Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 8, July 2021: 2908-2925

Vendor-Buyer Green's Inventory Model with Price Sensitive Demand under Inflation

Dr. Nitin Kumar Mishra[a], Dr. Seema Mishra[a], Dr. Rajni Bhalla[b]

[a] Department of Mathematics, Lovely Professional University, Phagwara 144411, Punjab(India).
[b] Department of Computer Application, Lovely Professional University, Phagwara 144411, Punjab(India)

Abstract

In the present scenario of globalization of products, it is nearly impossible for the organizations to rule the prices of their products. However, if the prices are lowered sales can be made to increase. Furthermore, the product's price and the ordering size dominates its demand. The present study incorporates demand which relies on the product's price. Considering the hazardous effect of the defective/waste products on the environment and with the increasing pressure on the firms from the government and various NGO's for saving environment, adapting to green's technology is a must for the firms. During the production process, there are numerous environment related issues which arise and need to be resolved throughout the inventory policy framing system. these issues not only decrease the financial development of the system but also tends to slow down the compatibility of the supply chain. The green's inventory module in a supply chain is considered for the study. Supply chain is increasingly gaining importance in the market as the vendor and buyer both realize cost benefits. The aim in developing this framework is to analyze the vendor-buyer association in consideration with the price dependent demand on the greens technology in an inflation based environment. The material and the related manufacturing/producing processes are aimed for the better usage of the raw materials. This work studies the strategies for the partial delay in paying back by the retailer. It includes reusing, remanufacturing and recycling of the waste and deteriorated products in the inventory system. Two different approaches for the delay in paying back time S are considered, (i) S < cycle time and (ii) S > cycle time. The study accomplishes two major objectives: (a) reduce the wastage of the material in turn heading towards a greener manufacturing process and (b) aiming for a minimized cost value for the vendor and the buyer intending an optimal period for refilling the raw material. Numerical illustrations are done by Mathematica 8.0 to characterize the results. Sensitivity analysis is conducted on respective values with table data and graph.

Keywords: Inflation, Supply chain, Credit term, Inventory.

1. Introduction

The choice of products and its reliability in the production process is examined for ensuring the reduction in the cost of manufacturing and also leads to less proportionate amount of inventory in remanufacturing. To achieve green effects of prolonged sustainability and renewable strengths at various scales, nanotechnology particularly nanophotonics is gaining paramount emphasis as discussed by Smith (2011). It is discussed in the study that nanomaterials can be taken to a low cost production value with high benefits for protecting the environment. Trade credit is gaining popularity amongst the retailer and the supplier. It has become a prominent source for the short term financing. The benefits of permissible delay accrued to the supplier includes (a) the supplier becomes aware of the responsible or trustworthy retailers (b) the supplier can initiate for a control over the market by controlling the supplies and (c) the merchandiser can negotiate on the price. From profiting the potential buyers to the merchandiser's self-operations credit time has an influential impact on increasing and capturing the market. An immense work is done with permissible delays in the theoretical context of the optimum value of the quantity ordered.

Improving the quality of environment is considered for safe and healthy food and drinking water, tidy physical systems, freedom from the toxicity prevailing in the communities, safe and proper waste disposal and management and the refurbishment of the polluted waste sites. At present, there is a rise in the public interest towards the cleanliness of the natural surroundings. This is happen due to the increase in the number of non -profitable organizations working in the interest of the consumers and the environment. The practice of involving the green concept of maintaining a healthy surrounding and clean environment into the inventory chain and research is of utmost importance. In the past decades the operational managers and the workforce at the firms were responsible only at lower levels for the environment management. There were other firms which were held responsible for ensuring the product quality, design, working and the waste administration due to the product. But, presently the conditions have changed. The government has issued certain norms and policies for every firm so as to have a proper hold over the environment management. With the revolution brought by the policies, it has become common to integrate the green environment management along with the running operations. The increase in the involvement of the green environment concept with the inventory operations have led several researchers to study and formulate various constraints and restrictions prevailing in the real-time problem into an inventory framework. The work done explains the inventory structure and the impact of the green inventory system on the cost and revenue of the inventory system. The significant interest to include the green inventory management is mainly driven by the deteriorated state of the environment due to over spelling of the waste and increase in the waste areas, production in the natural resources and increase in the pollution levels. Moreover, it is not just that the green concept should be included in the inventory system, but the organization should also run into a profit making condition.

Present model demonstrates an entirely new concept of re- manufacturing the waste products within cycle the time and adding them back to the market for revenue generation.

2. Review of Literature

In today's competitive market, it has become increasingly important for the supplier to keep hold of their potential retailers so that they can have a hold over the market in a passive way. During the credit period, the retailer is allowed to have income through two ways. Firstly, by selling the goods throughout the delay time and secondly by earning through the interest on the amount of money held. The supplier also gives the privilege to the retailer by not charging interest, if the payback is made within the delay time. In this way, credit time is a profitable strategy for the retailers. Although, the supplier too earns during the delay time. This is done by increasing the ordering size and reducing the replenishing orders. In this way the supplier reduces the increase in setup cost. Therefore this delay time is a win-win strategy for both the retailers and the suppliers. The research works done in this field also incorporates various other factors and figure out the optimum solutions for both the retailer and the supplier. The models are developed in the deterministic and stochastic frameworks. A general ordering model with predefined credit days by which the retailer can delay in the payment was established by Goyal (1985). A mathematical inventory ordering framework with time-variable increasing demand without deterioration was formulated by Teng et al. (2012). With permissible delay provided by the supplier a replenishment inventory model for environment conservation is developed by S. Saxena et al. (2020), V. Singh et al. (2019). In this research they have considered partially backlog shortages and deterioration. A review on permissible delay and the detailed scope for the upcoming research works in this field is presented by Seifert et al. (2013). Inflation is an important factor in today's economy. Inflation signifies the power of money. With the rising value of money, the buying price increases which results in reduction of the demand. Since inventory is amount to a significant figure in any firm inflation should be considered in determining every cost related to the inventory model. A constant inflation rate for every cost was taken by Buzacott (1975) and MISRA (1975). An optimal replenishment model evaluating inflated cost for perishable products was discussed by Moon et al. (2005) and Thangam & Uthayakumar (2010). Considering inflation Bhaula & Kumar (2014) formulated an optimal inventory ordering quantity framework with weibull deterioration, shortages and inventory associated demand. For time varying deterioration and demand, an EOQ simulation model is framed by P. Singh et al. (2017). For a supply chain inventory model, where a credit duration is offered to the buyer, V. Singh et al. (2018) frame the model to figure out the optimal refilling period under inflation with time-dependent deterioration. B. Sarkar (2012) incorporated the trade delays in payment in the study with time relying deterioration. M. Sarkar & Sarkar (2013) framed an inventory problem model for the probabilistc nature of deterioration. Further B. Sarkar et al. (2015) considered deterioration of variable nature with the credit delays for the product with fixed period of life. Taleizadeh, Noori-daryan, & Cardenas-Barron (2015) developed a model for optimal refilling level in a supply chain for deteriorated goods. Taleizadeh, Noori-daryan, & Tavakkoli-Moghaddam (2015) further studied an optimal ordering policy in a supply chain with rate of inspection and buying back of the defective goods are considered. B. Sarkar & Saren (2016) presented an ordering policy in an inventory problem model with warehouse. Taleizadeh et al. (2016) applied in their study a theoretical approach of stakelberg in a supply chain to determine the optimal refilling strategies. Taleizadeh & Noori-daryan (2016) incorporated the pharmacy goods in developing a reworking model. B. Sarkar et al. (2018) prsented a study on inventory problem with defective products and their impact on the environment for optimally determining the ordering size. Primarily, inventory model formulation aims at minimising the overall cost of the system. The focus is on the optimum quantity to be ordered and the optimum time to place an order. In turn optimizing various cost associated with the EOQ model. The products ordered are subject to deterioration with respect to time. An inventory model with exponential deterioration was framed by Aggarwal & Jaggi (1995) under credit time. This model was later extended by Jamal et al. (2000) with the inclusion 3 of shortages. A linear trend for deterioration of the products was considered by Tripathy & Mishra (2012). They formulated the economic inventory ordering model with credit term under non-increasing demand. Dye (2013) in his study showed the consequences of the technological yield for the deteriorating products. T. Sarkar et al. (2012), Wang & Liu (2018) and J. Wu et al. (2014) also considered the expiry date of the deteriorating products along with the optimal policy. An important study for the deteriorating and spoiled products is made by S. Saxena et al. (n.d.). They considered recycling, remanufacturing of the goods and guided the models towards green inventory. They also considered deterioration as Weibull.N. Saxena et al. (2019) developed a model to determine the optimal manufacturing time period for managing the waste. B. Sarkar (2019)in a multi-level product model applied a mathematical modelling for the defective items.

