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Analysis of a Complex Feedback Queue Network

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Abstract

The aim of the paper is to discuss a system of complex feedback queue model containing three subsystems; one is comprised with two biserial service channels and other with two parallel service channels. Both the subsystems are commonly connected with a central service channel. The customers have the facility to revisit any of the service channels at most once with different probability of leaving the server and moving from one server to another server. Mean queue length is derived in steady state condition. Particular values are given to the variables to check the validity of the model. The model is applicable to many real life situations such as in shopping malls, offices, communication networks etc.

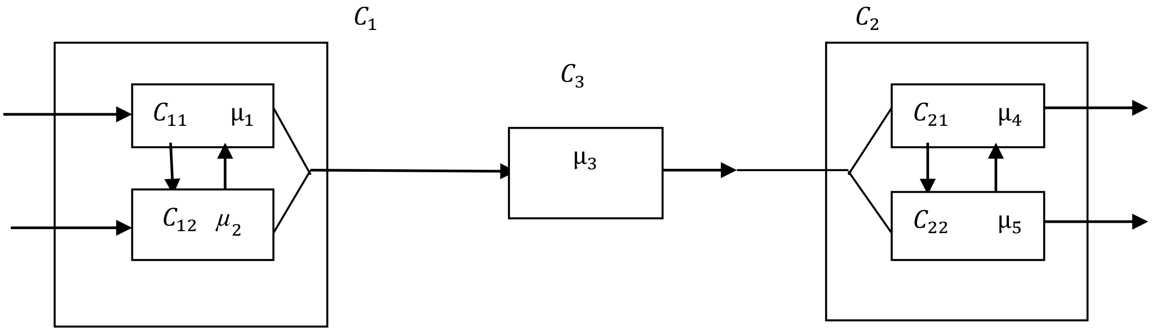
Keywords: Biserial channels, Parallel channel, common server, feedback queue, mean queue length

Introduction

Waiting line theory came in existence in 20th century. It was A.K. Erlang, Danish mathematician, statician and engineer who introduced this theory and created models to describe the system of Copenhagen Telephone Exchange Company. After that many mathematician and researchers contribute their work in the development of queuing theory. Maggu (1970) introduced a system of biserial queues. Singh T.P (2005) study the behavioural analysis of a network of queues with parallel biseries queues connected to a common server. Further Gupta Deepak (2006) analyzes a complex system of queue model comprised of two subsystems in which each subsystem is centrally linked with a common server. Singh T.P, Kusum, Gupta Deepak (2010) presents a network queue model centrally linked with a common feedback channel. TyagiArti, Singh T.P, Saroa M.S (2012) presents semi bitendem feedback queue network which is further an extension of work done by Singh T.P, Gupta Deepak. But in the above said model probabilities of revisiting the customers are taken same as of their first visit. After that Gupta Deepak & Gupta Renu (2020) present this type of model with batch arrival. In the present model we are further expanding this work by analysing a complex feedback queue model that is comprised with two biserial, two parallel servers and these are centrally connected with a common server. In the model it is assumed that the customers can revisit any of the server at most one time and the customer will revisit the server with changed probability .Because the probability of revisiting the system doesn't remain same always. Mean queue length and the waiting time of customers are derived from steady state differential equations.

MODEL DESCRIPTION:

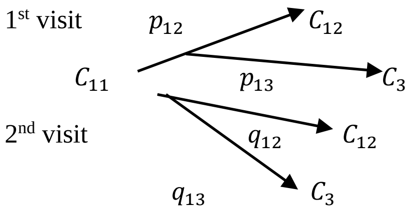
There are three subsystems C_1 ,C_2,C_3 in this model. C_{11}&C_{12} are two biserial service channels in subsystem C_1 and C_{21}&C_{22} are parallel service channels contained in subsystem C_2. The subsystems $C_1$and C_2 are linked in series with a common service channel C_3.At first the customer will arrive at subsystem C_1 in front of service channels C_{11}&C_{12} with poisson arrival rate λ_1&λ_2.



