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Research Article

Intuitionistic Fuzzy πgγ* Homeomorphisms and Intuitionistic Fuzzy Connectedness in intuitionistic fuzzy topological spaces

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Abstract

In this paper we have introduced intuitionistic fuzzy $\pi g\gamma*$ homeomorphisms and intuitionistic fuzzy $\pi g\gamma*$ connectedness in intuitionistic fuzzy topological spaces . Some of their properties are studied.

Key words and phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy $\pi g\gamma*$ closed set, intuitionistic fuzzy $\pi g\gamma*$ continuous mapping and intuitionistic fuzzy $\pi g\gamma*$ closed mappings, intuitionistic fuzzy $\pi g\gamma*$ homeomorphisms and intuitionistic fuzzy $\pi g\gamma*$ connected spaces

1. Introduction

The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy πgγ* homeomorphisms and studied some of their basic properties. We provide some interesting propositions and results on intuitionistic fuzzy πgγ* homeomorphisms. We have also introduced intuitionistic fuzzy πgγ* connectedness and studied some of their properties.

2. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form

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$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$.
Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X$. Also for the sake of simplicity, we shall use the notation $A = \{\langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.
(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,
$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$.

Definition 2.5: [6] An IFS $A = \{ \langle x, \mu_A, \nu_A \rangle \}$ in an IFTS (X, τ) is said to be an
(i) *Intuitionistic fuzzy semi open set* (IFSOS in short) if $A \subseteq cl(int(A))$,
(ii) *Intuitionistic fuzzy α-open set* (IFαOS in short) if $A \subseteq int(cl(int(A)))$,
(iii) Intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$,

Definition 2.6: [7] The union of IFROSs is called intuitionistic fuzzy π-open set (IFπOS in short) of an IFTS (X, τ). The complement of IFπOS is called intuitionistic fuzzy π -closed set (IFπCS in short).

Definition 2.7: [6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an
(i) *Intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$,

(ii) *Intuitionistic fuzzy α-closed set* (IFαCS in short) if cl(int(cl(A)) ⊆ A,
(iii) Intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int((A).

Definition 2.8:[5] An IFS A of an IFTS (X, τ) is an
(i) Intuitionistic fuzzy γ-open set (IFγOS in short) if A ⊆ int(cl(A)) ∪ cl(int(A)),
(ii) Intuitionistic fuzzy γ-closed set (IFγCS in short) if cl(int(A)) ∩ int(cl(A)) ⊆ A.

Definition 2.9: [11] Let A be an IFS in an IFTS (X, τ). Then
sint(A) = ∪ { G / G is an IFSOS in X and G ⊆ A },
scl(A) = ∩ { K / K is an IFSCS in X and A ⊆ K }.

Note that for any IFS A in (X, τ), we have scl(A^c)=(sint(A))^c and sint(A^c) = (scl(A))^c .
Definition 2.10: [10] An IFS A in an IFTS (X, τ) is an
(i) Intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X.
(ii) Intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFROS in X.

Definition 2.11: [11] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

Definition 2.12: [10] An IFS A in (X, τ) is said to be an intuitionistic fuzzy $\pi g\gamma$* closed set (IF$\pi g\gamma$*CS in short) if cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFπOS in (X, τ). The family of all IF$\pi g\gamma$*CSs of an IFTS (X, τ) is denoted by IF$\pi g\gamma$*C(X).

Result 2.13: [10] Every IFCS, IFGCS, IFRCS, IFαCS is an IF $\pi g\gamma$*CS but the converses may not be true in general.

Definition 2.14: [10] An IFS A is said to be an intuitionistic fuzzy $\pi g\gamma$*open set (IF$\pi g\gamma$*OS in short) in (X, τ) if the complement A^c is an IF$\pi g\gamma$*CS in X.
The family of all IF$\pi g\gamma$*OSs of an IFTS (X, τ) is denoted by IF$\pi g\gamma$*O(X).

Definition 2.15: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy contra continuous (IF contra continuous in short) if f^{-1}(B) ∈ IFCS(X) for every B∈ σ.

