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Certain Results on Pair Sum Labeling of Newly Constructed Graphs

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Abstract

For a (p,q) graph G, an injective map f from $V(G)$ to $\{\pm1,\pm2,\ldots,\pm p\}$ is said to be a pair sum labeling if the induced edge function f_e from $E(G)$ to $\{Z-0\}$ defined by $f_e(uv)=f(u)+f(v)$ is $1-1$ and $f_e(E(G))$ is either of the form $\{\pm m_1,\pm m_2,\ldots,\pm m_{\frac{q}{2}}\}$ or $\{\pm m_1,\pm m_2,\ldots,\pm m_{\frac{q-1}{2}}\}\cup\{m_{\frac{q+1}{2}}\}$ depending on q which is either even or odd. A graph that admits the behavior of pair sum labeling is named a pair sum graph. In this manuscript we study the pair sum labeling of uniform $(3,n)$-cyclic graph, n-cyclic copies of C_3 and path union of ladders.

Keywords: Pair Sum Labeling, Uniform, $(3,n)$-Cyclic Graph, n-Cyclic Copies of C_3, Path Union of Ladders.

1. Introduction

Among the miscellaneous types of graph labeling, pair sum labeling of graphs is a newest form of labeling strategy. The labeling concept of pair sum graphs was introduced by Ponraj et al. [1]. The considered graph G in this article is of simple, undirected, finite. Terms not characterized here are utilized in the feeling of Harary [4]. An investigation of Pair sum labeling for certain standard graphs like cycle, path, bi-star complete graph, and some more kinds of graphs are discussed in [1-3]. Also they have proved most of all categories of trees admits pair sum labeling up to order $n\geq 8$ [5].

Definition 1

For a (p,q) graph G, an injective map f from $V(G)$ to $\{\pm1,\pm2,\ldots,\pm p\}$ is said to be a pair sum labeling if the induced edge function f_e from $E(G)$ to $\{Z-0\}$ defined by $f_e(uv)=f(u)+f(v)$ is $1-1$ and $f_e(E(G))$ is either of the form $\{\pm m_1,\pm m_2,\ldots,\pm m_{\frac{q}{2}}\}$ or $\{\pm m_1,\pm m_2,\ldots,\pm m_{\frac{q-1}{2}}\}\cup\{m_{\frac{q+1}{2}}\}$ depending on q which is either even or odd. A graph that admits the behavior of pair sum labeling is named a pair sum graph.

2. Main Result

Definition 2

A uniform $(3,n)$-cyclic graph SC_3^n where n is even is obtained by attaching cycles of length 3 to all the pendent vertices of a star graph S_{n+1} (See Figure 2(a)).

Definition 3

Let the graphs $G_1, G_2, .., G_n$, $n \geq 2$ be all replicated copies of a constant cycle graph G. Adding an edge in between any two vertices of G_i and G_{i+1} consecutively for $i = 1,2, \dots, (n-1)$ and an edge between G_n and G_1 is named as a uniform n-cyclic graph.

Let $L_3^1, L_3^2, \dots, L_3^m$ be m copies of ladder graph L_3 are labeled as follows (See Figure. 1).

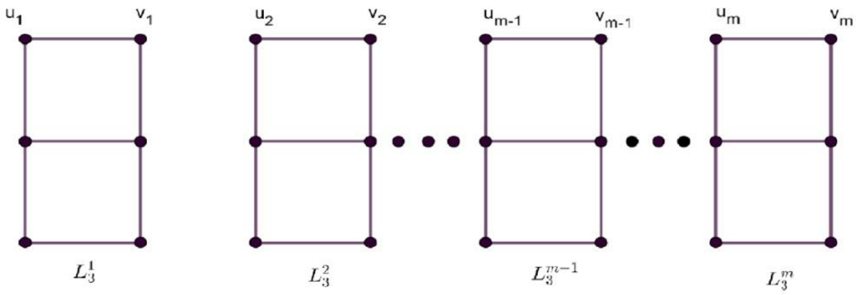


Figure. 1 m copies of ladder graph L_3

Definition 4

Let us consider the m-copies of ladder graphs L_3 where $m \geq 4$ is even. The graph $P(L_3^m)$ is obtained by joining an edge in between the vertices v_i of L_3^i and u_{i+1} of L_3^{i+1} where $i = 1,2, \dots, m-1$ consecutively is named as path union of ladders.

Theorem 1

The uniform $(3, n)$-cyclic graph SC_3^n is a pair sum graph if n is even.