Until the early 1990’s nearly the demand in inventory models is kept certain as it becomes easier to manage the inventories. However this assumption of constant demand was later superseded by time dependent demand and depending on price. While the mathematical formulation becomes simpler with constant demand, it is not practical in reality where the demand fluctuates owing to many constraints prevailing in the market. A linear variation of the demand implying increasing or decreasing was shown by C. Wu & Zhao (2014), Ghosh & Chaudhuri (2006), Dye & Hsieh (2011). The product’s demand rises if there is an abundant supply of the product in the market. But this in turn increases the issue of holding the inventory, as an extra cost is incurred to place the excess inventory. The problem becomes critical when the items are of deteriorating nature. This stock dependent nature of demand is also studied by several researches. V. Singh et al. (2017) have considered an inventory framework with inventory relying demand. They have processed a formulation for EOQ with credit delay and partially backlogged shortages. In actuality, demand of any product depends upon the stock and the price prevailing in the market. To have an understanding for this price dependent demand many researchers have made prominent studies. It is easily seen that when the price of the product is reduced, its demand is increased. However it is shown by Chern et al. (2008) that none of the firms can dictate over the prices of the product. In this study an optimal order quantity theory is formulated with demand being price sensible. The work considers two possibilities of permissible delay. First where the credit term offered by the vendor is within the cycle time and second when the buyer gets the credit time which is greater than the cycle. The model is developed under green inventory system. The government’s policies towards clean and green environment has led many firms to move in this direction. The researchers develop the model wherein the firms adopting the green inventory measures are making profits too. Srivastava (2007) in his study have shown how the firms can make remarkable profits adopting to the green supply chain. Richter (1997) generalized the theory of total disposal or total amendment that is bang bang policy amounting to a profit scheme. He considered in his study remanufacturing. The fuzzification of the parameters included in the inventory is amongst the significant way in providing a more realistic solution to the inventory problem. P. Singh et al. (2018),Mishra et al. (2019) formulated a model with fuzzing concept.

The present model is analysed in a framework involving inflation in an inventory system with the price-sensitive demand and a partial trade delay in payment. The innovation in the work lies with the inclusion of the green inventory system where the waste/defective goods are not in the surroundings rather they are recycled/remanufactured. The convexity of the total cost and the optimal refilling period for the two alternatives of credit delay period with price sensitive demand is proven. Proceeding further with the studies sensitivity of certain parameters is exercised and discussed to ascertain the logical insights.

The table below presents a review of work done by various reseachers including several different parameters in the field of green technology. The work progresses with the inclusion of all those parameters.

3.

Author	**Green concept**	**Price Sensitive**	**inflation**	**Supply Chain**	**Deterio-ration**	**Re-Manufac-turing**	**shortages**	**Inspection rates**	**%repair-able**
Moon Giri and k (2005)	…	…	Yes	…	yes	…	…	…	…
Ghoshand Chaudhuri(2006)	…	…	…	…	Yes	…	Yes	…	…
Chern, yang , Teng and PapaChristos(200	…	...	Yes	…	Yes	…	Yes	…	…
Dyeand Ouyang (2011)	…	Yes	…	Yes	yes	…	…	…	…
Sarkar, ghosh and Chaudhuri(2012)	…	…	…	…	yes	…	Yes	…	…
Dye(2013)	Yes	…	…	…	Yes	…	…	…	…
Bhaula and Kumar(2014)	…	…	Yes	…	Yes	…	Yes	…	…
Cobb(2016)	Yes	…	…	Yes	…	Yes	…	…	…
Singh ,Mishra ,sir And sexena(2107	..	…	…	Yes	…	…	Yes	…	…
Singh, sexena, sir And Mishra (2017	…	…	…	…	Yes	…	…	…	…
Sarkar(2017)	yes	...	….	Yes	Yes	yes	yes	…	…
Singh, sexena, Gupta , Mishra ar Singh(2018**)**	….	...	yes	Yes	Yes	…	yes	…	…
Singh Mishra ,Mishra , singh ar Saxena (2018	yes	...	…	Yes	Yes	yes	yes	…	…
Thispaper	yes	yes	yes	Yes	Yes	yes	yes	yes	Yes

Theoretical assumptions and Parametric Quantities

3.1. Theoretical Assumptions

The following assumptions are followed:

(1) Inflation rate g is constant on all inventory costs.

(2) Permissible delay period S is offered to the buyer with further two cases as S < cycle time and S > cycle time.

(3) Constant deterioration γ and shortages with no lead time is allowed.

(4) The model is framed in a finite planning horizon using SFI (Shortages follows Inventory) with I_{oi} units as the demand in the first cycle.

(5) The purchasing cost cc ($/unit) and the selling price (p $/unit) of the buyer are different.

