Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 8, July 2021: 3034-3045

On rnp-open sets in nano topological spaces

J.B.Toranagatti

Department of Mathematics, Karnatak University's Karnatak Arts College, Dharwad-580001, India.
jagadeeshbt2000@gmail.com

Abstract

The notion of rnp-open sets in nano topological spaces is introduced.Some properties and characterizations of rnp-open sets are established. Also,a new class of continuity called rnp-continuity is introduced and its properties are investigated.

Keywords: nano preopen, nano preclosed, rnp-open, rnp-closed, rnp-continuity, nano pre continuity.

2020 Mathematics Subject Classification:*54A05;54C10; 54B05*

1. Introduction

M.L.Thivagar and C.Richard[7] initiated the study of nano topology by using theory approximation and boundary region of a subset of an universe in terms of an equivalence relation on it. They have also defined nano-interior and nano-closure in a nano topological spaces. In this paper,we introduce and study a new class of sets called rnp-open sets. Also,some properties of rnp-continuous functions are obtained.

2. Preliminaries

Definition 2.1[4] Let U be a non-empty finite set of objects, called the universe, and $\Re$ be an equivalence relation on U named as the indiscernible relation. The pair (U,$\Re$) is said to be the approximation space. Let Y $\subseteq$ U.

(i)The lower approximation of Y with respect to $\Re$ is $L_{\Re}(Y)= \bigcup_{y\in U} \{ \Re(y):\Re(y) \subseteq Y\}$

where $\Re(y)$ denotes the equivalence class determined by y $\in$ U.

(ii) The upper approximation of Y with respect to $\Re$ is $H_{\Re}(Y)= \bigcup_{y\in U} \{ \Re(y):\Re(y) \cap Y\neq \phi\}$.

(iii)The boundary region of Y with respect to $\Re$ is $B_{\Re}(Y) = H_{\Re}(Y)/L_{\Re}(Y)$.

Definition 2.2[7] In the approximation space $(U,\mathfrak{R})$,let $X \subseteq U$. Then

$\mathfrak{I}_{\mathfrak{R}}(X)=\aleph^T=\{U,\phi,L_{\mathfrak{R}}(X),H_{\mathfrak{R}}(X),B_{\mathfrak{R}}(X)\}$ forms a topology on U and it is called as the nano topology with respect to X. The pair $(U,\aleph^T)$ is called nano topological space.

Elements of $\aleph^T$ are known as the nano open(briefly,n-open) sets and the relative complements of nano open sets are called nano closed(briefly,n-closed) sets.

Throughout this paper,the word "NTS" mean an arbitrary nano topological space $(U,\aleph^T)$.

Let $M_1 \subseteq U$, then $\aleph cl(M_1) = \cap\{G: M_1 \subseteq G \text{ and } G^c \in \mathfrak{I}_{\mathfrak{R}}(X)\}$ is the nano closure of M_1 and $\aleph int(M_1) = \cup \{H: H \subseteq M_1 \text{ and } H \in \mathfrak{I}_{\mathfrak{R}}(X) \}$ is the nano interior of M_1.

Definition 2.3[3,5,7] A subset M_1 in $(U,\aleph^T)$ is said to be:

(i) nano b-open(briefly,nb-open) if $M_1 \subseteq \aleph cl(\aleph int(M_1)) \cup \aleph int(\aleph cl(M_1))$,

(ii) nano preopen(briefly,np-open) if $M_1 \subseteq \aleph int(\aleph cl(M_1))$,

(iii) nano regular open(briefly,nr-open) if $M_1 = \aleph int(\aleph cl(M_1))$,

(iv) nano α-open(briefly,nα-open) if $M_1 \subseteq \aleph int(\aleph cl(\aleph int(M_1)))$.

(iv) nano semiopen(briefly,ns-open) if $M_1 \subseteq \aleph cl(\aleph int(M_1)$,

(v) nano β-open(briefly,nβ-open) if $M_1 \subseteq \aleph cl(\aleph int(\aleph cl(M_1)))$.

The complements of the above respective open sets are their respective closed sets.

The family of all n-open(resp.,n-closed,np-closed,np-open, nb-open and nb-closed) sets of $(U,\aleph^T)$ is denoted by $\aleph O(U)$(resp.,$\aleph C(U)$, $\aleph PC(U)$ (resp.,$\aleph PO(U)$, $\aleph BO(U)$ and $\aleph BC(U)$).