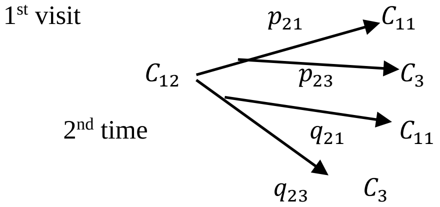
NOTATIONS:

Servers		C_{11}	C_{12}	C_3	C_{21}	C_{22}
Service Rate		μ_1	μ_2	μ_3	μ_4	μ_5
No. of customers		n_1	n_2	n_3	n_4	n_5
Probability of customers moving from one server to another	First visit	$C_{11} \rightarrow C_{12}$ p_{12} $C_{11} \rightarrow C_3$ p_{13}	$C_{12} \rightarrow C_3$ p_{23} $C_{12} \rightarrow C_{11}$ p_{21}	$C_3 \rightarrow C_{11}$ p_{31} $C_3 \rightarrow C_{12}$ p_{32} $C_3 \rightarrow C_{21}$ p_{34} $C_3 \rightarrow C_{22}$ p_{35}	$C_{21} \rightarrow C_3$ p_{43} $C_{21} \rightarrow exit$ p_4	$C_{22} \rightarrow C_3$ p_{53} $C_{22} \rightarrow$exit p_5
	Second visit	$C_{11} \rightarrow C_{12}$ q_{12} $C_{11} \rightarrow C_3$ q_{13}	$C_{12} \rightarrow C_3$ q_{23} $C_{12} \rightarrow C_{11}$ q_{21}	$C_3 \rightarrow C_{21}$ q_{34} $C_3 \rightarrow C_{22}$ q_{35}	$C_{21} \rightarrow exit$ q_4	$C_{22} \rightarrow$exit q_5
Probability of leaving the server	First visit	a	b	c	d	e
	Second visit	a_1	b_1	c_1	d_1	d_1

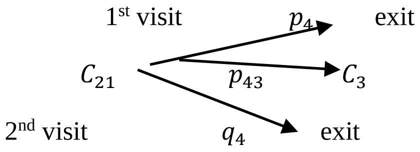
Possibilities of leaving the server C_{11}:



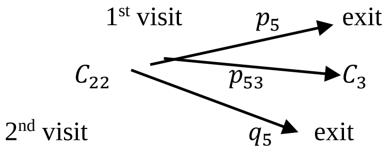
Possibilities of leaving the server C_{12}:



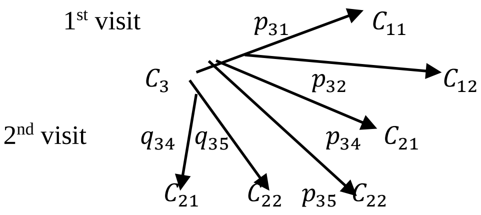
Possibilities of leaving the server C_{21}:



Possibilities of leaving the server C_{22}:



Possibilities of leaving the server C_3:



After being served at C_{11} then with leaving probability a the customer will either visit server C_{12} or server C_3 with probabilities p_{12} and p_{13}such that $p_{12} + p_{13} = 1$ .If the unsatisfied customer revisit the server C_{11},then after leaving the server with probability a_1 ,he will again visit either server C_{12}or server $C_3$for service with probability q_{12} and q_{13}such that $q_{12} + q_{13} = 1$ and $ap_{12} + ap_{13} + a_1q_{12} + a_1q_{13} = 1$

From server C_{12} the customer may either visit to server C_{11} or server C_3 with probabilitiesp_{21}, p_{23} and with probabilities of revisit as q_{21}, q_{23} such that $p_{21} + p_{23} = 1, q_{21} + q_{23} = 1$ and $bp_{21} + bp_{23} + b_1 q_{21} + b_1 q_{23} = 1$

For serverC_3, $p_{31} + p_{32} + p_{34} + p_{35} = 1, q_{34} + q_{35} = 1$ and $cp_{31} + cp_{32} + cp_{34} + cp_{35} + c_1 q_{34} + c_1 q_{35} = 1$, where, $p_{31}, p_{32}, p_{34}, p_{35}$ are probabilities of first visit to servers $C_{11}, C_{12}, C_{21}, C_{22}$ respectively q_{34}, q_{35} are the probabilities of second visit to servers C_{21}, C_{22}.

