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Research Article

# CYCLE RELATED GRAPHS - NEAR MEAN CORDIAL

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## ABSTRACT

Let G = (V,E) be a simple graph. A **Near Mean Cordial Labeling** of G is a function in f : $V(G) \rightarrow \{1, 2, 3, \ldots, p-1, p+1\}$ such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & \text{if}\big(f(u) + f(v)\big) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.

In this paper, It is to be proved that , Tortoise graph $T_n$ and Snail graph $S_n$are **Near Mean Cordial** graphs.

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***Keywords and Phrases:*** Cordial labeling, Near Mean Cordial Labeling and Near Mean Cordial Graph.

## I. INTRODUCTION

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology ,we referred Harary [4].For labeling of graphs, we referred Gallian[1]. A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge uv a label depending on the vertex labels of u and v.

A graph G is said to labeled if the n vertices are distinguished from one another by symbols such as $v_1, v_2, \ldots\ldots.. v_n$. In a labeling of a particular type, the vertices are assigned distinct values from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2]. In this paper, It is to be proved that Tortoise graph $T_n$ and Snail graph $S_n$ are **Near Mean Cordial** graphs.

## II.PRELIMINARIES

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**Definition 2.1** :

Let G= (V,E) be a simple graph. Let f:V(G) $\rightarrow \{0,1\}$ and the induced edge label,assigning $|f(u) - f(v)|$ is called a **Cordial Labeling** if the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the number of edges labeled 1differ by atmost 1 .A graph is called **Cordial** if it has a cordial labeling.

**Definition 2.2:**

Let G = (V,E) be a simple graph. G is said to be a **Mean Cordial Graph** if f:V(G)$\rightarrow$\{0,1,2\} such that for each edge uv the induced map f*defined by f*(uv) =$\left\lfloor\frac{f(u)+f(v)}{2}\right\rfloor$ where $\lfloor x\rfloor$ denote the least integer which is $\leq$ x and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with zero label. $e_f(1)$isthe number of edges with one label.

**Definition 2.3:**

Let G = (V,E) be a simple graph. A **Near Mean Cordial Labeling**of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1} such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & \text{if}\big(f(u)+f(v)\big) \equiv 0 \ (\text{mod}2) \\ 0 & \text{else} \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.

**Definition 2.4:**

A **snail** $\mathbf{S_n}$ $(n \geq 4)$is obtained from path $P_n = v_1, v_2, \ldots, v_n$by attaching two parallel edges between $v_i$ and $v_{n-i+1}$ for $i = 1,2, \ldots, \left\lfloor\frac{n}{2}\right\rfloor$

**Definition 2.5:**

A **tortoise** $T_n$ $(n > 4)$ is obtained from path $P_n = v_1, v_2, \ldots, v_n$by attaching one edges between $v_i$ and $v_{n-i+1}$ for $i = 1,2, \ldots, \left\lfloor\frac{n-1}{2}\right\rfloor$

## III. MAIN RESULTS

**Theorem 3.1**: Snail graph ($S_n$) is a Near Mean Cordial Graph.

**Proof:**

Let V(G) = $\{u_i : 1 \leq i \leq n\}$

Let E(G) = {{($u_i$ $u_{i+1}$): $1 \le i \le n-1$} U{2($u_i$ $u_{n-i+1}$) : $1 \le i \le \frac{n}{2}$ }

Definef : V(G) → {1,2,3, …., n-1, n+1} by

Let $f(u_i) = 2i-1$ , $1 \le i \le \frac{n}{2}$

$f(u_n) = n+1$

$f(u_{n/2+i}) = 2i$ , $1 \le i \le \frac{n-2}{2}$

The induced edge labelings are,

$f^*(u_i u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$ , $1 \le i \le n-1$

$2f^*(u_i u_{n-i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{n-i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$ ,$1 \le i \le \frac{n}{2}$

**Edge condition:-**

Here, $e_f(0) = n$ and $e_f(1) = n-1$

So, in all the cases, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $S_n$ is a Near Mean Cordial graph.