Theorem 2.4 If M_1 and M_2 be any subsets in $(U,\aleph^T)$.Then:

(1) $M_1 \cap \aleph cl(M_2) \subseteq \aleph cl(M_1 \cap M_2)$ if M_1 is n-open.

(2)$\aleph int(M_1 \cup M_2) \subseteq M_1 \cup \aleph int(M_2)$ if M_2 is n-closed.

Definition 2.5[1] If K is a subset in $(U,\aleph^T)$,then:

$\aleph pcl(K)= \cap\{B:B^c \in \aleph^T \text{ such that } K \subseteq B\}$

$\aleph pInt(K)= \cup \{G: G \in \aleph^T \text{ such that } G \subseteq K\}$

Theorem 2.6 For a subset K in $(U,\aleph^T)$,

(i) $\aleph pcl(K)$ is the smallest np-closed superset of K and $\aleph pint(K)$ is the largest np-open subset of K.

(ii) K is np-closed if and only if $K = \aleph pcl(K)$ and K is np-open if and only if $K = \aleph pint(K)$.

Theorem 2.7[3] For a subset K in $(U,\aleph^T)$,

(i) ℵbcl(K) is the smallest nb-closed superset of K.

(ii) K is nb-closed if and only if K = ℵbcl(K).

Definition 2.8[2, 3] A function $\ell:(U_1, \mathfrak{I}_{\mathfrak{R}}(X))) \rightarrow (U_2, \mathfrak{I}_{\mathfrak{R}*}(Y))$ is called:

(i) np-continuous if $\ell^{-1}(K) \in \aleph PO(U_1)$ for every $K \in \mathfrak{I}_{\mathfrak{R}*}(Y)$,

(ii) nb-continuous if $\ell^{-1}(K) \in \aleph BO(U_1)$ for every $K \in \mathfrak{I}_{\mathfrak{R}*}(Y)$.

3.More properties of nano pre-closed sets:

In this section,we give additional results on np-open and np-closed sets which would be useful in our later section.

Theorem 3.1 In a NTS $(U,\aleph^T)$,let $M \subseteq U$.Then:

(1) ℵpcl(M) = M ∪ ℵcl(ℵint(M)).

(2) ℵscl(M) = M ∪ ℵint(ℵcl(M)).

(3) ℵbcl(M) = M ∪ [ℵcl(ℵint(M)) ∩ ℵint(ℵcl(M))].

Proof:(1)Since ℵpcl(M) is np-closed,

ℵcl(ℵint(M)) ⊆ ℵcl(ℵint(ℵpcl(M)) ⊆ ℵpcl(M)..........(I)

On the otherhand we have

ℵcl(ℵint(M ∪ ℵcl(ℵint(M))) ⊆ ℵcl(ℵint(K) ∪ℵcl(ℵint(M))) by Theorem 2.17

= ℵcl(ℵcl(ℵint(M)))

= ℵcl(ℵint(M))

⊆ M ∪ ℵcl(ℵint(M)......(I)

Therefore M ∪ ℵcl(ℵint(M) is a np-closed superset of M it follows that

ℵpl(M) ⊆M ∪ ℵcl(ℵint(M)........(II)

By (I) and (II),ℵpcl(M) = M ∪ ℵcl(ℵint(M)).

The other results can be proved similarly.

Corollary 3.2 In a NTS $(U,\aleph^T)$,let $M \subseteq U$.Then:

(1) ℵpint(M) = M ∩ ℵint(Ncl(M)).

(2) $\aleph$sint(M) = M ∩ $\aleph$cl(Nint(M)).

(3) $\aleph$pint(M) = M ∩ [$\aleph$int($\aleph$cl(M)) ∪ $\aleph$cl($\aleph$int(M))].

Theorem 3.3 In a NTS $(U,\aleph^T)$,let M⊆U.Then:

$\aleph$int($\aleph$cl($\aleph$pcl(M))= $\aleph$int($\aleph$cl(M)).

Proof: We have $\aleph$int($\aleph$cl($\aleph$pcl(M)) = $\aleph$int($\aleph$cl(M ∪ $\aleph$cl($\aleph$int(M))))

= $\aleph$int(Ncl(M) ∪ $\aleph$cl($\aleph$int(M)))

= $\aleph$int($\aleph$cl(M))

Theorem 3.4 In a NTS $(U,\aleph^T)$,let M⊆U.Then $\aleph$pint($\aleph$pcl(M)) = $\aleph$pcl(M) ∩ $\aleph$int($\aleph$cl(M)).