From servers C_{21}, C_{22} the customer either moves to sever C_3 or he may exit the system such that for server $C_{21} dp_{43} + dp_4 + d_1 q_4 = 1$ and for server C_{22}, $ep_{53} + ep_5 + e_1 q_5 = 1$

FORMULATION OF MODEL:

Suppose $P_{n_{1,}n_{2,}n_3 n_{4,}n_5}$ (t) denotes the probability of n_1, n_2, n_3, n_4, n_5customers in front of the servers C_{11},C_{12},C_3, C_{21},C_{22} respectively, where $n_1, n_2, n_3, n_4, n_5 \geq 0$ .The equations for the complex queue network model in steady state are as follows:

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + +\mu_4 + \mu_5) P_{n_{1,}n_{2,}n_{3,}n_{4,}n_5} = \lambda_1 P_{n_1-1,n_{2,}n_{3,}n_{4,}n_5} + \lambda_2 P_{n_1,n_2-1,n_{3,}n_{4,}n_5} + \mu_1(ap_{12} + a_1 q_{12}) P_{n_1+1,n_2-1,n_{3,}n_{4,}n_5} + \mu_1(ap_{13} + a_1 q_{13}) P_{n_1+1,n_{2,}n_3-1,n_{4,}n_5} + \mu_2(bp_{21} + b_1 q_{21}) P_{n_1-1,n_2+1,n_{3,}n_{4,}n_5} + \mu_2(bp_{23} + b_1 q_{23}) P_{n_{1,}n_2+1,n_3-1,n_{4,}n_5} + \mu_3(cp_{31}) P_{n_1-1,n_{2,}n_3+1,n_{4,}n_5} + \mu_3(cp_{32}) P_{n_{1,}n_2-1,n_3+1,n_{4,}n_5} + \mu_3(cp_{34} + c_1 q_{34}) P_{n_{1,}n_{2,}n_3+1,n_4-1,n_5} + \mu_3(cp_{35} + c_1 q_{35}) P_{n_{1,}n_{2,}n_3+1,n_{4,}n_5-1} + \mu_4(dp_{43}) P_{n_{1,}n_{2,}n_3-1,n_4+1,n_5} + \mu_4(dp_4 + d_1 q_4) P_{n_{1,}n_{2,}n_{3,}n_4+1,n_5} + \mu_5(ep_{53}) P_{n_{1,}n_{2,}n_3-1,n_{4,}n_5+1} + \mu_5(ep_5 + e_1 q_5) P_{n_{1,}n_{2,}n_{3,}n_{4,}n_5+1} \quad (1)$$

There will be 31 more equations by considering all possible cases for n_1, n_2, n_3, n_4, n_5.

To solve these equations we define G.F as,

G(X,Y,Z,R,S) = $\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \sum_{n_5=0}^{\infty} P_{n_{1,}n_{2,}n_3 n_{4,}n_5} X^{n_1} Y^{n_2} Z^{n_3} R^{n_4} S^{n_5}$ where $|X|$=1,$|Y|$=1,$|Z|$=1,$|R|$=1,$|S|$=1, also partial generating functions are ,

$$G_{n_{2,}n_3 n_{4,}n_5}(X) = \sum_{n_1=0}^{\infty} P_{n_{1,}n_{2,}n_3 n_{4,}n_5} X^{n_1} \quad (2)$$

$$G_{n_{3,}n_{4,}n_5}(X,Y) = \sum_{n_2=0}^{\infty} G_{n_{2,}n_3 n_{4,}n_5}(X) Y^{n_2} \quad (3)$$

$$G_{n_{4,}n_5}(X,Y,Z) = \sum_{n_3=0}^{\infty} G_{n_{3,}n_{4,}n_5}(X,Y) Z^{n_3} \quad (4)$$

$$G_{n_5}(X,Y,Z,R) = \sum_{n_3=0}^{\infty} G_{n_{4,}n_5}(X,Y,Z) R^{n_4} \quad (5)$$

$$\text{G(X,Y,Z,R,S)} = \sum_{n_5=0}^{\infty} G_{n_5}(X,Y,Z,R) S^{n_5} \quad (6)$$