Definition 2.16: [6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) Intuitionistic fuzzy semi contra continuous (IFS contra continuous in short) if f^{-1}(B) ∈ IFSCS(X) for every B ∈ σ.
(ii) Intuitionistic fuzzy α contra continuous (IFα contra continuous in short) if f^{-1}(B) ∈ IFαCS(X) for every B ∈ σ.
(iii) Intuitionistic fuzzy pre contra continuous (IFP contra continuous in short) if f^{-1}(B) ∈ IFPCS(X) for every B ∈ σ.

Definition 2.17: [5] A mapping f: (X, τ) $\rightarrow$ (Y,σ) is called an *intuitionistic fuzzy γ contra continuous* (IFγ contra continuous in short) if $f^{-1}(B)$ is an IFγCS in (X, τ) for every B $\in$ σ.

Definition 2.18: [9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized contra continuous (IFG contra continuous in short) if $f^{-1}(B) \in$ IFGCS(X) for every IFOS B in Y.

Definition 2.19: [5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi\gamma^*cT_{1/2}$ (in short IF$\pi\gamma^*cT_{1/2}$) space if every IF$\pi g\gamma^*$CS in X is an IFCS in X.

Definition 2.20: [6] An IFTS (X, τ) is an intuitionistic fuzzy $\pi\gamma^*gT_{1/2}$ (IF$\pi\gamma^*gT_{1/2}$) space if every IF$\pi g\gamma^*$CS is an IFGCS in X.

Definition 2.21: [7] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi\gamma^*T_{1/2}$ (in short IF$\pi\gamma^*T_{1/2}$) space if every IF$\pi g\gamma^*$CS in X is an IFγCS in X.

3. Intuitionistic fuzzy $\pi g\gamma^*$ homeomorphisms

In this section we have introduced intuitionistic fuzzy $\pi g\gamma^*$ homeomorphisms and studied some of its properties.

Definition 3.1: A bijection mapping f: (X, τ) $\rightarrow$ (Y,σ) is called an *intuitionistic fuzzy* $\pi g\gamma^*$ homeomorphisms if f is both an intuitionistic fuzzy $\pi g\gamma^*$ continuous mapping and an intuitionistic fuzzy $\pi g\gamma^*$ closed mapping.

Example 3.2: Let X = {a, b}, Y = {u, v} and $T_1 = \langle$ x, (0.5, 0.4), (0.5, 0.6) $\rangle$, $T_2 = \langle$ y, (0.6, 0.6), (0.4, 0.4) $\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an *intuitionistic fuzzy* $\pi g\gamma^*$ homeomorphisms.

Theorem 3.3: Every intuitionistic fuzzy homeomorphisms is an intuitionistic fuzzy $\pi g\gamma^*$ homeomorphisms but not conversely.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be an IF homeomorphism. Then f is both an IF continuous mapping and an IF closed mapping and hence f is both an IF$\pi g\gamma^*$ continuous mapping and an IF$\pi g\gamma^*$ closed mapping. Therefore the mapping f is an IF$\pi g\gamma^*$ homeomorphism.

Example 3.4: Let X = {a, b}, Y = {u, v} and $T_1 = \langle$ x, (0.5, 0.4), (0.5, 0.6) $\rangle$, $T_2 = \langle$ y, (0.6, 0.6), (0.4, 0.4) $\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. By Example 3.2, f is an *intuitionistic fuzzy* $\pi g\gamma^*$ homeomorphisms. But f is not IF homeomorphism, since f is not an IF continuous mapping, as $T_2^c = \langle$y, (0.4, 0.4), (0.6, 0.6)$\rangle$ is an IFCS in Y but $f^{-1}(T_2^c)$ is not an IFCS in X, as the only IFCSs in X are 0^c, 1^c and T_1^c.

Theorem 3.5: Every IFα homeomorphism is an IFπgγ* homeomorphism but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα homeomorphism. Then f is both an IFα continuous mapping and an IFα closed mapping. As every IFα continuous mapping is an $IF\pi g\gamma*$ continuous mapping and every IFα closed mapping is an $IF\pi g\gamma*$ closed mapping, f is an $IF\pi g\gamma*$ homeomorphism.

