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Priority – Discipline Queue Model for Passport Office System

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Abstract

This paper presents a mathematical queue model for passport office system .It provides a more realistic description of queue model for passport office system on priority basis .Now a days almost in every system priority is given in some cases for example medical cases, to some officials and customers\applicants having some emergency .This model consists two rows, one for high priority and other for low priority customers. It is assumed that the arrival and service process follow the Poisson distribution. It is supposed that the service will be completed when the customer\applicant pass through all the service channels. This model can also be applicable in many real life situations like hospital management, in market, in offices etc.

Keywords:. Priority, Feedback, Queuing, Poisson distribution

1. Introduction

A lot of queuing models have been developed by the researchers, in which some queuing models only predict the system performance, other describes the real life situations and are directly applicable to that situations .In real life situations priority is given to some customers for fast service .Priority discipline queuing models are the systems having priority queues with customer having high priority and customer with low priority.

The first priority queue model was studied by Cobham (1954), “A Cobham priority assignment in waiting line problems”. After that a lot of work on feedback and priority queues is done by many researchers .Singh T.P and Bhardwaj Reeta (2019) presents a stochastic queue model for passport office system. The limitation of the model is that the customers on priority basis are not considered and the probabilities of revisit of customers are not changed. The present model is developed on the basis of above said work done by Singh T.P and Bhardwaj Reeta. In this queuing model we try to develop a queuing model for passport office system with two queues for high priority customers and low priority customers. After applying online\offline the applicant will enter the system for getting passport with an appointment slip. He has to come across the three service channels to get passport. If there is any lacuna remaining in his documents either he has to go back or he may leave the system. The model has been analysed in steady state .The differential difference equations are obtained and the probability generating function technique has been used to obtain queue characteristics. Behavioural analysis of the model is done for different numerical values of the parameters.

2. Notations used

λ_{1L}= arrival rate at low priority queue

λ_{1H}= arrival rate at high priority queue

μ_{1L}= service rate at low priority queue of 1st server

μ_{1H}= service rate at high priority queue of 1st server

μ_2= service rate of 2nd server

μ_3= service rate of 3rd server

a= probability of leaving the server 1st time

a_1= probability of leaving the server 1st time

p_1= probability of exit the system from 1st server

p_2= probability of exit the system from 2nd server

p_{12} =probability of moving from 1st server to 2nd server first time

q_{12} =probability of moving from 1st server to 2nd server second time

p_{21} =probability of moving from 2nd server to 1st server

p_{23} =probability of moving from 2nd server to 3rd server

L_1= length of low priority queue at 1st server

L_2= length of high priority queue at 1st server

L_3= length of queue in front of 2nd server

L_4= length of queue in front of 3rd server

3. Model Description

The model consists of three service channels. After applying online\offline the applicant will enter the system for getting passport with an appointment slip. He has to come across the three service channels to get the passport. Firstly the applicant will come at service channel $\boldsymbol{S_1}$.There are two queues $\boldsymbol{Q_{1L}}$ and $\boldsymbol{Q_{1H}}$ in front of the service channel $\boldsymbol{S_1}$.The applicants arrive with arrival rates $\boldsymbol{\lambda_{1L}}$, $\boldsymbol{\lambda_{1H}}$ **and** $\boldsymbol{\mu_{1L}}, \boldsymbol{\mu_{1H}}, \boldsymbol{\mu_2}, \boldsymbol{\mu_3}$ are service rates respectively. After checking the appointment slip at server $\boldsymbol{S_1}$ either the applicant will move to the server $\boldsymbol{S_2}$ with probability $\boldsymbol{p_{12}}$ or he will exit the server with probability $\boldsymbol{p_1}$.At server $\boldsymbol{S_2}$ he will go through the document verification process. If there is some problem in documents then either he may go back to the server $\boldsymbol{S_1}$ with probability $\boldsymbol{p_{21}}$ and if the problem is solved and all the documents are complete then the customer will visit again the server $\boldsymbol{S_2}$ (after feedback) with the probability $\boldsymbol{q_{12}}$or he will exit the server with probability $\boldsymbol{p_2}$ and if document verification is successful then he may move to the server $\boldsymbol{S_3}$ with probability $\boldsymbol{p_{23}}$ for other formalities and will finally get the passport.

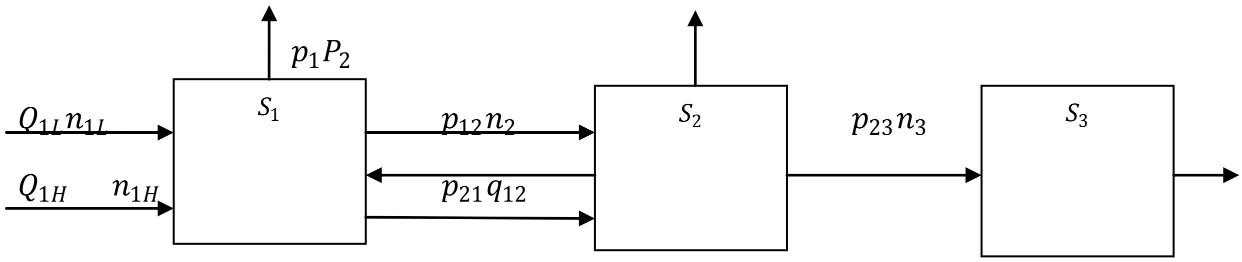

Figure 1: flow of customers in passport office

4. Formulation of Mathematical Model

Let $P_{n_{1L},n_{1H},n_2,n_3}(t)$ be the probability of n_{1L},n_{1H},n_2,n_3 applicants waiting for service in front of the serversS_1,S_2 and S_3 at time t respectively.The differential difference equations in steady state are,