And we get solution as,

$$\text{G(X,Y,Z,R,S)}=\frac{\mu_1\left(1-\frac{AY}{X}-\frac{BZ}{X}\right)G_1+\mu_2\left(1-\frac{CX}{Y}-\frac{DZ}{Y}\right)G_2+\mu_3\left(1-\frac{ER}{Z}-\frac{FS}{Z}-\frac{IX}{Z}-\frac{JY}{Z}\right)G_3+\mu_4\left(1-\frac{G}{R}-\frac{KZ}{R}\right)G_4+\mu_5\left(1-\frac{H}{S}-\frac{LZ}{S}\right)G_5}{\lambda_1(1-X)+\lambda_2(1-Y)+\mu_1\left(1-\frac{AY}{X}-\frac{BZ}{X}\right)+\mu_2\left(1-\frac{CX}{Y}-\frac{DZ}{Y}\right)+\mu_3\left(1-\frac{ER}{Z}-\frac{FS}{Z}-\frac{IX}{Z}-\frac{JY}{Z}\right)+\mu_4\left(1-\frac{G}{R}-\frac{KZ}{R}\right)+\mu_5\left(1-\frac{H}{S}-\frac{LZ}{S}\right)}$$

(7) here for convenience we denote , $G_1 = G_0(Y,Z,R,S) G_2 = G_0(X,Z,R,S) G_3 = G_0(X,Y,R,S) G_4 = G_0(X,Y,Z,S) G_5 = G_0(X,Y,Z,R)$, and

A= $(ap_{12} + a_1 q_{12})$ B=$(ap_{13} + a_1 q_{13})$, C=$(bp_{21} + b_1 q_{21})$, D=$(bp_{23} + b_1 q_{23})$, E=$(cp_{34} + c_1 q_{34})$, F=$(cp_{35} + c_1 q_{35})$, G=$(dp_4 + d_1 q_4)$, H=$(ep_5 + e_1 q_5)$, I=cp_{31}, J=cp_{32}, K=(dp_{43}), L=(ep_{53}) (**)

At X=Y=Z=R=S=1, G(X,Y,Z,R,S)=1 and equation (7) reduces to indeterminate form. By applying L'Hospital rule for indeterminate form we obtain,

$$\mu_1 G_1 - C\mu_2 G_2 - I\mu_3 G_3 = -\lambda_1 + \mu_1 - C\mu_2 - I\mu_3 \quad (8)$$

$-A\mu_1 G_1 + \mu_2 G_2 - J\mu_3 G_3 = -\lambda_2 + \mu_2 - A\mu_1 - J\mu_3$ (9)

$$-B\mu_1 G_1 - D\mu_2 G_2 + \mu_3 G_3 - K\mu_4 G_4 - L\mu_5 F_5 = -B\mu_1 - D\mu_2 + \mu_3 - K\mu_4 - L\mu_5 \quad (10)$$

$$-E\mu_3 G_3 + \mu_4 G_4 = -E\mu_3 + \mu_4 \quad (11)$$

$$-F\mu_3 G_3 + \mu_5 G_5 = -F\mu_3 + \mu_5 \quad (12)$$

On solving equations from (8) to (12) ,we get

$$G_1 = 1 - \frac{(\lambda_1 + C\lambda_2)}{\mu_1(1-AC)} + \frac{(I+CJ)(\lambda_1(DA+B)+\lambda_2(D+BC))}{\mu_1 A'} \quad (13)$$

$$G_2 = 1 - \frac{(\lambda_1 A + \lambda_2)}{\mu_2(1-AC)} + \frac{(IA+J)(\lambda_1(DA+B)+\lambda_2(D+BC))}{\mu_2 A'} \quad (14)$$

$$G_3 = 1 - \frac{(\lambda_1(DA+B)+\lambda_2(D+BC))}{\mu_3 A'} \quad (15)$$

$$G_4 = 1 - \frac{E}{\mu_4}\left(\frac{(\lambda_1(DA+B)+\lambda_2(D+BC))}{A'}\right) \quad (16)$$

$$G_5 = 1 - \frac{F}{\mu_5}\left(\frac{(\lambda_1(DA+B)+\lambda_2(D+BC))}{A'}\right) \quad (17)$$