(6) A portion of the deal is imperfect and is subjected to screening. After screening the imperfect goods are taken back by the vendor for recycling and remanufacturing. The perfect quality units are supplied within the cycle time.

(7) The price-sensitive demand function M(p) is $x + yp$ with $x,y > 0$.

(8) The buyer would pay for the goods received under the permissible delay period t = S given by the vendor. The buyer also pays interest at the rate I_p ($/unit) for the stock held within the time interval $[S, s_i+1]$. Contrary, if $S >= s_i+1$ then the interest is not payable by the buyer.

(9) The buyer also earns interest at the rate I_e ($/unit) and collects the revenue from the stock during the complete cycle time from t = 0 to t = S.

3.2. Parametric Quantities

(1) a: discount rate.

(2) g: rate of inflation

(3) c_o: Ordering cost ($/unit)

(4) c_h: Holding Cost ($/unit)

(5) c_c: Purchasing Cost ($/unit)

(6) c_d: Deterioration Cost ($/unit)

(7) c_s: Shortage Cost ($/unit)

(8) c_r: Screening Cost ($/unit)

4. Mathematical Process for Green Inventory Model Formulation

Model Specification

The model is developed for the removal of the imperfect quality items which are present in the lot at a uniform rate k. These imperfect goods pollute the environment and so the lot undergoes full screening and the defectives are picked by the vendor. The screening process continues till t''_i . The vendor recycles, repair and remanufactures the defective goods and transports it back to the buyer at time t''_i. . At si+1 the whole inventory is consumed and at this point of time the demand accumulates for the shortages.

The level of change in inventory for various intervals as shown in figure 1 is given by the following differential equations:

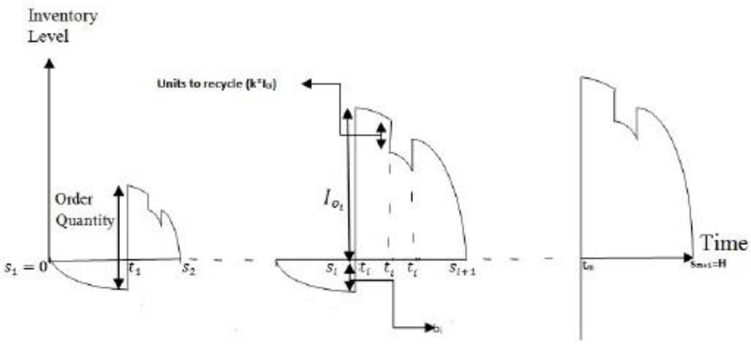

Figure 1. Graphical creation of Green Inventory Model

$\frac{d(I(t))}{dt} + \gamma I(t) = -M(p)$, $t_1 \leq t \leq t'_I$, (i=1,2,……,m) (1)

The boundary condition is $I(t_i) = I_{oi}$ and $I(t'_i) = I_{si}$

$\frac{d(y(t))}{dt} + \gamma y(t) = -M(p)$, $t'_i \leq t \leq t''_i$, (i=1,2,……,m) (2)

The boundary condition is $y(t'_i) = I_{si} - k. I_{oi}$ and $y(t''_i) = I_{fi}$

$\frac{d(z(t))}{dt} + \gamma z(t) = -M(p)$, $t''_i \leq t \leq s_{i+1}$, (i=1,2,……,m) (3)

The boundary conditions is $z(t''_i) = I_{fi} + k .I_{oi}$ and $z(s_{i+1}) = 0$

The amount of shortage at any point of time is given by:

$\frac{d(b(t))}{dt} = -M(p)$, $s_i \leq t \leq t_i$, (i=1,2,……,m) (4)

The boundary condition is $b(s_i)=0$ and $b(t_i)=b_i$

Solving the equation with the boundary conditions, we obtain

$I_{oi}=(s_{i+1}-t_i)*(x+y*p)$ (5)

$I_{si}=(s_{i+1}-t_i-CT)*(y*p +x)$ (6)

$I_{fi}= (y*p +x) * (s_{i+1}-t_i)(1 +k) - (y*p +x)*CT$ (7)

Using the above calculated values , the results for the various inventory level diff

initial equation are:

$I(t)=(\gamma(-t+t_i)+1)*(s_{i+1}-t_i)*(y*p+x)+(-t+t_i)*(y*p+x)$, $t_i \leq t \leq t'_i$, {i= 1,2,…..,m} (8)

Equation (2) solved as:

$Y(t)=(y*p+x)*(s_{i+1}-t_i)*((1-k)+(t_i+t)*\gamma-k*(t_i+ CT-t)*\gamma) +(y*p+ x)*(-t+t_i)$, $t_i' \leq t \leq t_i''$,(i = 1,2,…m} (9)

Equation (3) solved as:

$z(t)=((y*p+x)\div \gamma)*(s_{i+1}-t)*(1-\gamma * t)$, $t_i'' \leq t \leq s_{i+1}$, {i = 1,2,…m} (10)

the amount of shortage is given by:

$b(t) =(s_i +t)*(y*p+x)$, $s_i \leq t \leq t_i$, {1,2,…….m} (11)

the optimal ordering size is given by $Qr = I_{oi} + b_i$

the total cost for the retailer within the inflationary environment includes:

1) Net estimate of ordering cost $NO_c= e^{(a-g)*t_i} *m + c_o$

2) Net estimate of holding cost $NH_c=c_h[\int_{t_i}^{(t_i+CT)} e^{(a-g)*t} *I(t)dt + \int_{t_i+CT}^{(t_i+2*CT)} e^{(a-g)*t} *y(t)dt + \int_{t_i+2*CT}^{s_i} e^{(a-g)*t} *z(t)dt]$

3) Net estimate of deterioration cost $ND_c=c_d [\int_{t_i}^{(t_i+CT)} e^{(a-g)*t} *\gamma * I(t)dt + \int_{t_i+CT}^{(t_i+2*CT)} e^{(a-g)*t} *\gamma * y(t)dt + \int_{t_i+2*CT}^{s_i} e^{(a-g)*t} *\gamma * z(t)dt]$

4) Net estimate of screening cost $NS_c = c_r * e^{(a-g)*t_i}*I_{oi}$

5) Net estimate of purchasing cost $NP_c = c_r * e^{(a-g)*t_i}*[I_{oi}+ b_i]$

6) Net estimate of storage cost $NS_h= c_s\int_{s_i}^{t_i} e^{(a-g)*t_i} *(y*p+x)*(s_i-t)dt$

7) Interest Payable : The buyer pays for the interest on the amount of stock held with him depending upon the delay period S offered to him by the vendor.
The following two conditions are further considered for paying the interest amount:

Condition (i): $(S<=s_{i+1})$

For the stock remaining with the buyer after the delay period S,the interest paid by the buyer at the rate Ip on the amount of stock is given as:

$IP_1 = I_p* c_c * \int_{S}^{s_i+1}(y * p + x) * (s_i+1 - t)\, dt$

$IP_1= (I_p*c_c*(y*p+x)*(s_{i+1}-S)^2)/2$

Condition(ii): $(S>=s_{i+1})$

As the delay period is more than the cycle time no interest is payable for the amount of stock in the inventory.