Proof:

Let the vertex set of SC_3^n be $\{u_1, u_2, \dots, u_{\frac{3n}{2}}; 1 \leq i \leq n, v_1, v_2, \dots, v_{\frac{3n}{2}}; 1 \leq i \leq n\}$ and w be the hub vertex.

Now let us define $f: V(SC_3^n) \to \{\pm 1, \pm 2, \dots, \pm \frac{3n}{2}\}$

such that

$$f(w) = 1$$

$f(u_i) = 2i$ where $i = 1,2, .., \frac{3n}{2}$

$f(v_i) = -2i$ where $i = 1,2, .., \frac{3n}{2}$

Furtherly from the edge function which is induced, we have

$f_e: E(SC_3^n) \to \{Z - 0\}$,

$f_e(u_i u_j) = 2i + 2j, i \neq j$

$f_e(v_i v_j) = -2i - 2j, i \neq j$

$f_e(wu_i) = 1 + 2i,$

$f_e(wv_j) = 1 - 2j,$

then

$f(E(SC_3^n)) =$
$\{\{\pm 3, \pm 9, \pm 15, \dots, \pm(3n-3)\}, \cup \{\pm 6, \pm 18, \pm 30, \dots, \pm(6n-6)\}, \cup \{\pm 8, \pm 20, \pm 32, \dots, \pm(6n-4)\}, \cup \{\pm 10, \pm 22, \pm 34, \dots, \pm(6n-2)\}\}.$

(See Figure 2(b))

Hence the graph SC_3^n is a pair sum graph.

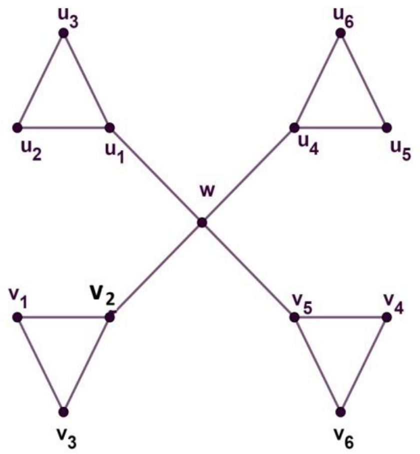


Figure 2(a) SC_3^4

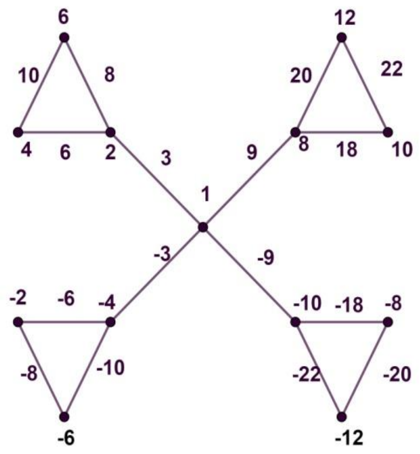


Figure 2(b) Pair sum labeling of SC_3^4

Theorem 2

The n-cyclic graph C_3^n, $n \geq 2$ is a pair sum graph if n is even.

Proof:

Let the vertex set of C_3^n be $\{u_1, u_2, \dots, u_{\frac{3n}{2}}; 1 \leq i \leq n, v_1, v_2, \dots, v_{\frac{3n}{2}}; 1 \leq i \leq n\}$

Now let us define $f: V(C_3^n) \to \{\pm 1, \pm 2, \dots, \pm \frac{3n}{2}\}$

such that

$f(u_i) = i$ where $i = 1,2, \dots, \frac{3n}{2}$

$f(v_i) = -i$ where $i = 1,2, \dots, \frac{3n}{2}$

Furtherly from the edge function which is induced, we have

$f_e: E(C_3^n) \rightarrow \{Z - 0\}$,

$f_e(u_i u_j) = i + j, i \neq j$

$f_e(v_i v_j) = -i - j, i \neq j f_e(u_i v_{i+1}) = -1$

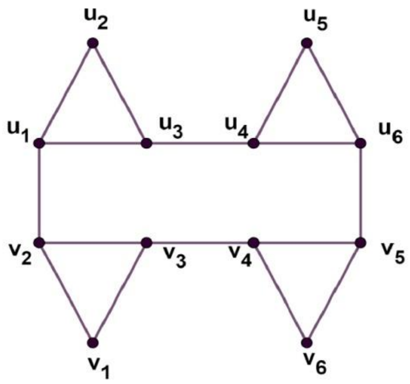
$$f_e(u_i v_{i-1}) = 1$$

then

$f(E(C_3^n)) =$
$\{\pm 1, \pm 7, \pm 13, \ldots, \pm(3n-5)\}, \cup \{\pm 3, \pm 9, \pm 15, \ldots, \pm(3n-3)\}, \cup \{\pm 4, \pm 10, \pm 16, \ldots, \pm(3n-2)\}, \{\pm 5, \pm 11, \pm 17, \ldots, \pm(3n-1)\}, \}$.