Here weassume,

$(A' = 1 - AC - BI - BCJ - DIA - DJ - KE - LF + ACKE + ACFL)$(18)

The solution of steady state differential equation is,

$P_{n_1,n_2,n_3 n_4,n_5}$= $(1-G_1)^{n_1}(1-G_2)^{n_2}(1-G_3)^{n_3}(1-G_4)^{n_4}(1-G_5)^{n_5} G_1 G_2 G_3 G_4 G_5$

=$(1-\rho_1)^{n_1}(1-\rho_2)^{n_2}(1-\rho_3)^{n_3}(1-\rho_4)^{n_4}(1-\rho_5)^{n_5}\rho_1\rho_2\rho_3\rho_4\rho_5$ where

$$\rho_1 = \frac{(\lambda_1 + C\lambda_2)}{\mu_1(1-AC)} + \frac{(I+CJ)(\lambda_1(DA+B)+\lambda_2(D+BC))}{\mu_1 A'} \quad (19)$$

$$\rho_2 = \frac{(\lambda_1 A + \lambda_2)}{\mu_2(1-AC)} + \frac{(IA+J)(\lambda_1(DA+B)+\lambda_2(D+BC))}{\mu_2 A'} \quad (20)$$

$$\rho_3 = \frac{(\lambda_1(DA+B)+\lambda_2(D+BC))}{\mu_3 A'} \quad (21)$$

$$\rho_4 = \frac{E}{\mu_4}\left(\frac{(\lambda_1(DA+B)+\lambda_2(D+BC))}{A'}\right) \quad (22)$$

$$\rho_5 = \frac{F}{\mu_5}\left(\frac{(\lambda_1(DA+B)+\lambda_2(D+BC))}{A'}\right) \quad (23)$$

PERFORMANCE MEASURES:

1. Mean Queue Length = $L_1 + L_2 + L_3 + L_4 + L_5 = \frac{\rho_1}{1-\rho_1} + \frac{\rho_2}{1-\rho_2} + \frac{\rho_3}{1-\rho_3} + \frac{\rho_4}{1-\rho_4} + \frac{\rho_5}{1-\rho_5}$ (24)
2. Mean Waiting Time = $\frac{L}{\lambda}$, where λ = $\lambda_1 + \lambda_2$ (25)
3. Variance = $\frac{\rho_1}{(1-\rho_1)^2} + \frac{\rho_2}{(1-\rho_2)^2} + \frac{\rho_3}{(1-\rho_3)^2} + \frac{\rho_4}{(1-\rho_4)^2} + \frac{\rho_5}{(1-\rho_5)^2}$ (26)

PARAMETRIC STUDY:

Let us take the numerical values as

a=0.4, b=0.6, c=0.3, d=0.7, e=0.6,$a_1 = 0.6$,$b_1 = 0.4$,$c_1 = 0.7$,d_1=0.5,e_1=0.8,$p_{12} = 0.4$,$p_{13} = 0.6$,$p_{21} = 0.7$,$p_{23} = 0.3$,$p_{31} = 0.2$,$p_{32} = 0.2$,$p_{34} = 0.6$,$p_{35} = 0.4$,$p_4 = 0.7$,$p_{43} = 0.3$,$p_5 = 0.8$,$p_{53} = 0.2$,$q_{12} = 0.3$,$q_{13} = 0.7$,$q_{21} = 0.6$,$q_{34} = 0.6$,$q_{35} = 0.4$,$q_4 = 0.6$,$q_5 = 0.5$

(from (**)) A=0.34, B=0.66, C= 0.66, D=0.34, E=0.51,F=0.33,G=0.79, H=0.88, I=0.06, J=0.06, K=0.21, L=0.12 and A'=0.6131 (from equation (18))

Table 1: utilization of servers, partial queue lengths, total queue length for different values of λ_1