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+\mu_2+\mu_3)P_{n_{1L},n_{1H},n_2,n_3} = \lambda_{1L}P_{n_{1L}-1,n_{1H},n_2,n_3}+\lambda_{1H}P_{n_{1L},n_{1H}-1,n_2,n_3}+\mu_{1H}p_1P_{n_{1L},n_{1H}+1,n_2,n_3}+\mu_{1H}(ap_{12}+a_1q_{12})P_{n_{1L},n_{1H}+1,n_2-1,n_3}+\mu_2p_2P_{n_{1L},n_{1H},n_2+1,n_3}+\mu_2p_{21}P_{n_{1L},n_{1H}-1,n_2+1,n_3}+\mu_2p_{23}P_{n_{1L},n_{1H},n_2+1,n_3-1}+\mu_3P_{n_{1L},n_{1H},n_2,n_3+1}\quad n_{1L},n_{1H},n_2,n_3>0$$
(1)

$n_{1L}=0, n_{1H},n_2,n_3>0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+\mu_2+\mu_3)P_{0,n_{1H},n_2,n_3} = \lambda_{1H}P_{0,n_{1H}-1,n_2,n_3}+\mu_{1H}p_1P_{0,n_{1H}+1,n_2,n_3}+\mu_{1H}(ap_{12}+a_1q_{12})P_{0,n_{1H}+1,n_2-1,n_3}+\mu_2p_2P_{0,n_{1H},n_2+1,n_3}+\mu_2p_{21}P_{0,n_{1H}-1,n_2+1,n_3}+\mu_2p_{23}P_{0,n_{1H},n_2+1,n_3-1}+\mu_3P_{0,n_{1H},n_2,n_3+1}$$
(2)

$n_{1H}=0, n_{1L},n_2,n_3>0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1L}+\mu_2+\mu_3)P_{n_{1L},n_{1H},n_2,n_3} = \lambda_{1L}P_{n_{1L}-1,0,n_2,n_3}+\mu_{1H}p_1P_{n_{1L},1,n_2,n_3}+\mu_{1H}(ap_{12}+a_1q_{12})P_{n_{1L},1,n_2-1,n_3}+\mu_{1L}p_1P_{n_{1L}+1,0,n_2,n_3}+\mu_{1L}p_{12}P_{n_{1L}+1,0,n_{2-1},n_3}+\mu_2p_2P_{n_{1L},0,n_2+1,n_3}+\mu_2p_{23}P_{n_{1L},0,n_2+1,n_3-1}+\mu_3P_{n_{1L},0,n_2,n_3+1}\quad (3)$$

$n_2=0, n_{1L},n_{1H},n_3>0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+\mu_3)P_{n_{1L},n_{1H},0,n_3} = \lambda_{1L}P_{n_{1L}-1,n_{1H},0,n_3}+\lambda_{1H}P_{n_{1L},n_{1H}-1,0,n_3}+\mu_{1H}p_1P_{n_{1L},n_{1H}+1,0,n_3}+\mu_2p_2P_{n_{1L},n_{1H},1,n_3}+\mu_2p_{21}P_{n_{1L},n_{1H}-1,1,n_3}+\mu_2p_{23}P_{n_{1L},n_{1H},1,n_3-1}+\mu_3P_{n_{1L},n_{1H},0,n_3+1}\quad (4)$$

$n_3=0,\ n_{1L},n_{1H},n_2>0$

$$(\lambda_{1L}+\lambda_{1H}+\mu_{1H}+\mu_2)P_{n_{1L},n_{1H},n_2,0} = \lambda_{1L}P_{n_{1L}-1,n_{1H},n_2,0}+\lambda_{1H}P_{n_{1L},n_{1H}-1,n_2,0}+\mu_{1H}p_1P_{n_{1L},n_{1H}+1,n_2,0}+$$

$\mu_{1H}(ap_{12} + a_1 q_{12})P_{n_{1L},n_{1H}+1,n_2-1,0} + \mu_2 p_2 P_{n_{1L},n_{1H},n_2+1,0} + \mu_2 p_{21} P_{n_{1L},n_{1H}-1,n_2+1,0} + \mu_3 P_{n_{1L},n_{1H},n_2,1}$
(5)

$n_{1L} = n_{1H} = 0, n_2, n_3 > 0$

$(\lambda_{1L} + \lambda_{1H} + \mu_2 + \mu_3)P_{0,0,n_2,n_3} =$
$\mu_{1H} p_1 P_{0,1,n_2,n_3} + \mu_{1H}(ap_{12} + a_1 q_{12})P_{0,1,n_2-1,n_3} + \mu_2 p_2 P_{0,0,n_2+1,n_3} + \mu_2 p_{23} P_{0,0,n_2+1,n_3-1} +$
$\mu_3 P_{0,0,n_2,n_3+1}$ (6)