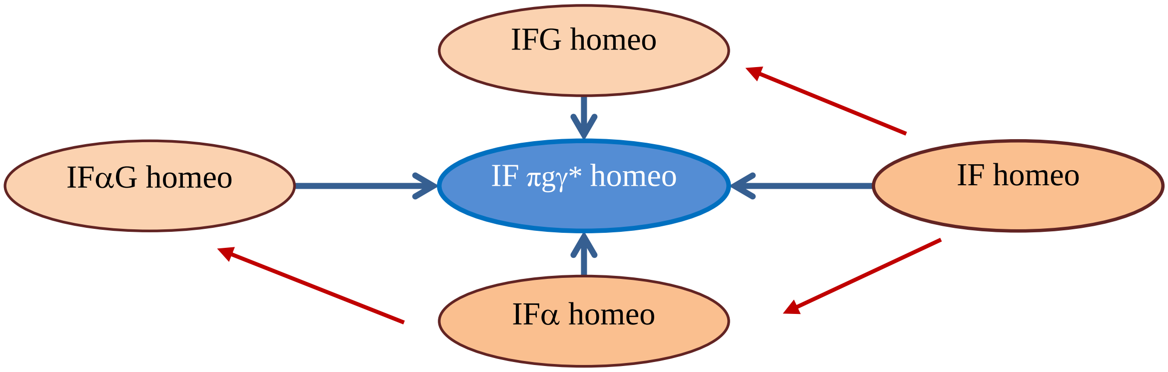
Example 3.6: Let X = { a, b }, Y = { u, v } and T_1 = ⟨ x, (0.5, 0.4), (0.5, 0.6) ⟩ and T_2 = ⟨ y, (0.5, 0.6), (0.4, 0.4) ⟩. Then τ = {$0_\sim$, T_1 ,$1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is an IFπgγ* homeomorphism. But f is not an IFα homeomorphism, since f is not an IFα continuous mapping, as T_2^c = ⟨y, (0.5, 0.6), (0.5, 0.4)⟩ is an IFCS in Y but $f^{-1}(T_2^c)$= ⟨ x, (0.5, 0.4), (0.5, 0.6) ⟩ is not an IFαCS in X.

Theorem 3.7: Every IFg homeomorphism is an IFπgγ* homeomorphism but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFg homeomorphism. Then f is both an IFg continuous mapping and an IFg closed mapping. As every IFg continuous mapping is an IFπgγ* continuous mapping and every IFg closed mapping is an IFπgγ* closed mapping, f is an IFπgγ* homeomorphism.

Example 3.8: Let X = { a, b }, Y = { u, v } and T_1 = ⟨ x, (0.5, 0.4), (0.5, 0.6) ⟩, T_2 = ⟨ x, (0.6, 0.6), (0.4, 0.4) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is an $IF\pi g\gamma*$ homeomorphism but not an IF g homeomorphism, since f is not an IFg continuous mapping, as T_2^c = ⟨y, (0.4, 0.4), (0.6, 0.6)⟩ is an IFCS in Y but $f^{-1}(T_2^c)$)= ⟨ x, (0.4, 0.4), (0.6, 0.6) ⟩ is not an IFGCS in X.

The relation between various types of intuitionistic fuzzy homeomorphism is given in the following diagram. In this diagram 'homeo' means homeomorphism.



The reverse implications are not true in general in the above diagram.

Theorem 3.9: Let f: (X, τ) → (Y, σ) be an $IF\,\pi g\gamma*$ homeomorphism, then f is an IF homeomorphism if X and Y are IFγ*c$T_{1/2}$ space.

Proof: Let f: (X, τ) → (Y,σ) be an $IF\,\pi g\gamma*$ homeomorphism. Then f is $IF\,\pi g\gamma*$ continuous mapping and is an $IF\,\pi g\gamma*$ closed mapping. Since X and Y are IFγ*c$T_{1/2}$ spaces, f is IF continuous mapping and IF closed mapping. Hence, f is IF homeomorphism.

Theorem 3.10: Let f: (X, τ) → (Y, σ) be an $IF\,\pi g\gamma*$ homeomorphism, then f is an IF generalized homeomorphism if X and Y are IFγ*g$T_{1/2}$space.

Proof: Let f: (X, τ) → (Y,σ) be an $IF\,\pi g\gamma*$ homeomorphism. Then f is $IF\,\pi g\gamma*$ continuous mapping and is an $IF\,\pi g\gamma*$ closed mapping. Since X and Y are IFγ*g$T_{1/2}$space, f is IF g continuous mapping and IF g closed mapping. Hence, f is IF g homeomorphism.

