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Research Article

Path Related Even Sum Graphs

Chinju Krishna. K[1], David Raj. C[2] and Rubin Mary. K[3]
[1]Research Scholar (Part Time- Internal), Reg. No: 18223232092004,
[1, 3]Department of Mathematics St. Jude's College, Thoothoor, Kanyakumari District, Tamilnadu, India
Affiliated to Manonmaniam Sundarnar University, Tirunelveli, TamilNadu, India
[2]Assistant Professor, Department of Mathematics, Malankara Catholic College, Mariagiri, Kaliyakkavilai, Kanyakumari District, Tamilnadu, India.
[1]chinjukrishna1991@gmail.com, [2]davidrajmccm@gmail.com, [3]rubyjudes@yahoo.com

ABSTRACT

Graph theory has become one of the most rapidly growing areas in Mathematics. Graph theory has independently discovered many timers through some puzzles that arose from the physical world consideration of chemical isomers, electrical networks, etc. There are several areas of Graph theory which have received good attention from mathematics. Labeling of graphs is one of the most interesting problem in the area of Graph theory. Graph Labeling was first introduced by Alexandra Rosa during 1960 where the vertices and edges are assigned real; values or subsets of a set subject to certain conditions. In Graph Labeling, vast amount of literature is available. Labeled graphs serve as useful models for a broad range of applications. The concept of Sum Graphs and Integral Sum Graphs was introduced by F. Harary. A sum graph is a graph for which there is a labeling of its vertices with distinct positive integers so that two vertices are adjacent if and only if the sum of their labels is the label of another vertex. The minimum number of isolated vertices required to make the graph G, a sum graph is called the sum number of G and is denoted by $\sigma(G)$. Integral sum graphs are defined similarly, except that the labels may be any integers. The minimum number of isolated vertices required to make the graph G, an integral sum graph is called the integral sum number of G and is denoted by $\xi(G)$

Keywords : even sum graph, path, cycle, union, pendent vertex, complete bipartite raph

1. INTRODUCTION

Labeling is an assignment of integers to the vertices or edges or both is subject to certain conditions motivated by practical problems .The concept of Sum Graphs & Integral Sum Graphs was introduced by F. Harary [5, 6, 7]. The properties of Sum Graphs are investigated by manyauthors, including Chen. Z [1], Mary Florida. L [9], Nicholas. T [8], Soma Sundaram. S [8], Vilfred.V [9, 10, 11], Surya Kala.V [11] and Rubin Mary. K [11]. A graph G is called an even sum graph if there is a labeling θ of its vertices with distinct non- negative even integers, so that for any two distinct vertices u and v; uv is an edge of G if and only if $\theta(u) + \theta(v) = \theta(w)$ for some vertex w in G: The minimum number of isolated vertices required to make the graph G ,an even sum graph is called the even sum number of G and is denoted by $\gamma(G)$ [2,3,12] In this paper we investigates different types of Even Sum Graphs. All graphs used in this paper are simple graphs. For all basic ideas, we refer [4, 5].

2. MAIN RESULTS

Theorem: 2.1. The graph $G = C_m \cup P_n$ is an even sum graph.

Proof:

Let $u_1, u_2, \ldots, u_m$ be the vertices of the cycle C_m and let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n. Define a function $\theta : V(G) \to 2Z^{+} \cup \{0\}$ by $\theta(u_1) = 2$; $\theta(u_2) = 4$; $\theta(u_i) = \theta(u_{i-2}) + \theta(u_{i-1})$, $3 \leq i \leq m$; $\theta(v_1) = \theta(u_n) + \theta(u_{n-1})$; $\theta(v_2) = \theta(v_1) + 2$; $\theta(v_i) = \theta(v_{i-2}) + \theta(v_{i-1})$, $3 \leq i \leq n-2$; $\theta(v_{n-1}) = 0$; $\theta(v_n) = \theta(v_{n-3}) + \theta(v_{n-4})$. Then the labels are distinct and $\theta(u) + \theta(v) = \theta(w)$ for some vertex w in G: Hence the graph G is an even sum graph.

Example: 2.2. The graph $G = C_6 \cup P_7$ is an even sum graph.

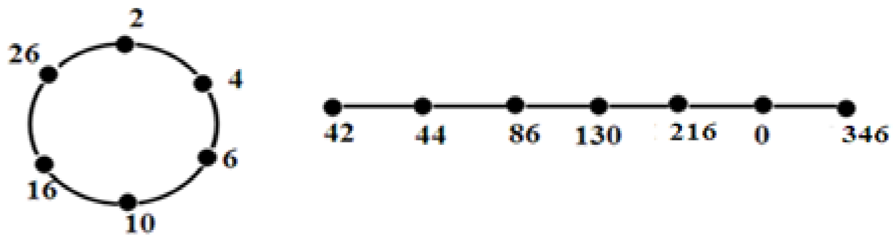

Figure : 2.1

Theorem: 2.3. The graph $G = (P_n \odot K_2) \cup nK_1$ is an even sum graph.