$n_{1L} = n_2 = 0, n_{1H}, n_3 > 0$

$(\lambda_{1L} + \lambda_{1H} + \mu_{1H} + \mu_3)P_{0,n_{1H},0,n_3} = \lambda_{1H} P_{0,n_{1H}-1,0,n_3} + \mu_{1H} p_1 P_{0,n_{1H}+1,0,n_3} + \mu_2 p_2 P_{0,n_{1H},1,n_3} +$
$\mu_2 p_{21} P_{0,n_{1H}-1,1,n_3} + \mu_2 p_{23} P_{0,n_{1H},1,n_3-1} + \mu_3 P_{0,n_{1H},0,n_3+1}$
(7)

$n_{1L} = n_3 = 0, n_{1H}, n_2 > 0$

$(\lambda_{1L} + \lambda_{1H} + \mu_{1H} + \mu_2)P_{0,n_{1H},n_2,0} = \lambda_{1H} P_{0,n_{1H}-1,n_2,0} + \mu_{1H} p_1 P_{n_{1L},n_{1H}+1,n_2,0} + \mu_{1H}(ap_{12} +$
$a_1 q_{12})P_{0,n_{1H}+1,n_2-1,0} + \mu_2 p_2 P_{0,n_{1H},n_2+1,0} + \mu_2 p_{21} P_{0,n_{1H}-1,n_2+1,0} + \mu_3 P_{0,n_{1H},n_2,1}$ (8)

$n_{1H} = n_2 = 0, n_{1L}, n_3 > 0$

$(\lambda_{1L} + \lambda_{1H} + \mu_{1L} + \mu_3)P_{n_{1L},0,0,n_3} = \lambda_{1L} P_{n_{1L}-1,0,0,n_3} + \mu_{1H} p_1 P_{n_{1L},1,0,n_3} + \mu_{1L} p_1 P_{n_{1L}+1,0,0,n_3} +$
$\mu_2 p_2 P_{n_{1L},0,1,n_3} + \mu_2 p_{23} P_{n_{1L},0,1,n_3-1} + \mu_3 P_{n_{1L},0,0,n_3+1}$
(9)

$n_{1H} = n_3 = 0, n_{1L}, n_2 > 0$

$(\lambda_{1L} + \lambda_{1H} + \mu_{1L} + \mu_2)P_{n_{1L},0,n_2,0} = \lambda_{1L} P_{n_{1L}-1,0,n_2,0} + \mu_{1H} p_1 P_{n_{1L},1,n_2,0} + \mu_{1L} p_1 P_{n_{1L}+1,0,n_2,0} +$
$\mu_{1H}(ap_{12} + a_1 q_{12})P_{n_{1L},1,n_2-1,0} + \mu_{1L} p_{12} P_{n_{1L}+1,0,n_2-1,0} + \mu_2 p_2 P_{n_{1L},0,n_2+1,0} + \mu_3 P_{n_{1L},0,n_2,1}$ (10)

$n_2 = n_3 = 0, n_{1L}, n_{1H} > 0$ $(\lambda_{1L} + \lambda_{1H} + \mu_{1H})P_{n_{1L},n_{1H},0,0} = \lambda_{1L} P_{n_{1L}-1,n_{1H},0,0} + \lambda_{1H} P_{n_{1L},n_{1H}-1,0,0} +$
$\mu_{1H} p_1 P_{n_{1L},n_{1H}+1,0,0} + \mu_2 p_2 P_{n_{1L},n_{1H},1,0} + \mu_2 p_{21} P_{n_{1L},n_{1H}-1,1,0} + \mu_3 P_{n_{1L},n_{1H},0,1}$
(11)

$n_{1L} = n_{1H} = n_2 = 0, n_3 > 0$ $(\lambda_{1L} + \lambda_{1H} + \mu_3)P_{0,0,0,n_3} = \mu_{1H} p_1 P_{0,1,0,n_3} + \mu_2 p_2 P_{n_{1L},n_{1H},n_2+1,n_3} +$
$\mu_2 p_{23} P_{0,0,1,n_3-1} + \mu_3 P_{0,0,0,n_3+1}$
(12)

$n_{1L} = n_{1H} = n_3 = 0,, n_2 > 0$

$(\lambda_{1L} + \lambda_{1H} + \mu_2)P_{0,0,n_2,0} = \mu_{1H} p_1 P_{0,1,n_2,0} + \mu_{1H}(ap_{12} + a_1 q_{12})P_{0,1,n_2-1,0} + \mu_2 p_2 P_{0,0,n_2+1,0} +$
$+\mu_3 P_{0,0,n_2,1}$ (13)

$n_{1L} = n_2 = n_3 = 0$ $n_{1H} > 0$

$(\lambda_{1L} + \lambda_{1H} + \mu_{1H})P_{0,n_{1H},0,0} =$
$\lambda_{1H}P_{0,n_{1H}-1,0,0} + \mu_{1H}p_1P_{0,n_{1H}+1,0,0} + \mu_2p_2P_{0,n_{1H},1,0} + \mu_2p_{21}P_{0,n_{1H}-1,1,0} + \mu_3P_{0,n_{1H},0,1}$
(14)

$n_{1H} = n_2 = n_3 = 0, n_{1L} > 0$

$$(\lambda_{1L} + \lambda_{1H} + \mu_{1L})P_{n_{1L},0,0,0} = \lambda_{1L}P_{n_{1L}-1,0,0,0} + \mu_{1H}p_1P_{n_{1L},1,0,0} + \mu_{1L}p_1P_{n_{1L}+1,0,0,0} + \mu_2p_2P_{n_{1L},0,1,0} + \mu_3P_{n_{1L},0,0,1}$$