$IP_2 =0$

(8) Interest Earned : The buyer also earns from the amount of stock held depending upon the delay time S. The interest earned is at the rate Ie for the above mentioned two conditions under consideration.

Condition(i): ($S<=s_{i+1}$)

The buyer not only collects the amount by selling the product, but also earns the interest on the amount at the rate Ie till the cycle period. The interest earned from time t = 0 to t = s_i+1 is given by:
$IE_1=I_e *c_c*\int_0^{s_{i+1}}(y*p+x)*(s_{i+1}\text{-}t)dt$
$IE_1= (I_e*c_c*(y*p+x)*(s^2_{i+1}))/2$

Condition(ii): ($S>=s_{i+1}$)
When the delay period S is more than the cycle duration, the buyer earns interest on the amount of stock from time t = 0 to t = S given by:

$IE_2=I_e *c_c*\int_0^{s_{i+1}}(y*p+x)*(S\text{-}t)dt$
$IE_1 = (I_e*c_c*(y*p+x)* s_{i+1} *(S-(s_{i+1})/2)$

Net Value of the total cost with the partial delayed payment option available is:
NT C(t_i 0, s_i , p) = $NO_c + NH_c + ND_c + NS_c + NP_c$ + IP – IE
NT C(t_i , s_i , p) = NTC_1(t_i , s_i , p), if S <= s_i+1 and
NT C(t_i , s_i , p) = NTC_2(t_i , s , p), if S >= s_i+1

where the cost values are shown in Appendix 1
The optimality of the net total cost is tested for the model in both the conditions of permissible time lag in the payment.

5. Optimality Check for Total Cost
In this segment net total cost for both the conditions is optimized by calcula

In this segment net total cost for both the conditions is optimized by calculating the optimal time intervals. The requisite for the optimality is :
$\frac{\partial NTC_1}{\partial t_i}=0$ and $\frac{\partial NTC_2}{\partial s_i}=0$

where, the values are shown in Appendix 2
The equation (14) and (15) in Appendix 2 are solved for the optimal values of t'_i s and s'_i s for different cycle m.

The time period determined from the two equations is then applied to determine the two different cost values for the different time lags.

Following two propositions demonstrates the strategical initiatives taken for the convexity of the cost values with the price sensitive demand and inflation along with green technology.

Proposition 1: When t_i and s_i are taken as constants, then the net total cost NTC_1 is convex with respect to p.

The first derivative with respect to p is equated to zero

$\frac{\partial NTC_1}{\partial p}=0$

The optimal values of p are determined from the above equation for which the second order derivative of the net total cost with respect to p is given by:

$\partial^2 NTC_1/\partial p^2 = c_s * (e^{(a-g)*t_i} * y * (s_i - t_i) + (c_h + \gamma * c_d) * ((e^{(a-g)*t"_i} * y * (s_i+1 - t_i) * ((1 - k) - 2 * \gamma * CT + k * \gamma * CT) + k* (e^{(a-g)*t'_i} * y * (s_i+1 - t_i) - e^{(a-g)*t"_i} * (y/\gamma) * (s_{i+1} - t"_i) * (1 - \gamma * t"_i)) > 0$ (12)

The convexity of the total cost with respect to p is shown by the following numerical illustration followed by the graph.

Proposition 2: When t_i and s_i are taken as constants, then the net total cost NTC_2 is convex with respect to p.

The first derivative with respect to p is equated to zero.

$\frac{\partial NTC_2}{\partial p}=0$

The optimal values of p for the net cost are determined and the second order derivative of the net total cost with respect to p is given by:

$\partial^2 NTC_2/\partial p^2 = c_s * e^{(a-g)*t_i} * y * (s_i - t_i) + (c_h + \gamma * c_d) * e^{(a-g)*} t"_i * y * (s_{i+1} - t_i) * ((1 - k) + (k - 2) * \gamma * CT) + k\, e^{(a-g)*t'_i} * y * (s_{i+1} - t_i) - e^{(a-g)*t"_i} * (y/\gamma) * (-t"_i + s_{i+1}) * (-\gamma * t"_i + 1)) > 0$ (13)

6. Algorithm

(1) Inputting the values to all the parameters.

(2) For net total cost in condition (1) of permissible delay NTC_1:

(a) t'_i s is and s'_is are determined by assigning a value to t_1 and keeping $s_1 = 0$. Calculating s_2 from equation(15).

(b) Calculating t_2 from equation (14) in the next step as t_1 , s_1 and s_2 are now known.

(c) Repeating the above steps until all the t 0 i s and s 0 i s are known.

(d) Calculate NT C_1. If NT $C_1(m)$ <= NT $C_1(m + 1)$, then optimal total cost $NTC^*_1 = NTC_1(m)$ and optimal time period m* = m.

(3) Optimal) t'_i s'_i s and net total cost NTC^*_1 are known.

(4) Repeat the same solution procedure for the second condition of permissible delay NTC_2. The optimal values of the respective) t'_i s'_i s are determined and the total cost is convex for the above calculated optimal values.

7. Numerical illustration for the two conditions of trade credit to minimize net total cost

The numerical solved validate the green model with price varied demand under inflation. The conjectural values to all the parameters are taken. To have the simplicity in the mathematical solvings $-t'_i + t''_I = t'_I - t_i =$ CT.

Example: 1 Taking the parameter values as: a=0.12, g=0.05, y=100 ,p=20, k=0.5, γ = 0.00001, CT = 0.016, co = \$10/unit, c_h = \$35/unit, c_d = \$10/unit, c_c = \$27/unit, c_r = \$1/unit, c_s = \$14/unit, I_e = \$0.05/unit, I_p = \$0.03/unit, S = 0.6.

The numerical is solved with Mathematica 8.0. The following table gives the net total amount for the buyer when the delay period is less than the cycle time. The data shows the convexity of the net total cost. The ordering schedule 5th cycle,4th cycle and 5th cycle are optimal with the optimal value of the cost being \$8370.58, \$9007.58 and \$7708.49 respectively. The optimal ordering quantity for the corresponding optimal replenishment schedules are \$9750, \$10100 and \$9400 respectively. The graph following the table depicts the convex nature of the cost

The convex nature of the net total cost can be seen from the above graph for a condition of delayed payment. In the similar way the convexity can be proved for the second condition as well.

