forall x_b, \eta_b \in \mathcal{W} \end{aligned}$$

Hence $A_B = (M_T^A, \mathbb{B}_I^A, J_F^A)$ be a \mathbb{BMBJ} - s algebra of \mathcal{W} .

Theorem 3. 14: Let $A_B = (M_T^A, \mathbb{B}_I^A, J_F^A)$ be a \mathbb{BMBJ} - s algebra of \mathcal{W} satisfying the condition(3.3). Then $A_B = (M_T^A, \mathbb{B}_I^A, J_F^A)$ be a $\mathbb{BMBJ}\mathcal{W}$ - filter of \mathcal{W} .

Proof: For any $x_b \in \mathcal{W}$, we have

$$\begin{aligned} M_T^A(1_b) &= M_T^A(x_b \sim x_b) \geq \min \{M_T^A(x_b), M_T^A(x_b)\} = M_T^A(x_b), \\ \mathbb{B}_I^{A-}(1_b) &= \mathbb{B}_I^{A-}(x_b \sim x_b) \leq \max \{\mathbb{B}_I^{A-}(x_b), \mathbb{B}_I^{A-}(x_b)\} = \mathbb{B}_I^{A-}(x_b), \\ \mathbb{B}_I^{A+}(1_b) &= \mathbb{B}_I^{A+}(x_b \sim x_b) \geq \min \{\mathbb{B}_I^{A+}(x_b), \mathbb{B}_I^{A+}(x_b)\} = \mathbb{B}_I^{A+}(x_b), \end{aligned}$$

and

$$\mathbb{J}_F^A(1_b) = \mathbb{J}_F^A(x_b \sim x_b) \leq \max \{ \mathbb{J}_F^A(x_b) \mathbb{J}_F^A(x_b) \} = \mathbb{J}_F^A(x_b).$$

Since $\eta_b \leq x_b \sim (x_b \sim \eta_b)$ for all $x_b, \eta_b \in \mathbb{W}$, it follows from condition(3.3) that

$$\mathbb{M}_F^A(\eta_b) \geq \min \{ \mathbb{M}_F^A(x_b \sim \eta_b), \mathbb{M}_F^A(x_b) \},$$

$$\mathbb{B}_F^{A-}(\eta_b) \leq \max \{ \mathbb{B}_F^{A-}(x_b \sim \eta_b), \mathbb{B}_F^{A-}(x_b) \},$$

$$\mathbb{B}_F^{A+}(\eta_b) \geq \min \{ \mathbb{B}_F^{A+}(x_b \sim \eta_b), \mathbb{B}_F^{A+}(x_b) \},$$

$$\mathbb{J}_F^A(\eta_b) \leq \max \{ \mathbb{J}_F^A(x_b \sim \eta_b), \mathbb{J}_F^A(x_b) \}$$

for all $x_b, \eta_b \in \mathbb{W}$. Then $A_B = (\mathbb{M}_F^A, \mathbb{B}_F^A, \mathbb{J}_F^A)$ be a \mathbb{BMBJ}_w - filter of \mathbb{W} .

4. Conclusion

We defined the \mathbb{BMBJ}_w - filter of Lattice wajsberg algebras and proved the properties of \mathbb{BMBJ}_w - filter. We derived some relation between fuzzy ideals, interval valued fuzzy ideals to \mathbb{BMBJ}_w - filter. Further we provide that cut sets of $\mathfrak{MB}\mathfrak{S}$ - sets formed \mathbb{BMBJ}_w - filter. Finally defined the \mathbb{BMBJ} - subalgebra and proved every \mathbb{BMBJ}_w - filter is a \mathbb{BMBJ} - subalgebra and converse is need not be true. Further we investigate the point filters of Lattice wajsberg algebras.

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