Proof: We have $\aleph$pint($\aleph$pcl(M)) = $\aleph$pcl(M) ∩ $\aleph$int($\aleph$cl($\aleph$pcl(M))

By Theorem 3.3, $\aleph$pint($\aleph$pcl(M)) = $\aleph$pcl(M) ∩ $\aleph$int($\aleph$cl(M)

Theorem 3.5 [3]In a NTS $(U,\aleph^T)$,let M⊆U.Then $\aleph$bcl(M)) = $\aleph$pcl(M) ∩$\aleph$scl(M)).

Theorem 3.6 In a NTS $(U,\aleph^T)$,let M⊆U.Then $\aleph$pint($\aleph$pcl(M)) = $\aleph$pint($\aleph$bcl(M)).

Proof: By Theorem 3.4,we have $\aleph$pint($\aleph$pcl(M))= $\aleph$pcl(M) ∩ $\aleph$int($\aleph$cl(M)

⊆ $\aleph$pcl(M) ∩ (M ∪ $\aleph$int($\aleph$cl(M)).

= $\aleph$pcl(M) ∩ ($\aleph$scl(M)).

= $\aleph$bcl(M).

Therefore, $\aleph$pint($\aleph$pcl(M)) ⊆ $\aleph$pint($\aleph$bcl(M)) and reverse inclusion is obvious

Definition 3.7 [6] A subset K in $(U,\aleph^T)$ is said to be nano dense if $\aleph$cl(K)=U

Theorem 3.8 In a NTS $(U,\aleph^T)$,every nano dense set is np-open but not conversely.

Proof: Let K be any nano dense set in $(U,\aleph^T)$ Then $\aleph$cl(K)=U. This implies

that $\aleph$int($\aleph$cl(K))=U so that K ⊆ $\aleph$int($\aleph$cl(K)). Hence K is np-open.

Example 3.9 Let U = $\{c_1,c_2,c_3,c_4\}$ with U\R= $\{\{c_1\}, \{c_3\}, \{c_2,c_4\}\}$ and let X = $\{c_1,c_2\}$,

$\aleph^T = \{U, \phi, \{c_1\}, \{c_1,c_2,c_4\}, \{c_2,c_4\}\}$.Then the set $\{c_1\}$ is np-open but not nano dense.

4.Nano regular pre-open sets:

Definition 4.1 A subset H in $(U,\aleph^T)$ is said to be:

(i)regular nano preopen(briefly,rnp-open) if H = $\aleph$pint($\aleph$pcl(H)), (ii)regular nano preclosed(briefly,rnp-closed) if H = ℵpcl(ℵpint(H)).

The family of all rnp-open(resp.,rnp-closed) sets of $(U,ℵ^T)$ is denoted by RℵPO(U) (resp.,RℵPC(U)).

Theorem 4.2 In a NTS $(U,ℵ^T)$,the following hold for any $M_1,M_2 \subseteq U$:

(i) If $M_1 \subseteq M_2$, then $ℵpint(ℵpcl(M_1) \subseteq ℵpint(ℵpcl(M_2))$.

(ii) If M_1 is np-open, then $M_1 \subseteq ℵpint(ℵpcl(M_1))$.

(iii) If M_1 is np-closed, then $ℵpcl(ℵpint(M_1)) \subseteq M_1$.

(iv) $ℵpint(ℵpcl(M_1))$ is rnp-open.

(v) If M_1 is np-closed, then $ℵpint(M_1)$ is rnp-open.

(vi) If M_1 is np-open, then $ℵpcl(M_1)$ is rnp-closed.

Proof:(i)Obvious.

(ii) Let M_1 be nano preopen and since $M_1 \subseteq ℵpcl(M_1)$,then $M_1 \subseteq ℵpint(ℵpcl(M_1))$.

(iii) Let M_1 be np-closed and since $ℵpint(M_1) \subseteq M_1$,then $ℵpcl(ℵpint(M_1) \subseteq M_1$.

(iv) We have $ℵpint(ℵpcl(ℵpint(ℵpcl(M_1)) \subseteq ℵpint(ℵpcl(ℵpcl(M_1)) = ℵpint(ℵpcl(M_1)$

and $ℵpint(ℵpcl(ℵpint(ℵpcl(M_1))) \supseteq ℵpint(ℵpint(ℵpcl(M_1)) = ℵpint(ℵpcl(M_1)$.