Theorem 3.11: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following are equivalent:
(i) f is an IF πgγ* closed mapping,
(ii) f^{-1} is IF πgγ* continuous mapping,
(iii) f is an IF πgγ* open mapping.

Proof:

(i) ⇒ (ii) Let B be an IFCS in X. Since f is an $IF\,\pi g\gamma*$ closed mapping f(B) = $(f^{-1})^{-1}$(B) is an $IF\,\pi g\gamma*$ CS in Y. This implies f^{-1} is an $IF\,\pi g\gamma*$ continuous mapping.
(ii) ⇒(iii) Let A be an IFOS in X. Then by hypothesis $(f^{-1})^{-1}$(A) = f(A) is an $IF\,\pi g\gamma*$ OS in Y. That is f is an $IF\,\pi g\gamma*$ open mapping.

(iii)⇒(i) Let f be an $IF\,\pi g\gamma*$ open mapping. Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis f(A^c) = $f(A)^c$ is an $IF\,\pi g\gamma*$ OS in Y as f is bijective. Therefore f(A) is an $IF\,\pi g\gamma*$ CS in Y. Hence f is an $IF\,\pi g\gamma*$ closed mapping.

4 Intuitionistic fuzzy M-πgγ* homeomorphisms

In this section we have introduced intuitionistic fuzzy M-πgγ* homeomorphisms and investigated some of their properties.

Definition 4.1: A bijective mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy M-***πgγ* (IFM-πgγ*) ***homeomorphism*** if f is both an $IF\,\pi g\gamma*$ irresolute mapping and an $IF\,\pi g\gamma*$ closed mapping.

Example 4.2: Let X = {a, b}, Y= {u, v}, G_1 = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ and G_2 = ⟨y, (0.4, 0.4), (0.6, 0.6)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively, where Then (X, τ) is an IFTS. Define a mapping f: (X, τ) →(Y, σ) by f(a) = u and f(b) = v. Here f is both an $IF\,\pi g\gamma*$ irresolute mapping and an $IF\,\pi g\gamma*$ closed mapping. Hence f is an IFM-πgγ* homeomorphism.

Proposition 4.3: Every IFM-πgγ* homeomorphism is an $IF\ \pi g\gamma*$ homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be an IFM-πgγ* homeomorphism.
Therfore, f is both an $IF\ \pi g\gamma*$ irresolute mapping and an $IF\ \pi g\gamma*$ closed mapping. To Prove : f is $IF\ \pi g\gamma*$ continuous mapping.
Let B be an IFCS in Y. This implies B is an $IF\ \pi g\gamma*$ CS in Y. By hypothesis $f^{-1}(B)$ is an $IF\ \pi g\gamma*$ CS in X. Hence f is an $IF\ \pi g\gamma*$ continuous mapping. Hence f is an $IF\ \pi g\gamma*$ homeomorphism.

5 Intuitionistic fuzzy $\pi g\gamma*$ connected spaces

In this section we have introduced intuitionistic fuzzy $\pi g\gamma*$ connected space and investigated some of their properties.

Definition 5.1: An IFTS (X, τ) is said to be an *$IF\ \pi g\gamma*$ connected space* if the only IFSs which are both IF πgγ* OS and an IF πgγ* CS are $0_\sim$ and $1_\sim$.

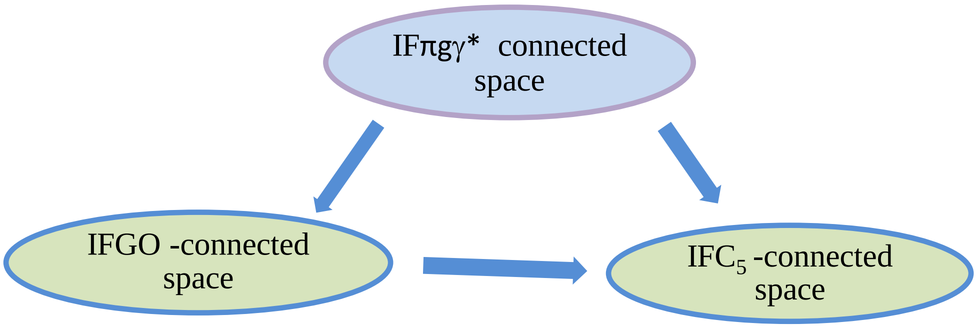
Proposition 5.2: Every IF πgγ* connected space is an IFC$_5$-connected space but not conversely in general.