(See Figure 3(b))

Hence the graph C_3^n is a pair sum graph.



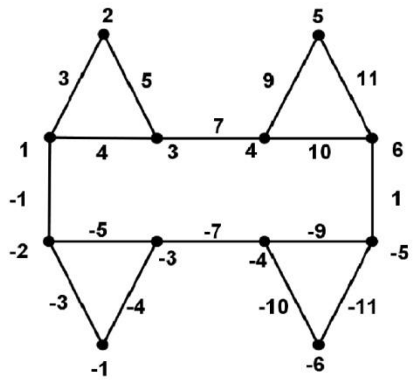


Figure 3(a) C_3^4 *Figure 3(b) Pair sum labeling of* C_3^4

Theorem 3

The graph $P(L_3^m)$ is a pair sum graph when $m \geq 4$ is even.

Proof:

Let the vertex set of $P(L_3^m)$ be $\{u_1, u_2, \ldots, u_{3m}, 1 \leq i \leq n, v_1, v_2, \ldots, v_{3m}, 1 \leq i \leq n\}$

Now let us define $f: V(P(L_3^m)) \rightarrow \{\pm 1, \pm 2, \ldots, \pm 3m\}$

such that

$f(u_i) = i$ where $i = 1,2,..,3m$

$f(v_i) = -i$ where $i = 1,2,..,3m$

Furtherly from the edge function which is induced, we have

$f_e: E(P(L_3^m)) \rightarrow \{Z - 0\}$,

$f_e(u_i u_j) = i + j, i \neq j$

$f_e(v_i v_j) = -i - j, i \neq j$

$$f_e(v_i u_{i+1}) = 1$$

then

$f(E(P(L_3^m)) = 1 \cup \{\pm 3, \pm 15, \pm 27, \ldots, \pm(6m-9)\} \cup \{\pm 4, \pm 16, \pm 28, \ldots, \pm(6m-8)\} \cup \{\pm 6, \pm 18, \pm 30, \ldots, \pm(6m-6)\} \cup \{\pm 7, \pm 19, \pm 31, \ldots, \pm(6m-5)\} \cup \{\pm 8, \pm 20, \pm 32, \ldots, \pm(6m-4)\} \cup \{\pm 10, \pm 22, \pm 34, \ldots, \pm(6m-2)\} \cup \{\pm 11, \pm 23, \pm 35, \ldots, \pm(6m-1)\} \cup \{\pm 9, \pm 21, \pm 33, \ldots, \pm(6m-15)\}$.

(See Figure (4(b))

Hence the graph $P(L_3^m)$ is a pair sum graph.

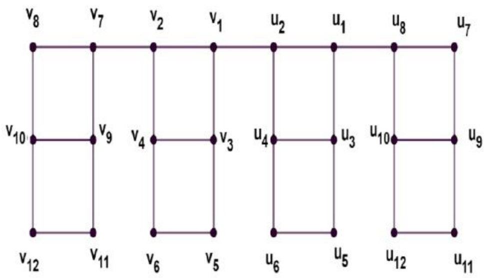


Figure 4(a) L_3^4

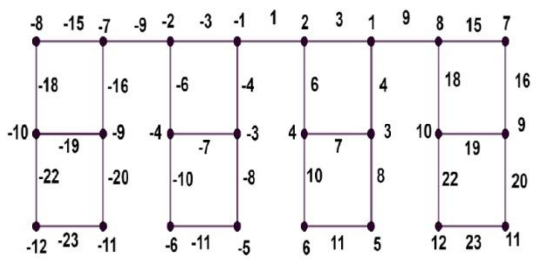


Figure 4(b) Pair sum labeling of L_3^4

3.Conclusion

Therefore as a progress in this manuscript, we have determined pair sum labeling for certain graphs such as, uniform $(3, n)$-cyclic graph, uniform n-cyclic graph, path union of ladder graphs. Pair sum labeling for interconnection networks is under examination.

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