(15) $n_{1L} = n_{1H} = n_2 = n_3 = 0$

$(\lambda_{1L} + \lambda_{1H})P_{0,0,0,0} = \mu_{1H}p_1P_{0,1,0,0} + \mu_2p_2P_{0,0,1,0} + \mu_3P_{0,0,0,1}$ (16)

5. Solution Process

To solve the steady state differential difference equations from (1) to (16), introducing generating function and partial generating function as follows:

F(X,Y,Z,R)=$\sum_{n_{1L}=0}^{\infty}\sum_{n_{1H}=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} P_{n_{1L},n_{1H},n_2,n_3}\, X^{n_{1L}}Y^{n_{1H}}Z^{n_2}R^{n_3}$
(17)

Where $|X| = |Y| = |Z| = |R| = 1$

Also, $F_{n_{1H},n_2,n_3,}$(X) = $\sum_{n_{1L}=0}^{\infty} P_{n_{1L},n_{1H},n_2,n_3}\, X^{n_{1L}}$
(18)

$F_{n_2,n_3,}$(X,Y) = $\sum_{n_{1H}=0}^{\infty} P_{n_{1H},n_2,n_3}\, Y^{n_{1H}}$ (19)

, F_{n_3}(X,Y,Z) = $\sum_{n_2=0}^{\infty} F_{n_2,n_3}$ (X, Y) Z^{n_2}
(20)

F(X,Y,Z) =$\sum_{n_3=0}^{\infty} F_{n_3}$ (X, Y, Z) R^{n_3} (21)

On solving equations from (1) to (16) and using equations from (17) to (21) we derive the solution as,

F(X,Y,Z,R) =

$$\frac{\mu_{1H}\left[1-\frac{(p_1)}{Y}-\frac{(ap_{12}+a_1q_{12})}{Y}Z\right]C+\mu_{1L}\left[-1+\frac{(p_1)}{X}+\frac{(ap_{12}+a_1q_{12})}{X}Y\right](C-D)+\mu_2\left[1-\frac{(p_2)}{Z}-\frac{(p_{21})Y}{Z}-\frac{(p_{23})R}{Z}\right]B+\mu_3\left[1-\frac{1}{R}\right]A}{\lambda_{1L}(1-X)+\lambda_{1H}(1-Y)+\mu_{1H}\left[1-\frac{(p_1)}{Y}-\frac{(ap_{12}+a_1q_{12})}{Y}Z\right]+\mu_2\left[1-\frac{(p_2)}{Z}-\frac{(p_{21})Y}{Z}-\frac{(p_{23})R}{Z}\right]+\mu_3\left[1-\frac{1}{R}\right]}$$
(22)

Where F_0(X,Y,Z) = A , F_0(X,Y,R) = B ,F_0(X,Z,R) = C , $F_{0,0}$(Z,R) = D

At X=Y=Z=R=1, F(X,Y,Z,R) = 1 and considering X→1 & Y=Z=R=1 and applying L' Hospital rule we get,

$-\lambda_{1L} = -\mu_{1L}(C - D)$ (23)

At Y→1 & X=Z=R=1, we get, $-\lambda_{1H} + \mu_{1H} - \mu_2p_{21} = \mu_{1H}C - \mu_{1L}(p_{12} + q_{12})(C - D) - \mu_2p_{21}B$
(24)

At Z→1 & X=Y=R=1, we get,- $\mu_{1H}(p_{12}+q_{12})$+μ_2= - $\mu_{1H}(p_{12}+q_{12})C$+$\mu_2 B$ (25)

At R→1 & X=Y=Z=1, we get, - $\mu_2 p_{23}$+μ_3= - $\mu_2 p_{23}B$+$\mu_3 A$
(26)

On solving above equations we get, A= 1-$\frac{p_{23}(\lambda_{1L}(ap_{12}+a_1q_{12})(ap_{12}+a_1q_{12})+\lambda_{1H}(ap_{12}+a_1q_{12}))}{\mu_3(1-(ap_{12}+a_1q_{12})p_{21})}$
(27)

B= 1-$\frac{(\lambda_{1L}(ap_{12}+a_1q_{12})(ap_{12}+a_1q_{12})+\lambda_{1H}(ap_{12}+a_1q_{12}))}{\mu_2(1-(ap_{12}+a_1q_{12})p_{21})}$ (28)

C= 1-$\frac{(\lambda_{1L}(ap_{12}+a_1q_{12})+\lambda_{1H})}{\mu_{1H}(1-(ap_{12}+a_1q_{12})p_{21})}$ (29)

D = 1-$\left(\frac{(\lambda_{1L(ap_{12}+a_1q_{12})}+\lambda_{1H})}{\mu_{1H}(1-(ap_{12}+a_1q_{12})p_{21})}+\frac{\lambda_{1L}}{\mu_{1L}}\right)$ (30)

The solution of differential difference equations in steady state is given by,

$$P_{n_{1L},n_{1H},n_2,n_3}=\rho_1{}^{n_{1L}}\rho_2{}^{n_{1H}}\rho_3{}^{n_2}\rho_4{}^{n_3}(1-\rho_1)(1-\rho_2)(1-\rho_3)(1-\rho_4)\text{, where}$$