Hence $ℵpint(ℵpcl(ℵpint(ℵpcl(M_1))) = ℵpint(ℵpcl(M_1)$. Hence $ℵpint(ℵpcl(M_1))$ is rnp-open.

(v) Suppose that $M_1 \in ℵPC(U)$. By (iii), $ℵpint(ℵpcl(ℵpint(M_1)) \subseteq ℵpint(M_1)$.

On the other hand, we have $ℵpint(M_1) \subseteq ℵpcl(ℵpint(M_1)$ so that

$ℵpint(M_1) \subseteq ℵpint(ℵpcl(ℵpint(M_1))$.Therefore $ℵpint(ℵpcl(ℵpint(M_1))=ℵpint(M_1)$.

This shows that $ℵpint(M_1)$ is rnp-open.

(vi)Similar to (v).

Theorem 4.3 In a NTS $(U,ℵ^T)$,every rnp-open set is (i) np-open, (ii)nb-open, (iii)nβ -open,

(iv) nb-closed.

Proof:(i)If K is rnp-open, then

$$K = ℵpint(ℵpcl(K) = ℵpcl(K) \cap ℵint(ℵcl(K)) \subseteq ℵint(ℵcl(K)).$$

Hence K is np-open.

(ii)Let K be rnp-open, then K = ℵpint(ℵpcl(K)

$$= ℵpcl(K) \cap ℵint(ℵcl(K))$$

$\subseteq ℵint(ℵcl(K))$

$\subseteq ℵint(ℵcl(K)) \cup ℵcl(ℵint(K))$.

Hence K is nb-open.

(iii) If K is rnp-open, then K = ℵpint(ℵpcl(K)

$= ℵpcl(K) \cap ℵint(ℵcl(K))$

$\subseteq ℵint(ℵcl(K))$

$\subseteq ℵcl(ℵint(ℵcl(K)))$.

Therefore, K is nβ-open.

(iv) Let K be rnp-open, then K= ℵpint(ℵpcl(K)

$= ℵpcl(K) \cap ℵint(ℵcl(K)$

$= [K \cup ℵcl(ℵint(K)] \cap ℵint(ℵcl(K)$

$= [K \cap ℵint(ℵcl(K)] \cup [ℵcl(ℵint(K) \cap ℵint(ℵcl(K)]$

$= K \cup [ℵcl(ℵint(K) \cap ℵint(ℵcl(K)]$ since K is np-open

$= ℵbl(K)$

Hence K is nb-closed.

The following Example shows that every np-open(hence nb-open and nβ-open) set need not be a rnp-open set.

Example 4.4 The set $\{c_1,c_2\}$ in Example 3.9 is np-open but it is not rnp-open.

Theorem 4.5 In $(U,ℵ^T)$,every nr-open set is rnp-open but not conversely.

Proof: Let K be any nr-open set. By Theorem 3.4, $ℵpint(ℵpcl(K) = ℵpcl(K) \cap ℵint(ℵcl(K)) = ℵpcl(K) \cap K = K$. Hence K is rnp-open.

Example 4.6 The set $\{c_2\}$ in Example 3.9 is rnp-open but it is not nr-open.

Definition 4.7 A NTS $(U,ℵ^T)$ is called nano partition if NO(U) = NC(U).

Theorem 4.8 Let $(U,ℵ^T)$ be a a nano partition space, then every np-open set is rnp-open.

Remark 4.9 The class of rnp-open sets is not closed under finite union as well as finite intersection as shown in Example 4.10.

Example 4.10 Consider $(U,ℵ^T)$ as in Example 3.9.

Here $\{c_1\}$ and $\{c_2\} \in RℵPO(U)$ but $\{c_1\} \cup \{c_2\} = \{c_1,c_2\} \notin RℵPO(U)$.

Moreover, $\{c_1,c_2,c_3\}$ and $\{c_1,c_3,c_4\} \in$ RℵPO(U) but $\{c_1,c_2,c_3\} \cap \{c_1,c_3,c_4\}$ =

$\{c_1,c_3\} \notin$ RℵPO(U).

Theorem 4.11 Let K be a np-closed in (U,$ℵ^T$),then K is np-open if and only if K is rnp-open.

Proof: Let K be a np-open set and by hypothesis,K is np-closed. Then K = ℵpint(K) and K = ℵpcl(K). Therefore,ℵpint(ℵpcl(K)) = ℵpint(K) = K. Hence K is rnp-open.