Table 1. Buyer's inflated total cost for varied values for parameter 'x'

$\frac{\downarrow}{x} \rightarrow m$	1	2	3	4	5	6
437.3	9605.73	8986.51	8588.51	8585.17	8370.58	8538.81
525	10014.8	9460.16	9133.88	9007.58	9075.13	9330.38
350	9203.27	8528.5	8067.81	7796.37	7708.49	7798.48

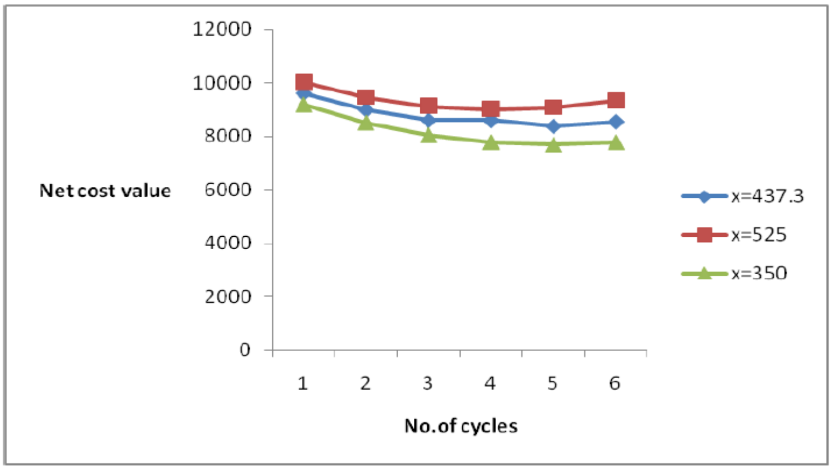

Figure 2. Net cost value for 'x'

Table 2. Buyer's inflated total cost for varied values for parameter 'p'

$\frac{\downarrow}{p} \rightarrow m$	1	2	3	4	5	6
25	11596.	11365.8	11390.3	11633.7	12089.2	12749.7
30	14241.2	14800.8	15654.5	16753.7	18090.1	19655.7
20	9203.27	8528.5	8067.81	7796.37	7708.49	7798.48

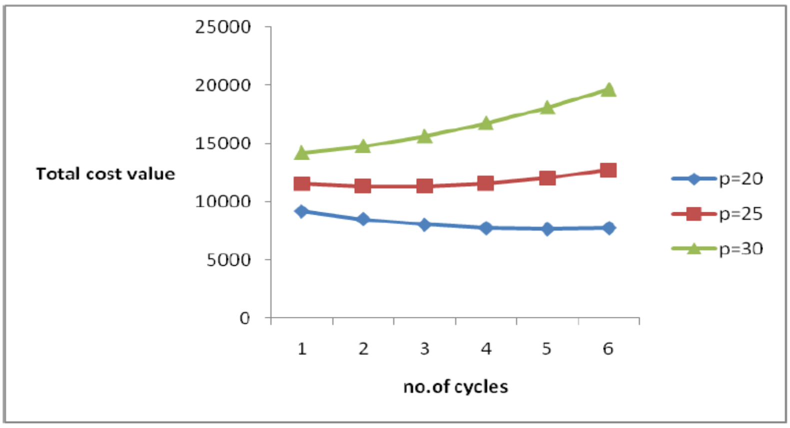

Figure 3. Net cost value for 'p'

7.1. Sensitivity Analysis and Logical Significances:

The cost parameters as taken in the numerical are changed to certain dimensions and their effect is studied on the net total cost of the green inventory model under both the conditions of delayed payment. Following table shows the change in the parameters and the optimal value of the net total cost.

7.2. Logical Significances

(1) Considering the two cases for delay in payment, it is observed from the numerical that the cost value when the paying back by the retailer exceeds the delay time is more than the cost value from the first case. This indicates the addition of interest paid on the amount being held back to the total cost.

(2) The cost value for the second case is inversely proportional to the parameters purchasing cost Cc and the inspection rate k. With the increase in these parameters the cost value decreases.

The optimized net total cost is obtained for a value of the parameter and follows a convex pattern. This is shown by the graph following the table (Figure 4) for one of the conditions of delayed payment. The table also shows the net total cost for the second alternative of trade credit interval which is again convex in nature.

The following results are derived from the sensitivity analysis table shown below:

(1) The two cost values for the two cases of delay period behave differently as seen in the sensitivity analysis. The cost value in first case NTC_1 is highly sensitive to the parameters a , c_c, c_s. The cost value increases with the increase in the valuation of these parameters.

(2) The cost value NTC_1 is inversely sensitive to the parameters g, c_h, CT and k. The cost value decreases with the increase in the valuation of these parameters.

(3) The cost value in the second case of delay period NTC_2 is highly sensitive to the parameters a, c_h, c_s ,CT. The cost value increases with the increase in their valuation.

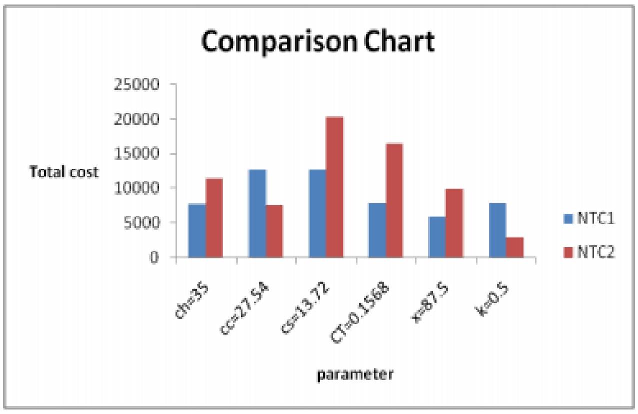

Figure 4. Cost Comparison in both cases

(4) The cost value NTC_2 is inversely sensitive to the parameters g, c_s, k. The cost value decreases with the increase in their valuation.