ρ_1 = 1- D = $\frac{(\lambda_{1L}(ap_{12}+a_1q_{12})+\lambda_{1H})}{\mu_{1H}(1-(ap_{12}+a_1q_{12})p_{21})}+\frac{\lambda_{1L}}{\mu_{1L}}$ (31)

ρ_2 = 1- C = $\frac{(\lambda_{1L}(ap_{12}+a_1q_{12})+\lambda_{1H})}{\mu_{1H}(1-(ap_{12}+a_1q_{12})p_{21})}$ (32)

ρ_3 = 1- B =$\frac{(\lambda_{1L}(ap_{12}+a_1q_{12})(ap_{12}+a_1q_{12})+\lambda_{1H}(ap_{12}+a_1q_{12}))}{\mu_2(1-(ap_{12}+a_1q_{12})p_{21})}$ (33)

ρ_4 = 1- A = $\frac{p_{23}(\lambda_{1L}(ap_{12}+a_1q_{12})(ap_{12}+a_1q_{12})+\lambda_{1H}(ap_{12}+a_1q_{12}))}{\mu_3(1-(ap_{12}+a_1q_{12})p_{21})}$(34)

The condition for which the solution of the model exists is,$\rho_{1,}\rho_2,\rho_3,\rho_4<1$

6. Queue Performance Measures

1. Mean Queue Length(L) = $L_1+L_2+L_3+L_4$= $\frac{\rho_1}{1-\rho_1}$+ $\frac{\rho_2}{1-\rho_2}$+ $\frac{\rho_3}{1-\rho_3}$+ $\frac{\rho_4}{1-\rho_4}$

2. Variance (V_{ar}) = $\frac{\rho_1}{(1-\rho_1)^2}+\frac{\rho_2}{(1-\rho_2)^2}+\frac{\rho_3}{(1-\rho_3)^2}+\frac{\rho_4}{(1-\rho_4)^2}$

7. Behavioural Analysis

In this section we will discuss the behaviour of partial queues and mean queue for the different values of low and high priority arrival rates and service rates in following manners:

1. Behaviour of partial queue lengths, mean queue length and variance of the system for different values of λ_{1L} and λ_{1H}.
2. Behaviour of mean queue length and variance of the system for different values of μ_{1L} and μ_{1H}.
3. Graphical analysis of partial queue lengths and mean queue length with respect to λ_{1L} , λ_{1H},μ_{1L} ,μ_{1H}

Table.1. **partial queue lengths, mean queue length and variance of the whole system with respect to $\boldsymbol{\lambda_{1L}}$**

λ_{1L}	λ_{1H}=1, p_1=0.3,p_{12}=0.7,p_2=0.1,p_{21}=0.15,p_{23}=0.75,q_{12}=0.3,a=0.5,a_1=0.5, μ_{1L}=12,μ_{1H}=12,μ_2=7,μ_3=8									
	ρ_1	ρ_2	ρ_3	ρ_4	L_1	L_2	L_3	L_4	L	V_{ar}
2	0.8332	0.6666	0.5714	0.375	4.995204	1.9994	1.333178	0.6	8.927782	40.01481
2.1	0.8583	0.6833	0.5857	0.3843	6.057163	2.157562	1.41371	0.624168	10.2526	53.98506
2.2	0.8833	0.7	0.6	0.3937	7.56898	2.333333	1.5	0.649349	12.05166	77.45722
2.3	0.9082	0.7166	0.6142	0.403	9.893246	2.528582	1.592017	0.675042	14.68889	121.9491
2.4	0.9333	0.7333	0.6285	0.4124	13.9925	2.749531	1.69179	0.701838	19.13566	225.8405
2.5	0.9583	0.75	0.6428	0.4218	22.98082	3	1.799552	0.729505	28.50987	569.3983
2.6	0.9833	0.7666	0.6571	0.4312	58.88024	3.28449	1.916302	0.758087	64.83912	3546.757