Other part follows from the Theorem 4.3(i)

Theorem 4.12 For a subset K in (U,$ℵ^T$),the following statements are equivalent:

(i)K is rnp-open;

(ii) K is np-open and nb-closed.

Proof:(i) → (ii):From Theorem 4.3(i,iv).

(ii) → (i):Let K be both nb-closed and np-open. Then K = ℵbcl(K) and K = ℵpint(K).

By Theorem 3.6, ℵpint(ℵpcl(K)) = ℵpint(ℵbcl(K)) = ℵpint(K) = K. Hence K is rnp-open.

Definition 4.13 [6] A NTS (U,$ℵ^T$) is called nano submaximal if every nano dense subset of U is n-open

Theorem 4.14 The following are equivalent for a NTS (U,$ℵ^T$):

(i) (U,$ℵ^T$) nano submaximal;

(ii) ℵPO(U)=ℵO(U).

Proof: (i) → (ii):Let K ⊆ U be np-open. Then K ⊆ ℵint(ℵcl(K) = M,say.

This implies ℵcl(M) = ℵcl(K), so that

(ℵcl((U\M) ∪ K) = ℵcl(U\M) ∪ ℵcl(K)

= ℵcl(U\M) ∪ ℵcl(M)

= U and thus (U\M) ∪ K is nano dense in U.

By (i), (U\M) ∪ K is n-open. Now, K = ((U\M) ∪ K) ∩ M which is n-open.

(ii) → (i): Let K be a nano dense subset of U. Then ℵint(ℵcl(K) = U, then K ⊆ ℵint(ℵcl(K) and K is np-open and hence by (ii), K is n-open.

Theorem 4.15 If a NTS $(U,\aleph^T)$ is nano submaximal, then any finite intersection of np-open sets is np-open.

Proof: Obvious since $\aleph O(X)$ is closed under finite intersection.

Theorem 4.16 If a NTS $(U,\aleph^T)$ is nano submaximal, then any finite in tersection of rnp-open sets is rnp-open.

Proof: Let $\{A_i : i=1,2,...,n\}$ be a finite class of rnp-open sets. Since the space $(U,\aleph^T)$ is nano submaximal, then by Theorem 4.15, we have $\bigcap_{i=1}^{n} A_i \in \aleph PO(U)$. By Theorem 4.2(ii),

$\bigcap_{i=1}^{n} A_i \subseteq \aleph pint(\aleph pcl(\bigcap_{i=1}^{n} A_i))$. For each i, we have $\bigcap_{i=1}^{n} A_i \subseteq A_i$ and thus $\aleph pint(\aleph pcl(\bigcap_{i=1}^{n} A_i)) \subseteq \aleph pint(\aleph pcl(A_i) = A_i$ as $\aleph pint(\aleph pcl(A_i) = A_i$. Therefore $\aleph pint(\aleph pcl(\bigcap_{i=1}^{n} A_i)) \subseteq \bigcap_{i=1}^{n} A_i$.

In consequence, $\bigcap_{i=1}^{n} A_i$ is rnp-open in U.

Theorem 4.17 If $(U,\aleph^T)$ is nano partition, then the arbitrary union of rnp-open sets is rnp-open.

Proof: It follows from the Theorem 4.8

Definition 4.18 A subset M in $(U,\aleph^T)$ is called nano ε-open if $\aleph int(\aleph cl(M)) \subseteq \aleph cl(\aleph int(M))$.

Theorem 4.19 In a NTS $(U,\aleph^T)$,every ns-open set is nano ε-set but not conversely.

Proof: Let K be a ns-open set, then $K \subseteq \aleph cl(\aleph int(K)) \rightarrow \aleph int(\aleph cl(K)) \subseteq \aleph cl(\aleph int(K))$.

Hence K is nano ε-set.

Example 4.20 The set $\{c_3\}$ in Example 3.9 is ε-set but it is not ns-open.

Theorem 4.21 In a NTS $(U,\aleph^T)$,every ns-closed set is nano ε-set but not conversely.

Proof: Let K be ns-closed,then $\aleph int(\aleph cl(K)) \subseteq K$. Therefore $\aleph int(\aleph cl(K)) \subseteq \aleph cl(\aleph int(K))$. Hence K is nano ε-set.