Sensitivity Analysis of cost parameter on Green Inventory Model

	% change in parameter	values	Net total cost (NTC_1)	Net total cost(NTC_2)
a	-98 -99 10.2 1.05	0.1176 0.1139 0.1224 0.1260	0299.84 4174.29 9.32.72 9877.45	40183.9 38468.3 42526 118381
g	-98 -99 10.2 1.05	0.049 0.0495 0.051 0.0525	8300.01 9192.72 7119.67 18420.15	41836.4 42575.5 40860.5 21043.7
c_h	-98 -99 10.2 1.05	34.3 34.65 35.7 36.75	8706.19 9164.94 0270.42 4095.44	11383.3 11360.3 11414 11437
c_e	-98 -99 10.2 1.05	26.46 25.65 27.54 28.35	2247.19 6380.81 12744.2 20162.8	2120.58 9386.95 7567.91 14834.3
c_s	-98 -99 10.2 1.05	13.72 13.3 14.28 14.7	12710 20057.8 2377.63 6414.59	20285.3 29656 7791.02 1579.7

CT	-98 -99 10.2 1.05	0.01568 0.0152 0.01632 0.0168	7783.95 7867.76 7636.55 7508.42	16347.1 16156.9 16602.8 16796.1
k	-98 -99 10.2 1.05	0.049 0.0475 0.51 0.525	8206.49 8691.27 7491.59 6948.99	3335.06 4085.81 2488.79 1966.49

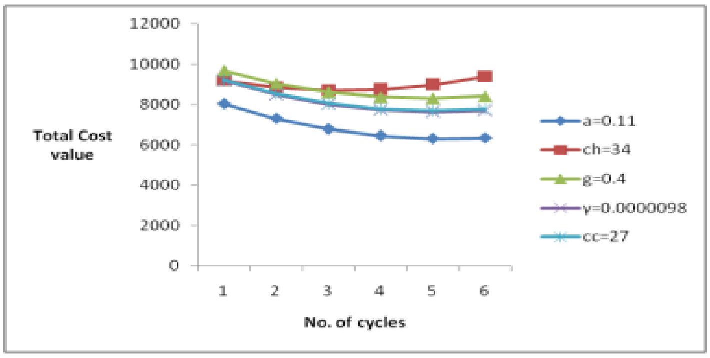

Figure 5. Sensitivity Analysis for various parameters

8. Analysis and Comparison

An inventory production model is developed by B. Sarkar (2013a) for a two staged supply chain, incorporating three types of probabilistic deterioration and transportation cost with constant demand in an infinite horizon. Taking the valuation for the cost parameters (c_h, c_d, c_o, demand D) as in the said paper and including the credit delays for the two mentioned cases with shortages, screening cost and interest paying, there is a slight increase in the cost value.

Example 2: Taking the parameter values as:

a=0.12, g=0.05, x=800 units, y=100, p=40, k=0.5, $\gamma = 0.00001$, CT = 0.09, co=\$25/unit, ch=\$7/unit, c_d=\$50/unit, c_c = \$27/unit, c_r = \$1/unit, c_s = \$14/unit, I_e = \$0.05/unit, I_p = \$0.03/unit, S = 0.6.

Table 3. Analysis and Comparison

Total cost	
Sarkar(2013)	\$15757.9
Self- paper	\$18538

The increase in the cost valuation in on the account of green concept as screening cost and the interest valuation in included in the final cost value. The goods remanufactured are also added to the system within the cycle time.

9. Conclusion

The research work investigates the partial permissible time lag for the retailer. The results derived shows that the retailer has a strong hold in decision making in an iventory system. The model incorporates the demand as price variant. However, much of the research work is done with the price variating demand and credit. The innovation in the inventory system lies with the incorporation of the inflation and green technology. The progress of the model for the optimal values to the cost and the relenishing schedule is done in an inflationary environment keeping in mind the hazards thrown in the environment in the form of waste products. The model proceeds with the re-manufacturing and recycling of the defective goods. The lot received is screened at retailer's end. An optimal present worth of the net total cost under the two scenarios of the supply chain contract is derived with the help of the numerical. The convexity of the Net total cost is verified graphically also for the two delayed payment options. Also optimal replenishment period is determined for the green's inventory study with inflation. With the partial trade delays the retailer earns through different modes. The cost of the system is derived taking into account the interest paid and earned. The two propositions give an important basis for the strategies to be framed by the retailer. Also a comparitive study with B. Sarkar (2013b) shows that there is an increase in the cost but the change is obvious following a green inventory system with inflation, partial delays and price stimulating demand. The present study can be extended to fuzzy logic (Batra et al. 2020, Singh & Verma 2019, Gaba & Verma 2019, Vijayalakshmi et al. 2021, Yang et al. 2020, Batra et al. 2020, Kumar et al. 2018, Rani et al. 2020, Ahmad Jan et al. 2021) and other computation methods. Fuzzy logics for various parameters are done as their values tend to fluctuate with the market. The paper can also be made to incorporate partially backlogged shortages.

10. Acknowledgement

The authors sincerely thank the anonymous reviewers for their guiding comments and the suggestions which helped us to formulate the model more precisely. The authors are indebted to the Editor-in-chief and the board of editors for their consistent assistance.

11. Appendix 1

The cost values under the two conditions of the partial permissible lag in payment is given as:

$NTC_1 = e^{(a-g)*t_i} * m * c_o + (c_h + \gamma * c_d) * \left(\int_{t_i}^{t_i+CT} I(t) * e^{(a-g)*t} dt + \int_{t_i+CT}^{(t_i+2*CT)} y(t) * e^{(a-g)*t} dt + \int_{t_i+2*CT}^{s_{i+1}} e^{(a-g)*t} * z(t)dt\right) + c_c * e^{(a-g)*t_i} * (y * p + x) * (s_{i+1} - s_i) + c_r * e^{(a-g)*t_i} * (y * p + x) * (s_{i+1} - t_i) + c_s \int_{s_i}^{t_i} e^{(a-g)*t} * (s_i - t) * (y * p + x)dt + (I_p + c_c * (y*p*x)*(s_{i+1} - S)^2)/2) - (I_e * c_c * (y*p+x)*(s^2_{i+1}))/2$

$NTC_2 = e^{(a-g)*t_i} * m * c_o + (c_h + \gamma * c_d) \left(\int_{t_i}^{t_i+CT} I(t) * e^{(a-g)*t} dt + \int_{t_i+CT}^{(t_i+2*CT)} y(t) * e^{(a-g)*t} dt + \int_{t_i+2*CT}^{s_{i+1}} e^{(a-g)*t} * z(t)dt\right) + c_c * e^{(a-g)*t_i} * (y * p + x) * (s_{i+1} - s_i) + c_r * e^{(a-g)*t_i} * (y * p + x) * (s_{i+1} - t_i) + c_s \int_{s_i}^{t_i} e^{(a-g)*t} * (s_i - t) * (y * p + x)dt + I_e + c_c + s_{i+1} * (y*p*x)*(S-(s_{i+1})/2)$

12. Appendix 2

$\partial NTC_1/\partial t_i = (a - g) * e^{(a-g)*t_i} * m * c_o + (c_h + \gamma * c_d) * \int_{t_i}^{t'_i} e^{(a-g)*t} * (s_{i+1} - t) * (y * p + x) * \gamma dt + \int_{t'_i}^{t''_i} (y * p + x) * e^{(a-g)*t} * ((1 - k) * (\gamma * (s_{i+1} - t) - 2 * \gamma * t_i) + k(1 + \gamma * CT)) dt + e^{(a-g)*t''_i} * (s_{i+1} - t_i)$