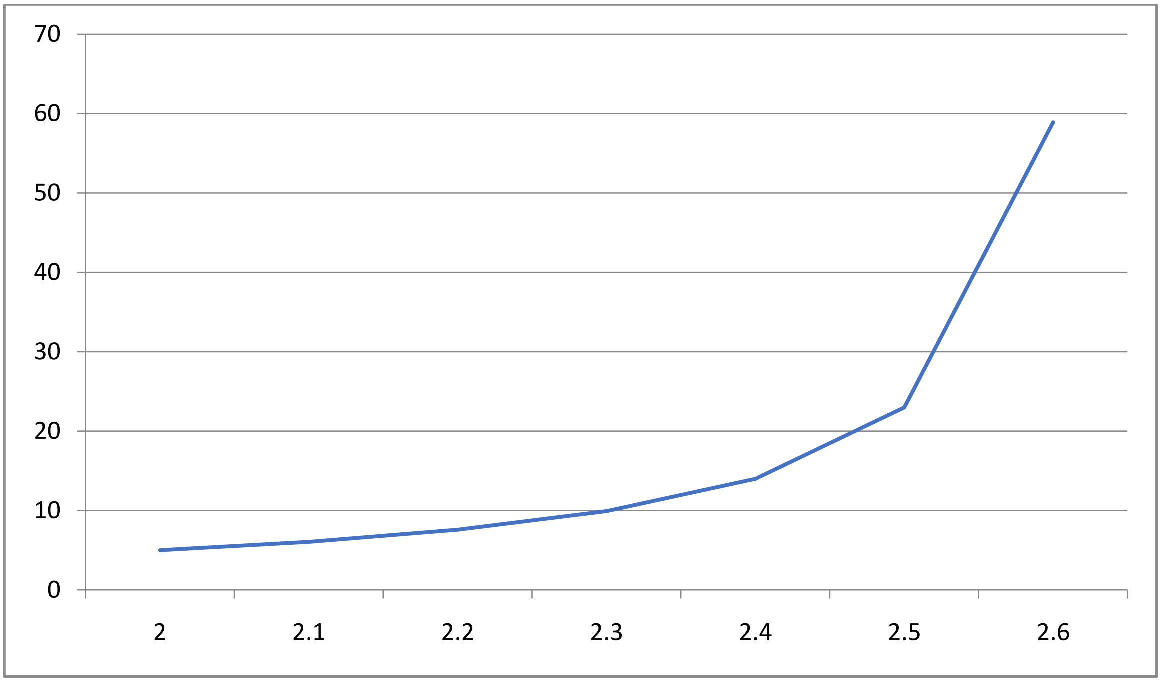

Figure 1. $\boldsymbol{L_1}$ **vs.** $\boldsymbol{\lambda_{1L}}$**(x-axis represents different values of** $\boldsymbol{\lambda_{1L}}$**and y-axis of**$\boldsymbol{L_1}$**)**

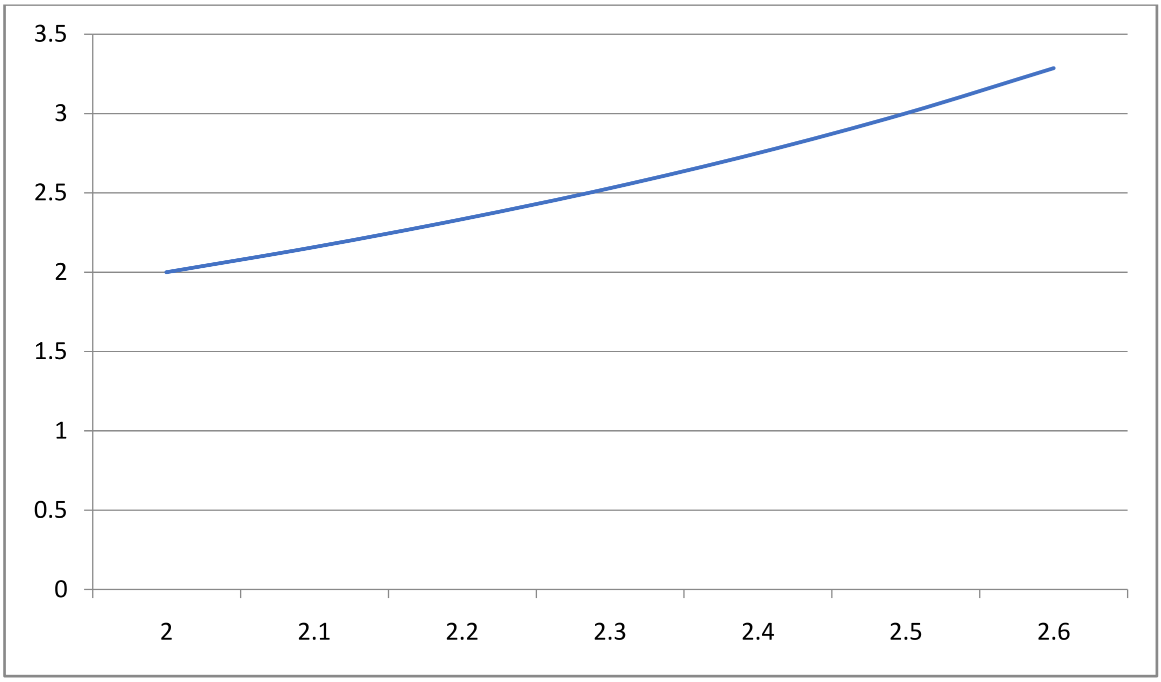

Figure 2. L_2 **vs.** λ_{1L}**(x-axis represents different values of** λ_{1L}**and y-axis of**L_2**)**

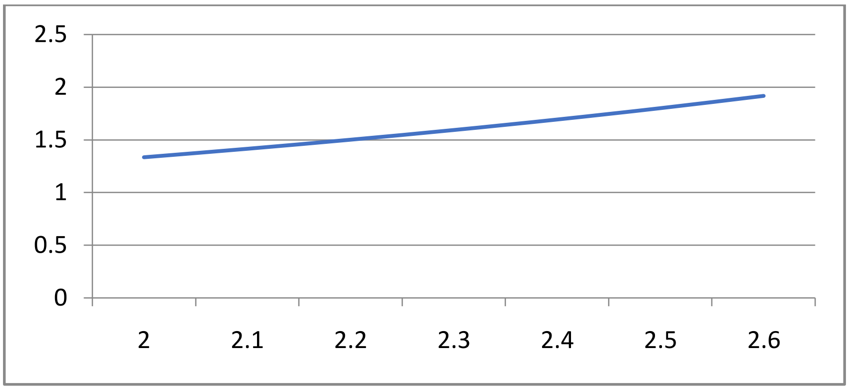

Figure 3. L_3 **vs.** λ_{1L}**(x-axis represents different values of** λ_{1L}**and y-axis of**L_2**)**