Example 4.22 The set $\{c_1,c_3\}$ in Example 3.9 is nano ε-set but it is not ns-closed.

DIAGRAM

nr-open → nα-open → ns-open → nano ε-set

↓ ↓ ↓

rnp-open → np-open → nb-open

Remark 4.23 The notions of nano ε-sets and rnp-open(hence np-open,nb-open)sets are independent of each other.

Example 4.24 Let $(U,\aleph^T)$ be a NTS as in Example 3.9. Then $\{c_3\}$ is nano ε-set but not a

nb-open set and the set $\{c_1\}$ is rnp-open but it is not a ε-set.

Theorem 4.25 The following are equivalent for any subset K in $(U,\aleph^T)$:

(i) K is ns-open;

(ii) K is both nb-open and nano ε-set.

Proof: (i) → (ii):Obvious

(ii) → (i):Let K be both nb-open and nano ε-set.

$\aleph int(\aleph cl(K)) \cap \aleph cl(\aleph int(K)) \subseteq K$ and $\aleph int(\aleph cl(K)) \subseteq \aleph cl(\aleph int(K))$.

Then $\aleph int(\aleph cl(K)) \subseteq K$ and hence K is ns-open.

Theorem 4.26 The following are equivalent for any subset K in $(U,\aleph^T)$:

(i) K is nr-open;

(ii) K is rnp-open and nano ε-set.

Proof: (i) → (ii):Obvious

(ii) → (i):Let K be rnp-open and nano ε-set.Then,by Theorems 3.1 and 3.4,

we obtain $K = \aleph pint(\aleph pcl(K))$

$= (K \cup \aleph cl(\aleph int(K)) \cap \aleph int(\aleph cl(K))$

$= (K \cap \aleph int(\aleph cl(K)) \cup (\aleph cl(\aleph int(K)) \cap \aleph int(\aleph cl(K)))$

$= (K \cap \aleph int(\aleph cl(K)) \cup \aleph int(\aleph cl(K))$

$= \aleph int(\aleph cl(K))$

Therefore, $K = \aleph int(\aleph cl(K))$ and hence M is nr-open

Definition 4.27 In $(U,\aleph^T)$,let $M \subseteq U$.

(1)The rnp-interior of M, denoted by $int^{\aleph}_{rp}(M)$ is defined as

$int^{N}_{rp}(M) = \cup\{ K:K \subseteq M \text{ and } M \in R\aleph PO(U) \}$;

(2)The rnp-closure of M, denoted by $cl^{\aleph}_{rp}(M)$ is defined as

$cl^{\aleph}_{rp}(M) = \cap \{ F: M \subseteq F \text{ and } F \in R\aleph PC(U) \}$.

Theorem 4.28 In $(U,\aleph^T)$,let $M \subseteq U$.Then the following hold:

(i) $int^{\aleph}_{rp}(M) \subseteq M \subseteq cl^{\aleph}_{rp}(M)$.

(ii) If M is rnp-open(rnp-closed),then $int^{\aleph}_{rp}(M) = M$(resp, $cl^{\aleph}_{rp}(M) = M$).

Corollary 4.29 If in addition $(U,\aleph^T)$ is nano partition,then the converse of Theorem 4.28(ii) is true.

5.rnp-continuous function:

In this section, the concept of rnp-continuity and their properties are investigated.

Definition 5.1 A function $\ell:(U_1, \mathfrak{I}_{\mathfrak{R}}(X)) \to (U_2, \mathfrak{I}_{\mathfrak{R}*}(Y))$ is said to be rnp-continuous if $\ell^{-1}(H)$ is rnp-open in $(U_1, \mathfrak{I}_{\mathfrak{R}}(X))$ for each $H \in \mathfrak{I}_{\mathfrak{R}*}(Y)$.

Example 5.2 Let $U_1 = \{d_1,d_2,d_3,d_4\}$ with $U_1\backslash\mathfrak{R} = \{\{d_1\}, \{d_3\}, \{d_2,d_4\}\}$

and let $X = \{d_1,d_2\}$, $\mathfrak{I}_{\mathfrak{R}}(X) = \{U_1, \phi, \{d_1\}, \{d_1,d_2,d_4\}, \{d_2,d_4\}\}$.

Then nano rnp-open sets are $U_1, \phi, \{d_1\},\{d_2\},\{d_4\},\{d_2,d_4\},\{d_1,d_2,d_3\},\{d_1,d_3,d_4\}$.