Proof: Let (X, τ) be an IF πgγ* connected space. Suppose (X, τ) is not an IFC$_5$-connected space, then there exists a proper IFS A which is both an IFOS and an IFCS in (X, τ). That is A is both an IFπgγ*OS and an IFπgγ*CS in (X, τ). This implies that (X, τ) is not an IF πgγ* connected space, a contradiction. Therefore (X, τ) must be an IFC$_5$-connected space.

Proposition 5.4: Every IF πgγ* connected space is an IFGO-connected space but not conversely in general.

Proof: Let(X,τ) be an IF πgγ* connected space. Suppose(X,τ) is not an IFGO-connected space, then there exists a proper IFS A which is both an IFGOS and an IFGCS in (X,τ). That is A is both an IF πgγ* OS and an IF πgγ* CS in (X,τ). This implies that (X,τ) is not an IFγ*G connected space, a contradiction. Therefore (X, τ) must be an IFGO-connected space.

Example 5.5: Let X = {a, b}, G_1= ⟨x, (0.2, 0.2), (0.7, 0.8)⟩ and G_2 = ⟨x, (0.6, 0.5), (0.4, 0.4)⟩.Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} be an IFT on X. Here (X, τ) is an IFGO connected space but not an IF πgγ* connected space, since the IFS A = ⟨x, (0.5, 0.4), (0.5, 0.5)⟩ is both an IF πgγ* OS and an IF πgγ* CS in (X, τ).
The relation between various types of intuitionistic fuzzy connectedness is given in the following diagram.



In the above diagram the reverse implications are not true in general.

Proposition 5.6: The IFTS (X, τ) is an IF πgγ* connected space if and only if there exists no non zero IF πgγ* OSs A and B in (X, τ) such that $A = B^c$.

Proof: Necessity: Let A and B be two IF πgγ* OSs in (X, τ) such that $A \neq 0_\sim$, $B \neq 0_\sim$ and $A = B^c$. Therefore $A = B^c$ is an IF πgγ* CS. Since $B \neq 0_\sim$, $A = B^c \neq 1_\sim$. Hence A is a proper IFS which is both an IF πgγ* OS and an IF πgγ* CS in (X, τ). Hence (X, τ) is not an IF πgγ* connected space, a contradiction to our hypothesis. Hence there exists no non-zero IF πgγ* OSs A and B in (X, τ) such that $A = B^c$.

Sufficiency: Suppose (X, τ) is not an IF πgγ* connected space. Then there exists an IFS which is both an IF πgγ* OS and an IF πgγ* CS in (X, τ) such that $0_\sim \neq A \neq 1_\sim$. Now let $B = A^c$. Then B is an IF πgγ* OS and $B \neq 1_\sim$. This implies $B^c = A \neq 0_\sim$, which is a contradiction to our hypothesis. Hence (X, τ) is an IF πgγ* connected space.

Proposition 5.7: Let (X, τ) be an IF πgγ* $T_{1/2}$ space, then the following are equivalent:

(i) (X, τ) is an IF πgγ* connected space,
(ii) (X, τ) is an IFGO – connected space,
(iii) (X, τ) is an IFC_5- connected space.

Proof: (i) ⇒ (ii) is obvious from the Proposition 6.2.4.
(ii) ⇒ (iii) is obvious.
(iii) ⇒ (i) Let (X, τ) be an intuitionistic fuzzy C_5- connected space. Suppose (X, τ) is not an IF πgγ* connected space, then there exists a proper IFS A in (X, τ) which is both an IF πgγ* OS and an IF πgγ* CS in (X, τ). But since (X, τ) is an IF πgγ* $T_{1/2}$ space, A is both an IFOS and an IFCS in (X, τ). This implies that (X, τ) is not an IFC_5-connected, which is a contradiction to our hypothesis. Therefore (X, τ) must be an IF πgγ* connected space.