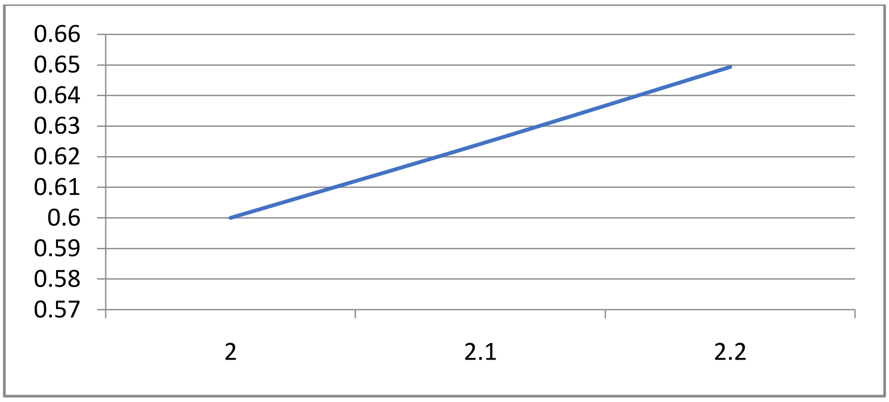

Figure 4. L_4 **vs.** λ_{1L}**(x-axis represents different values of** λ_{1L}**and y-axis of**L_4**)**

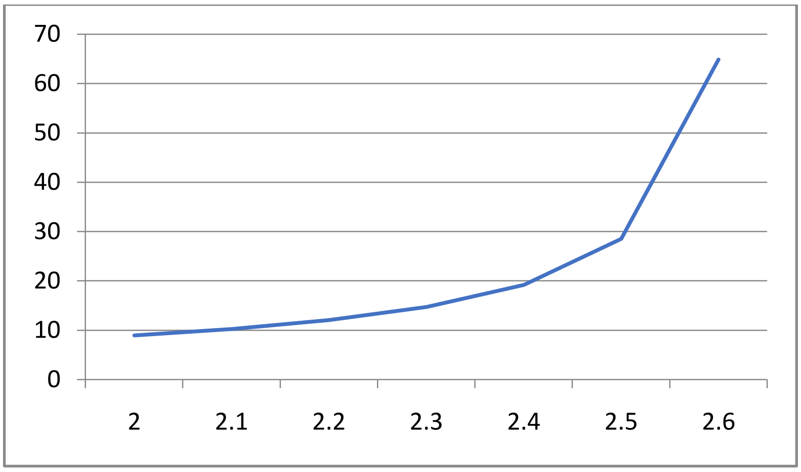

Figure 5. L vs. λ_{1L}(x-axis represents different values of $\lambda_{1L}$and y-axis of L)

Table2: partial queue lengths, mean queue length and variance of the whole system with respect to λ_{1H}

λ_{1H}	λ_{1L}=2, p_1=0.3,p_{12}=0.7,p_2=0.1,p_{21}=0.15,p_{23}=0.75,q_{12}=0.3,a=0.5,a_1=0.5, μ_{1L}=12,μ_{1H}=12,μ_2=7,μ_3=8									
	ρ_1	ρ_2	ρ_3	ρ_4	L_1	L_2	L_3	L_4	L	V_{ar}
1	0.8332	0.666	0.5714	0.375	4.995204	1.994012	1.333178	0.6	8.922394	39.9879
1.1	0.8666	0.7	0.6	0.3937	6.496252	2.333333	1.5	0.649349	10.97893	61.29632
1.2	0.8999	0.7333	0.6285	0.4124	8.99001	2.749531	1.69179	0.701838	14.13317	105.8681
1.3	0.9332	0.7666	0.6571	0.4312	13.97006	3.28449	1.916302	0.758087	19.92894	230.1263
1.4	0.9666	0.8	0.6857	0.4499	28.94012	4	2.181674	0.817851	35.93964	894.8988

Table3: mean queue length and variance of the whole system with respect to μ_{1L} and μ_{1H}

λ_{1L}=2, p_1=0.3,p_{12}=0.7,p_2=0.1,p_{21}=0.15,p_{23}=0.75 q_{12}=0.3, a=0.5,a_1=0.5, λ_{1H}=1μ_{1L}=12,,μ_{1H}=12,μ_2=7,μ_3=8					
μ_{1L}	L	V_{ar}	μ_{1H}	L	V_{ar}
12	8.927782	40.01481	12	8.927782	40.01481
12.5	7.881586	30.5736	12.5	7.710956	29.00881
13	7.117659	24.6658	13	6.863485	22.65377

13.5	6.538266	20.71914	13.5	6.242216	18.64497
14	6.083149	17.9322	14	5.765306	15.92478
14.5	5.713703	15.86743	14.5	5.385476	13.97306
15	5.408104	14.2905	15	5.075282	12.51602