$* ((y * p+x) * ((1-k)-2 * \gamma * CT + k * \gamma * CT) - 2 * (y * p+x) * CT) + k * e^{(a-g)*t''_i} * (s_{i+1} - t_i) * (y * p + x) - e^{(a-g)*t''_i} * ((y * p+x)/\gamma) * (s_{i+1} - t''_i) * (1 - \gamma * t''_i)) + c_c * (a-g) * e^{(a-g)*t_i} * (y * p+x) * (s_{i+1} - s_i) + c_r * (a-g) * e^{(a-g)*t_i} * (s_{i+1} - t_i) * (y * p+x) - c_r * e^{(a-g)*t_i} * (y * p + x) + c_s * e^{(a-g)*t_i} * (s_i - t_i) * (y * p + x)$
(14)

$\partial NTC_1/\partial s_i = (c_h + \gamma * c_d) * (\int_{t_{i-1}}^{t'_{i-1}} e^{(a-g)*t} * (y * p + x) * (1 + \gamma * (t_{i-1} - t))\,dt + * (\int_{t'_{i-1}}^{t''_{i-1}} e^{(a-g)*t} * (y * p + x) * (\gamma * (t_{i-1} - t)) + (1-k) - k * \gamma * (t'_{i-1} - t))dt + * (\int_{t''_{i-1}}^{s_{i-1}} e^{(a-g)*t} * ((y * p + x)/\gamma) * (1 - \gamma * t)dt + k * e^{(a-g)*t'(i-1)} * (s_i - t_{(i-1} - t) * (y * p + x) + e^{(a-g)*t''(i-1)} (((s_i - t_{(i-1)}) * (y * p + x) + ((1-k) - 2 * \gamma * CT + k * \gamma * CT) - 2 * (y * p + x) * CT))$

$-(s_i - t''_{(i-1)}) * ((y * p + x/\gamma) * (1 - \gamma * t''_{(i-1)})) + c_r * (y * p + x) * e^{(a-g)*t_i} + c_s * (\int_{s_i}^{t_i} (y * p + x) * e^{(a-g)*t} dt + (y * p + x) * e^{(a-g)*t_i} * (s_i - t_i)) + (I_p/2) * (y * p + x) * c_c - I_e * (y * p + x) * c_c * s_{i+1}$ (15)

13.References

[1]. Aggarwal, S. P., & Jaggi, C. K. (1995). Ordering policies of deteriorating items under permissible delay in payments. *Journal of the operational Research Society*, *46*(5), 658-662.

[2]. B. Vijayalakshmi, K. Ramar, NZ Jhanjhi, Sahil Verma, M. Kaliappan, K. Vijayalakshmi, Kavita, Uttam Ghosh S. Vimal.(2021) "An Attention Based Deep Learning Model For Traffic Flow Prediction Using Spatio Temporal Features Towards Sustainable Smart City" IJCS, Wiley

[3]. Batra, Isha, Sahil Verma, and Mamoun Alazab.(2020) "A lightweight IoT-based security framework for inventory automation using wireless sensor network." International Journal of Communication Systems 33, no. 4 (2020): e4228.

[4]. Bhaula, B., & Kumar, M. R. (2014). An economic order quantity model for weibull deteriorating items with stock dependent consumption rate and shortages under inflation. International Journal of Computing and Technology, 1 (5), 196–204.

[5]. Buzacott, J. (1975). Economic order quantities with inflation. Journal of the Operational Research Society, 26 (3), 553–558.

[6]. Chern, M.-S., Yang, H.-L., Teng, J.-T., & Papachristos, S. (2008). Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation. European Journal of Operational Research, 191 (1), 127–141.

[7]. Dye, C.-Y. (2013). The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. Omega, 41 (5), 872–880. 18.

[8]. Dye, C.-Y., & Hsieh, T.-P. (2011). Deterministic ordering policy with price-and stockdependent demand under fluctuating cost and limited capacity. Expert Systems with Applications, 38 (12), 14976–14983.

[9]. Gaba, Seema, and Sahil Verma.(2019) "Analysis on Fog Computing Enabled Vehicular Ad hoc Networks." Journal of Computational and Theoretical Nanoscience 16, no. 10 (2019): 4356-4361.

[10]. Ghosh, S., & Chaudhuri, K. (2006). An eoq model with a quadratic demand, time proportional deterioration and shortages in all cycles. International Journal of Systems Science, 37 (10), 663–672.

[11]. Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. Journal of the operational research society, 36 , 335–338.

[12]. Guojie Yang, Mian Ahmad Jan, Ateeq Ur Rehman, Muhammad Babar, Mian Muhammad Aimal and Sahil Verma(2020) "Interoperability and Data Storage in Internet of Multimedia Things: Investigating Current Trends, Research Challenges and Future Directions" IEEE Access. DOI: 10.1109/ACCESS.2020.3006036, 2020

[13]. Isha Batra, Sahil Verma, Kavita, Uttam Ghosh, Joel J. P. C. Rodrigues, Gia Nhu Nguyen, A.S.M. Sanwar Hosen and Vinayagam Mariappan.(2020) "Hybrid Logical Security Framework for Privacy Preservation in the Green Internet of Things" MDPI-Sustainability, DOI: 10.3390/su12145542

[14]. Jamal, A., Sarker, B. R., & Wang, S. (2000). Optimal payment time for a retailer under permitted delay of payment by the wholesaler. International Journal of Production Economics, 66 (1), 59–66.

[15]. Mishra, S., Mishra, N. K., Singh, V., Singh, P., & Saxena, S. (2019). The fuzzyfied supply chain finite planning horizon model. Journal of Computional and Theoretical Nanosciences, 16, 41354142 (2019).

[16]. MISRA, R. B. (1975). Optimum production lot size model for a system with deteriorating inventory. The International Journal of Production Research, 13 (5), 495–505.

[17]. Mian Ahmad Jan, Bin Dong, Syed Rooh Ullah Jan, Zhiyuan Tan, Sahil Verma, Kavita.(2021) "A Comprehensive Survey on Machine Learning-based Big Data Analytics for IoT-enabled Smart Healthcare System" MONET, Springer, https://doi.org/10.1007/s11036-020-01700-6

[18]. Moon, I., Giri, B. C., & Ko, B. (2005). Economic order quantity models for ameliorating/deteriorating items under inflation and time discounting. European Journal of Operational Research, 162 (3), 773–785.

[19]. Pooja Rani, Kavita, Sahil Verma, Nhu Gia Nguyena.(2020) "Mitigation of Black hole and Gray hole Attack using Swarm Inspired Algorithm with Artificial Neural Network" IEEE Access. DOI: 10.1109/ACCESS.2020.3004692

[20]. Richter, K. (1997). Pure and mixed strategies for the eoq repair and waste disposal problem. Operations-Research-Spektrum, 19 (2), 123–129.