For example, the Near Mean cordial labeling of $S_8$ is shown in Figure 3.1.1.

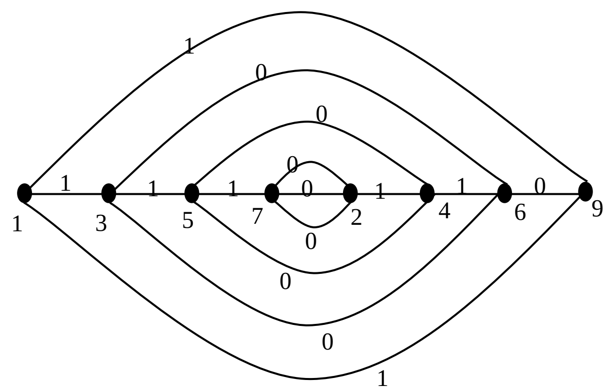


*Figure 3.1.1*

**Theorem 3.2**: Tortoise graph ($T_n$) is a Near Mean Cordial Graph.

**Proof:**

Let V(G) = { $u_i$: $1 \le i \le n$ }

Let E(G) = {{($u_i$ $u_{i+1}$): $1 \le i \le n-1$} U{ ($u_i$ $u_{n-i+1}$) : $1 \le i \le \frac{n-1}{2}$ }

Define f : V(G) → {1,2,3, …., n-1, n+1} by

Let $f(u_{2i-1}) = i$ , $1 \le i \le \frac{n-1}{2}$

$f(u_n) = n+1$

$f(u_{2i}) = n-2+(i-1)$ , $1 \leq i \leq \frac{n-1}{2}$

The induced edge labelings are,

$f^*(u_i u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$ , $1 \leq i \leq n-1$

$f^*(u_i u_{n-i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{n-i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$ , $1 \leq i \leq \frac{n-1}{2}$

**Edge condition:-**

(i) If $n \equiv 1 \pmod 4$

Here, $e_f(0) = e_f(1) = \frac{3n-3}{4}$

(ii) If $n \equiv 3 \pmod 4$

Here, $e_f(0) = \frac{3n-1}{4}$ and $e_f(1) = \frac{3n-5}{4}$

So, in all the cases, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, $T_n$ is a Near Mean Cordial graph.

For example, the Near Mean cordial labeling of $T_7$ and $T_9$ are shown in Figure 3.2.1.and 3.2.2.

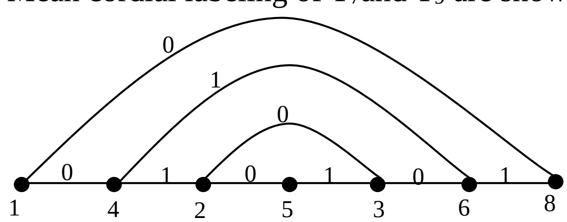


*Figure 3.2.1.*

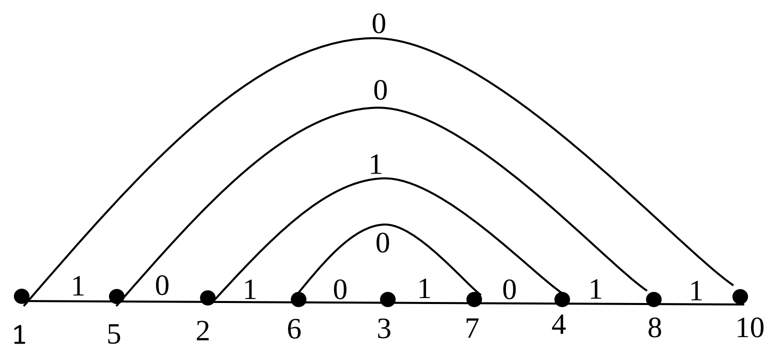


*Figure 3.2.2.*

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