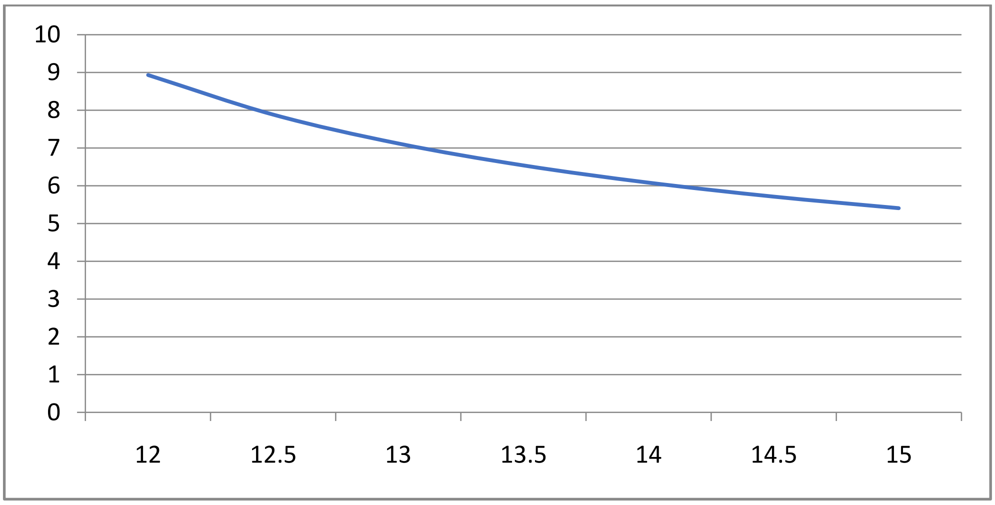

Figure 6. L vs. λ_{1L}(x-axis represents different values of $\lambda_{1L}$and y-axis of L)

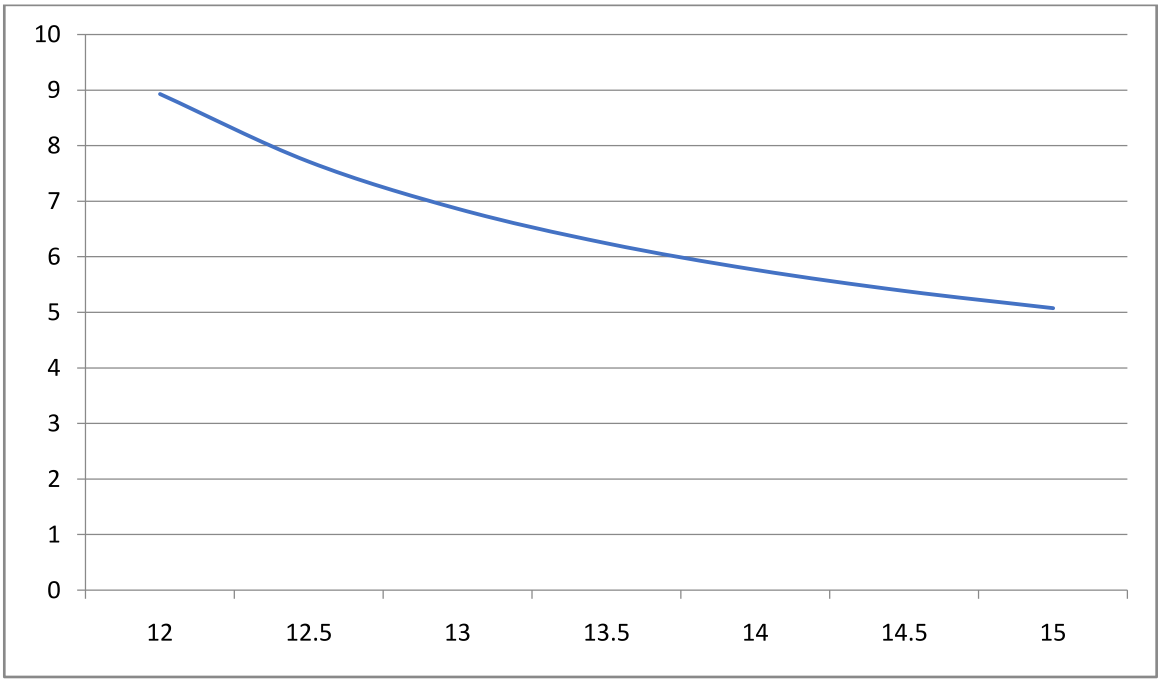

Figure 7. L vs. λ_{1L}(x-axis represents different values of $\lambda_{1L}$and y-axis of L)

8. **Result & Discussion**

From above tables and graphs it can easily seen that with the increase of low arrival rate and high arrival rate, partial queue length L_1 and total length of queue is increasing very fast. We can say that the increase in high priority customers don't affect the arrival of low priority customers. Also increase in high priority service rate and low priority service rate does not affect so much on the total length of queue. Queue is dispersing all most at the same speed.

8. Particular case

If we take λ_{1L}= 0, λ_{1H} = λ, q_{12} =p_{12}then the solution of the model looks like the model presented by Singh T.P. & Bhardwaj Reeta [6]

9. Conclusion

The present model is already solved by Singh T.P. & Bhardwaj Reeta [6].In our model we consider the case of those applicants also who has given the priority on the basis of some particular reasons. Mean queue length is obtained with the help of generating function technique and classical formulas. Behavioural analysis of the mean queue length is also discussed for different values ofλ_{1L} , λ_{1H},μ_{1L} ,μ_{1H} .A particular case has also been discussed to check the validity of the model.

10. Acknowledgement

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11. Conflict of Interest

The authors have no conflict of interests to declare.

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