Proposition 5.8: If f: (X, τ) → (Y, σ) is an IF πgγ* continuous mapping and (X, τ) is an IF πgγ* connected space, then (Y, σ) is an IFC_5- connected space.

Proof: Let (X, τ) be an IF πgγ* connected space. Suppose (Y, σ) is not an IFC_5-connected space, then there exists a proper IFS A which is both an IFOS and an IFCS in (Y, σ). Since f is an IF πgγ* continuous mapping, $f^{-1}(A)$ is both an IF πgγ* OS and an IF πgγ* CS in (X, τ). But it is a contradiction to our hypothesis. Hence (Y, σ) must be an IFC_5- connected space.

Proposition 5.9: If f: (X, τ) → (Y, σ) is an IF πgγ* irresolute surjection mapping and (X, τ) is an IF πgγ* connected space, then (Y, σ) is an IF πgγ* connected space.

Proof: Suppose (Y, σ) is not an IF πgγ* connected space, then there exists a proper IFS A such that A is both an IF πgγ* OS and an IF πgγ* CS in (Y, σ). Since f is an IF πgγ* irresolute mapping, $f^{-1}(A)$ is both an IF πgγ* OS and an IF πgγ* CS in (X, τ). But this is a contradiction to our hypothesis. Hence (Y, σ) must be an IF πgγ* connected space.

Definition 5.10: An IFTS (X, τ) is an IF πgγ* connected between two IFSs A and B if there is no IF πgγ* OS E in (X, τ) such that A ⊆ E and $E\,\bar{q}\,B$.

Proposition 5.11: An IFTS (X, τ) is IF πgγ* connected between two IFSs A and B if and only if there is no IF πgγ* OS and IF πgγ* CS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Proof: Necessity: Let (X, τ) be IF πgγ* connected between two IFSs A and B. Suppose that there exists an IF πgγ* OS and IF πgγ* CS E in (X, τ) such that $A \subseteq E \subseteq B^c$, then $E\,\bar{q}\,B$ and A ⊆E. This implies (X, τ) is not IF πgγ* connected between A and B, by a contradiction to our hypothesis. Therefore there is no IF πgγ* OS and an IF πgγ* CS E in (X, τ) such that $A \subseteq E \subseteq B^c$.

Sufficiency: Suppose that (X, τ) is not IF πgγ* connected between A and B. Then there exists an IF πgγ* OS E in (X, τ) such that A ⊆ E and $E\,\bar{q}\,B$. This implies that there is no IF πgγ* OS E in (X, τ) such that $A \subseteq E \subseteq B^c$. But this is a contradiction to our hypothesis. Hence (X, τ) is IF πgγ* connected between A and B.

Proposition 5.12: If an IFTS (X, τ) is IF πgγ* connected between A and B and $A \subseteq A_1$, $B \subseteq B_1$, then (X, τ) is an IF πgγ* connected between A_1 and B_1.

Proof: Suppose that (X, τ) is not IF πgγ* connected between A_1 and B_1, then by Definition of 'IF πgγ* connected between two IFSs A and B', there exists an IF πgγ* OS E in (X, τ) such that $A_1 \subseteq E$ and $E\,\bar{q}\,B_1$. This implies $E \subseteq B_1^c$ and $A_1 \subseteq E$. That is $A \subseteq A_1 \subseteq E$. Hence A ⊆ E. Since $E \subseteq B_1^c$, $B_1 \subseteq E^c$, that is $B \subseteq B_1 \subseteq E^c$. Hence $E \subseteq B^c$. Therefore (X, τ) is not IF πgγ* connected between A and B, which is a contradiction to our hypothesis. Hence (X, τ) must be IF πgγ* connected between A_1 and B_1.

Proposition 5.13: Let (X, τ) be an IFTS and A and B be IFSs in (X, τ). If $A\,{}_{q}\,B$, then (X, τ) is IF πgγ* connected between A and B.

Proof: Suppose (X, τ) is not an IF πgγ* connected between A and B. Then there exists an IF πgγ* OS E in (X, τ) such that A ⊆ E and $E \subseteq B^c$. This implies that $A \subseteq B^c$. That is $A\,\bar{q}\,B$. But this is a contradiction to our hypothesis. Hence (X, τ) must be IF πgγ* connected between A and B.

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