[21]. Sarkar, B. (2012). An eoq model with delay in payments and time varying deterioration rate. Mathematical and Computer Modelling, 55 (3-4), 367–377.

[22]. Sarkar, B. (2013a). A production-inventory model with probabilistic deterioration in two-echelon supply chain management. Applied Mathematical Modelling, 37 (5), 3138–3151.

[23]. Sarkar, B. (2013b). A production-inventory model with probabilistic deterioration in two-echelon supply chain management. Applied Mathematical Modelling, 37 (5), 3138–3151.

[24]. Sarkar, B. (2019). Mathematical and analytical approach for the management of defective items in a multi-stage production system. Journal of cleaner production, 218 , 896–919.

[25]. Sarkar, B., Ahmed, W., Choi, S.-B., & Tayyab, M. (2018). Sustainable inventory management for environmental impact through partial backordering and multi-tradecredit-period. Sustainability, 10 (12), 4761.

[26]. Sarkar, B., & Saren, S. (2016). Ordering and transfer policy and variable deterioration for a warehouse model. Hacettepe Journal of Mathematics and Statistics, 46 (5), 985–1014.

[27]. Sarkar, B., Saren, S., & C´ardenas-Barr´on, L. E. (2015). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. Annals of Operations Research, 229 (1), 677–702.

[28]. Sarkar, M., & Sarkar, B. (2013). An economic manufacturing quantity model with probabilistic deterioration in a production system. Economic Modelling, 31 , 245– 252.

[29]. Sarkar, T., Ghosh, S., & Chaudhuri, K. (2012). An optimal inventory replenishment policy for a deteriorating item with time-quadratic demand and time-dependent par-tial backlogging with shortages in all cycles. Applied Mathematics and Computation, 218 (18), 9147–9155.

[30]. Saxena, N., Sarkar, B., & Singh, S. (2019). Selection of remanufacturing/production cycles with an alternative market: A perspective on waste management. Journal of Cleaner Production, 118935. Saxena, S., Singh, V., Gupta, R. K., Mishra, N. K., & Singh, P. (n.d.). Green inventory supply chain model with inflation under permissible delay in finite planning horizon.

[31]. Saxena, S., Singh, V., Gupta, R. K., Singh, P., & Mishra, N. K. (2020). A supply chain replenishment inflationary inventory model with trade credit. In International conference on innovative computing and communications (pp. 221–234).

[32]. Seifert, D., Seifert, R. W., & Protopappa-Sieke, M. (2013). A review of trade credit literature: Opportunities for research in operations. European Journal of Operational Research, 231 (2), 245–256.

[33]. Singh, Puneeta, and Sahil Verma.(2019) "Analysis on Different Strategies Used in Blockchain Technology." Journal of Computational and Theoretical Nanoscience 16, no. 10 (2019): 4350-4355.

[34]. Singh, P., Mishra, N. K., Kumar, M., Saxena, S., & Singh, V. (2018). Risk analysis of flood disaster based on similarity measures in picture fuzzy environment. Afrika Matematika, 29 (7-8), 1019–1038.

[35]. Singh, P., Mishra, N. K., Singh, V., & Saxena, S. (2017). An eoq model of time quadratic and inventory dependent demand for deteriorated items with partially backlogged shortages under trade credit. In Aip conference proceedings (Vol. 1860, p. 020037).

[36]. Singh, V., Mishra, N. K., Mishra, S., Singh, P., & Saxena, S. (2019). A green supply chain model for time quadratic inventory dependent demand and partially backlogging with weibull deterioration under the finite horizon. In Aip conference proceedings (Vol. 2080, p. 060002).

[37]. Singh, V., Saxena, S., Gupta, R. K., Mishra, N. K., & Singh, P. (2018). A supply chain model with deteriorating items under inflation. In 2018 4th international conference on computing sciences (iccs) (pp. 119–125).

[38]. Singh, V., Saxena, S., Singh, P., & Mishra, N. K. (2017). Replenishment policy for an inventory model under inflation. In Aip conference proceedings (Vol. 1860, p. 020035).

[39]. Smith, G. B. (2011). Green nanotechnology. In Nanostructured thin films iv (Vol. 8104, p. 810402).

[40]. Srivastava, S. K. (2007). Green supply-chain management: a state-of-the-art literature review. International journal of management reviews, 9 (1), 53–80.

[41]. Sumit Kumar, Ravishankar and Sahil Verma.(2018) A research paper on "Context Aware Dynamic Permission Model: A Retrospect of Privacy and Security in Android System" in International Conference on Intelligent Circuits and Systems (ICICS2018), IEEE Xplore

[42]. Taleizadeh, A. A., & Noori-daryan, M. (2016). Pricing, inventory and production policies in a supply chain of pharmacological products with rework process: a game theoretic approach. Operational Research, 16 (1), 89–115.

[43]. Taleizadeh, A. A., Noori-daryan, M., & C´ardenas-Barr´on, L. E. (2015). Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items. International Journal of Production Economics, 159 , 285–295.

[44]. Taleizadeh, A. A., Noori-Daryan, M., & Govindan, K. (2016). Pricing and ordering decisions of two competing supply chains with different composite policies: A stackelberg game-theoretic approach. International Journal of Production Research, 54 (9), 2807–2836.

[45]. Taleizadeh, A. A., Noori-daryan, M., & Tavakkoli-Moghaddam, R. (2015). Pricing and ordering decisions in a supply chain with imperfect quality items and inspection under buyback of defective items. International Journal of Production Research, 53 (15), 4553–4582.

[46]. Teng, J.-T., Min, J., & Pan, Q. (2012). Economic order quantity model with trade credit financing for non-decreasing demand. Omega, 40 (3), 328–335.

[47]. Thangam, A., & Uthayakumar, R. (2010). An inventory model for deteriorating items with inflation induced demand and exponential partial backordersa discounted cash flow approach. International Journal of management Science and Engineering management, 5 (3), 170–174.

[48]. Tripathy, C. K., & Mishra, U. (2012). Ordering policy for linear deteriorating items for declining demand with permissible delay in payments. International Journal of Open Problems in Computer Science and Mathematics, 238 (1388), 1–10.

[49]. Wang, Z., & Liu, S. (2018). Supply chain coordination under trade credit and quantity discount with sales effort effects. Mathematical Problems in Engineering, 2018 .

[50]. Wu, C., & Zhao, Q. (2014). Supplier–retailer inventory coordination with credit term for inventory-dependent and linear-trend demand. International Transactions in Operational Research, 21 (5), 797–818.

[51]. Wu, J., Ouyang, L.-Y., Cardenas-Barron, L. E., & Goyal, S. K. (2014). Optimal credit period and lot size for deteriorating items with expiration dates under two level trade credit financing. European Journal of Operational Research, 237 (3), 898–908. 2