Proof:

Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n. Let v_i, x_i be the vertices of K_2 which are joined to the vertex u_i of path P_n, $1 \leq i \leq n$. The resultant graph is $P_n \odot K_2$ whose edge set is $E = \{ u_iu_{i+1} / 1 \leq i \leq n-1\} \cup \{ u_iv_i, u_ix_i, v_iw_i / 1 \leq i \leq n\}$. Let $a_1, a_2, \ldots, a_n$ be the isolated vertices of nK_1. Define a function θ: $V(G) \to 2Z^{+} \cup \{0\}$ by $\theta(u_1) = 0$; $\theta(u_2) = 2$; $\theta(u_3) = 4$; $\theta(u_i) = \theta(u_{i-2}) + \theta(u_{i-1})$, $3 \leq i \leq n$; $\theta(v_1) = \theta(u_n) + \theta(u_{n-1})$; $\theta(v_i) = \theta(u_i) + \theta(w_i)$, $2 \leq i \leq n$; $\theta(w_1) = \theta(v_1) + 2$; $\theta(w_2) = \theta(v_1) + \theta(w_1)$; $\theta(w_i) = \theta(v_{i-1}) + \theta(w_{i-1})$, $3 \leq i \leq n$; $\theta(a_1) = \theta(w_n) + \theta(v_n)$; $\theta(a_i) = \theta(v_i) + \theta(u_i)$, $2 \leq i \leq n$; Then the labels are distinct and $\theta(u) + \theta(v) = \theta(w)$ for some vertex w in G. Hence the graph G is an even sum graph.

Example: 2.4. The graph $G = (P_4 \odot K_2) \cup 4K_1$ is an even sum graph.

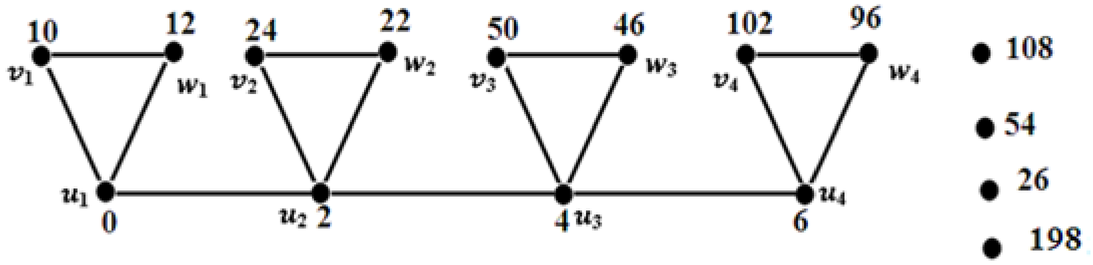

Figure : 2.2

Theorem: 2.5. A graph G obtained by attaching the central vertex of $K_{1,2}$ at each pendent vertex of a comb and addition of an isolated vertex is an even sum graph.

Proof:

Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n and let v_i be the vertex adjacent to u_i, $1 \leq i \leq n$. The resultant graph is $P_n \odot K_1$. Let x_i, w_i, y_i be the vertices of the i^{th} copy of $K_{1,2}$ with the central vertex. Identify the vertex w_i with v_i, $1 \leq i \leq n$. Then we get the required graph G whose edge set is $E = \{ u_iu_{i+}$

$_1$ / $1 \leq i \leq n-1\}$ ∪{ u_iv_i, v_ix_i, v_iy_i / $1 \leq i \leq n\}$. Let a_0 be the isolated vertex of G. Define a function θ: V(G) → $2Z^+ \cup \{0\}$ by $\theta(u_1) = 2$; $\theta(u_2) = 4$; $\theta(u_i) = \theta(u_{i-2}) + \theta(u_{i-1})$, $3 \leq i \leq n$; $\theta(v_1) = \theta(u_n) + \theta(u_{n-1})$; $\theta(v_i) = \theta(y_{i-1}) + \theta(v_{i-1})$, $2 \leq i \leq n$; $\theta(x_1) = 0$; $\theta(x_i) = \theta(v_i) + \theta(u_i)$, $2 \leq i \leq n$; $\theta(y_i) = \theta(x_i) + \theta(v_i)$, $2 \leq i \leq n$; $\theta(a_0) = \theta(y_n) + \theta(v_n)$. Then the labels are distinct and $\theta(u) + \theta(v) = \theta(w)$ for some vertex w in G. Hence the graph G is an even sum graph.

Example: 2.6. An even sum graph of G when n = 4.

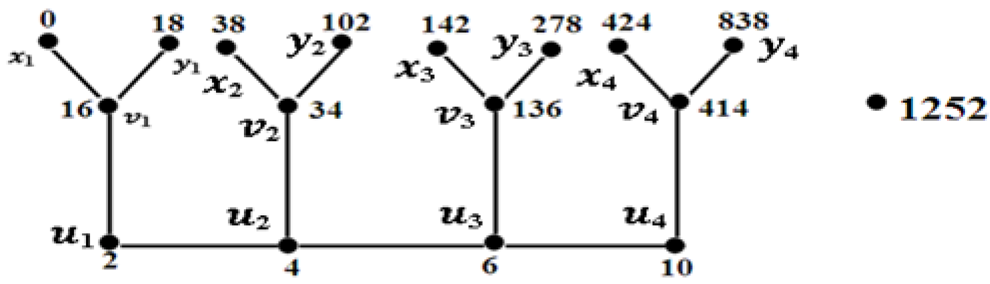

Figure : 2.3

3. CONCLUSION

In this paper, we have explored the concept of even sum graphs and we investigated some path related graphs to be an even sum graph. This paper motivates to derive similar results on other types of graphs to be an even sum graphs and to investigate the characterization of even sum graphs.

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