$\lambda_2 = 3, \mu_1 = 10, \mu_2 = 6, \mu_3 = 8, \mu_4 = 7, \mu_5 = 5$											
λ_1	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	L_1	L_2	L_3	L_4	L_5	L
2	0.576	0.8756	0.7907	0.4608	0.4174	1.358491	7.038585	3.777831	0.854599	0.716444	13.74595
2.2	0.6044	0.8935	0.8222	0.4793	0.4341	1.527806	8.389671	4.624297	0.920492	0.767097	16.22936
2.4	0.6332	0.9115	0.8539	0.4977	0.4509	1.726281	10.29944	5.844627	0.990842	0.821162	19.68235
2.6	0.661	0.9289	0.8855	0.5161	0.4676	1.949853	13.0647	7.733624	1.066543	0.878287	24.693
2.8	0.6894	0.9475	0.9171	0.5346	0.4842	2.219575	18.04762	11.06273	1.148689	0.938736	33.41735
3	0.7177	0.9655	0.9487	0.553	0.5009	2.542331	27.98551	18.49318	1.237136	1.003606	51.26176

Table 2: utilization of servers, partial queue lengths, total queue length for different values of λ_2

$\lambda_1 = 2, \mu_1 = 10, \mu_2 = 6, \mu_3 = 8, \mu_4 = 7, \mu_5 = 5$											
λ_2	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	L_1	L_2	L_3	L_4	L_5	L
2	0.4784	0.6436	0.6325	0.3686	0.3339	0.917178	1.805836	1.721088	0.583782	0.501276	5.529161
2.2	0.498	0.6897	0.6641	0.3871	0.3418	0.992032	2.222688	1.977077	0.631588	0.519295	6.342679
2.4	0.5175	0.7363	0.6957	0.4052	0.3673	1.072539	2.792188	2.286231	0.681237	0.580528	7.412723
2.6	0.537	0.7827	0.7274	0.4132	0.384	1.159827	3.601933	2.668379	0.704158	0.623377	8.757673
2.8	0.5566	0.8291	0.759	0.4424	0.4007	1.2553	4.851375	3.149378	0.7934	0.668613	10.71807
3	0.576	0.8756	0.7907	0.4608	0.4174	1.358491	7.038585	3.777831	0.854599	0.716444	13.74595

Table 3: queue length for different values of $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$

$\lambda_1 = 2, \lambda_2 = 3, \mu_2 = 6, \mu_3 = 8, \mu_4 = 7, \mu_5 = 5$									
μ_1	L	μ_2	L	μ_3	L	μ_4	L	μ_5	L
10	13.74594	6	13.74594	8	13.32731	7	13.70656	5	13.74594
10.5	13.63313	7	9.713765	9	12.33285	8	13.56695	6	13.56277
11	13.51285	8	8.618563	10	11.6892	9	13.44995	7	13.45421
11.5	13.41338	9	8.114673	11	11.32105	10	13.36736	8	13.3825
12	13.33033	10	7.930565	12	11.08272	11	13.30617	9	13.33141
12.5	13.25976	11	7.686769	13	10.91553	12	13.25896	10	13.29324

It can be clearly observed from above tables that when arrival rate increases then partial queue lengths and total queue length for the system increases. In the starting point when we increase arrival rate, the length of queue increases with slow speed after sometime the length of queue increases faster than before. With the increase in service rate at different service channels, the total queue length of the system is decreasing.

PARTICULAR CASE:

If we take, a= b=c= d= e=$a_1 = b_1 = c_1 = d_1$=e_1=1,$p_{31} = 0, p_{32} = 0, p_4 = p_{43} = p_5 = p_{53} = q_{12} = q_{13} = q_{21} = q_{34} = q_{35} = q_4 = q_5 = 0$

then the result telly with the model derived by Gupta Deepak (2006).

CONCLUSION:

In the above model we analyze a complex system of biserial and parallel server, with the concept of feedback. The probabilities of revisiting the different service channels are changed in the sense that the probabilities may not remain same always. The model becomes much real according to real life situations because the customer is not always satisfied in one visit. The developed model is applicable to many real life situations. Mean queue length, expected waiting time and variance of queue have been obtained after solving time independent equations. A particular case has been taken for validity check.

CONFLICT OF INTERESTS: We declare that there is no conflict of interests.

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