Let $U_2 = \{e_1,e_2,e_3,e_4\}$ with $U_2\backslash\mathfrak{R} = \{\{e_1,e_3\}, \{e_3\}, \{e_4\}\}$ and let $Y = \{e_1,e_2\}$,

$\mathfrak{I}_{\mathfrak{R}*}(Y) = \{U_2, \phi, \{e_2\}, \{e_1,e_2,e_3\}, \{e_1,e_3\}\}$.

Define $h: (U_1, \mathfrak{I}_{\mathfrak{R}}(X)) \to (U_2 , \mathfrak{I}_{\mathfrak{R}*}(Y))$ as $h(d_1) = e_1$, $h(d_2) = e_2$, $h(d_3) = e_3 = h(d_4)$.

Then $h^{-1}(\{e_2\}) = \{d_2\}$, $h^{-1}(\{e_1,e_2,e_3\}) = U_1$ and $h^{-1}(\{e_1,e_3\}) = \{d_1,d_3,d_4\}$ and hence h is rnp-continuous.

Theorem 5.3 A function $\ell: (U_1, \mathfrak{I}_{\mathfrak{R}}(X)) \to (U_2, \mathfrak{I}_{\mathfrak{R}*}(Y))$ is rnp-continuous if and only if $\ell^{-1}(D)$ is rnp-closed in $(U_1, \mathfrak{I}_{\mathfrak{R}}(X))$ for every $D \in \aleph C(U_2)$.

Proof: Let $D \in \aleph C(U_2)$, then $U_2\backslash D \in \aleph O(U_2)$. Since ℓ is rnp-continuous, $\ell^{-1}(U_2\backslash D) = U_1\backslash \ell^{-1}(D)$ is rnp-open in U_1. Therefore, $\ell^{-1}(D)$ is rnp-closed in $(U_1, \mathfrak{I}_{\mathfrak{R}}(X))$.

Conversely,let $K \in \aleph O(U_2)$,then $(U_2\backslash K) \in \aleph C(U_2)$. By assumtion, $\ell^{-1}(U_2\backslash K) = U_1\backslash \ell^{-1}(K)$ is rnp-closed in U_1 which implies $\ell^{-1}(K)$ is rnp-open in U_1.

Therefore, ℓ is rnp-continuous.

Remark 5.4 The following implications hold and none of its implications is reversible.

rnp-continuity $\rightarrow$ np-continuity $\rightarrow$ nb-continuity.

Example 5.5 Consider $(U_1, \Im_{\Re}(X))$ and $(U_2, \Im_{\Re*}(Y))$ as in Example 5.2.

Define $\hbar : (U_1, \Im_{\Re}(X)) \rightarrow (U_2, \Im_{\Re*}(Y))$ as $\hbar(d_1) = e_1$, $\hbar(d_2) = e_3$, $\hbar(d_3) = e_4$ and

$\hbar(d_4) = e_2$. Then $\hbar^{-1}(\{e_2\}) = \{d_4\}$ $\hbar^{-1}(\{e_1,e_2,e_3\}) = \{d_1,d_2,d_3\}$ and $\hbar^{-1}(\{e_1,e_3\}) = \{d_1,d_2\}$. Therefore, $\hbar$ is np-continuous(hence nb-continuous) but there exists $\{e_1,e_3\} \in \Im_{\Re*}(Y)$ such that $\hbar^{-1}(\{e_1,e_3\}) = \{d_1,d_2\} \notin R\aleph PO(U)$. Hence $\hbar$ is not rnp-continuous.

Theorem 5.6 The following statements are equivalent for a function

$\ell: (U_1, \Im_{\Re}(X)) \rightarrow (U_2, \Im_{\Re*}(Y))$ where $(U_1, \Im_{\Re}(X))$ is nano partition:

(i) ℓ is rnp-continuous;

(ii)For each $B \subseteq U_2$, $cl^{\aleph}_{rp}((\ell-1(B)) \subseteq \ell^{-1}(\aleph cl(B))$;

(iii)For each $A \subseteq U_1$, $\ell(cl^{\aleph}_{rp}((A)) \subseteq \aleph cl(\ell(A))$;

(iv)For each $B \subseteq U_2$, $\ell^{-1}(\aleph int(B)) \subseteq int^{\aleph}_{rp}(\ell^{-1}(B))$.

Proof:(i) $\rightarrow$ (ii): Let $B \subseteq U_2$ and since $\aleph cl(B) \in \aleph C(U_2)$. Then by (i),

$\ell^{-1}(\aleph cl(B)) \in R\aleph PC(U_1)$ which implies $cl^{\aleph}_{rp}(\ell^{-1}(B)) \subseteq cl^{\aleph}_{rp}(\ell^{-1}(\aleph cl(B))) = \ell^{-1}(\aleph cl(B)$.

(ii) $\rightarrow$ (i): Let $M \in \aleph C(U_2)$.Then by (ii), $cl^{\aleph}_{rp}(\ell^{-1}(M)) \subseteq \ell^{-1}(\aleph cl(M)) = \ell^{-1}(M)$ which implies

$cl^{\aleph}_{rp}(\ell^{-1}(M)) = \ell^{-1}(M)$ and since U_1 is nano partition, then by Corollary 4.29, $\ell^{-1}(M)$ is rnp-closed in U_1.

(ii) $\rightarrow$ (iii): Let $A \subseteq U_1$.Then $\ell(A) \subseteq U_2$. By (ii), we get $\ell^{-1}(\aleph cl(\ell(A))) \supseteq cl^{\aleph}_{rp}(\ell^{-1}(\ell(A))) \supseteq cl^{\aleph}_{rp}(A)$. Therefore, $\ell(cl^{\aleph}_{rp}(A)) \subseteq \ell(\ell^{-1}(\aleph cl(\ell(A))) \subseteq \aleph cl(\ell(A))$.

(iii)$\rightarrow$ (iv): Let $B \subseteq U_2$ and $\ell^{-1}(B) \subseteq U_1$. Then by (iii),

$\ell(cl^{\aleph}_{rp}(\ell^{-1}(B)) \subseteq \aleph cl(\ell(\ell^{-1}(B)) \subseteq \aleph cl(B) \rightarrow cl^{\aleph}_{rp}(\ell^{-1}(B)) \subseteq \ell^{-1}(\aleph cl(B))$.

(ii) → (iv): Replace B by $U_2 \setminus B$ in (ii), we get $cl^{\aleph}_{rp}(\ell^{-1}(U_2 \setminus B)) \subseteq \ell^{-1}(\aleph cl(U_2 \setminus B))$.

It implies that $cl^{\aleph}_{rp}(U_1 \setminus \ell^{-1}(B)) \subseteq \ell^{-1}(U_2 \setminus \aleph int(B))$.

Therefore, $\ell^{-1}(\aleph int(B)) \subseteq int^{\aleph}_{rp}(\ell^{-1}(B))$ for each $B \subseteq U_2$.

(iv) → (i): Let $B \in \aleph O(U_2)$.Then, $\ell^{-1}(B) = \ell^{-1}(\aleph int(B)) \subseteq int^{\aleph}_{rp}(\ell^{-1}(B)$ which implies

$int^{\aleph}_{rp}(\ell^{-1}(B) = \ell^{-1}(B)$ and since $(U_1, \mathfrak{I}_{\mathfrak{R}}(X))$ is nano partition then by Corollary 4.29,

$\ell^{-1}(B)$ is rnp-open in U_1.

References

1. K.Bhuvaneswari and K.Mythili Gnanapriya,On nano generalised pre closed sets and nano pre generalised closed sets in nano topological spaces,Int.J. Inven. Res. in Sci., Eng. and Technology 3(2014), 16825-16829.
2. D.A. Mary and I. Arockiarani,On characterizations of nano rgb-clased sets in nano topological spaces, Int. J. Mod. Eng. Res. 5(1)(2015),68–76.
3. D.A. Mary and I. Arockiarani,On b-open sets and b-continuous functions in nano topological spaces, Int. J. Innov. Res. Stud. 3(11)(2014),98–116.
4. Z.Pawlak,Rough sets, Int. J. Comput. Inf. Sci. 11(5)(1982), 341–356.
5. A.Revathy, G.Ilango,On nano β-open sets, Int. J. Eng. Contemp.Math. Sci. 1(2)(2015), 1–6.
6. P.Sathishmohan,V.Rajendran,C.Vignesh Kumar and P.K.Dhanasekaran, On nano semipre neighbourhoods in nano topological spaces,Malaya Journal of Matematik, 6(1)(2018), 294-298.
7. M.L.Thivagar and C.Riclord,On nano forms of weakly open sets, Int.J.Math.Stat.Inven., 1(1)(2013),31-37.