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Research Article

Case Study for Workshop Queuing Model

Dr. Vipin Kumar Solanki
Department of Mathematics
Sanskriti University
Mathura
solankivipin11@gmail.com

Dr. Saloni Srivastava
Department of Mathematics
RBSETC
Bichpuri, Agra
salonisaloni17@gmail.com

Ms. Shivali Shrivastava
Department of Mathematics
Sanskriti University
Mathura
shivali.mini11@gmail.com

Abstract
Workshops would avoid losing their customers due to a long wait on the line. Some workshops initially provide more waiting areas than they actually need to put them in the safe side, and reducing the areas as the time goes on safe space. However, waiting areas alone would not solve a problem when customers withdraw and go to the competitor's door; the service time may need to be improved. This shows a need of a numerical model for the worshop management to understand the situation better. This paper aims to show that queuing theory satisfies the model when tested with a real-case scenario. We obtained the data from a workshop in Agra. We then derive the arrival rate, service rate, utilization rate, waiting time in queue and the probability of
potential customers to balk based on the data using Little's Theorem and M/M/1 queuing model. The arrival rate at Ford workshop during its busiest period of the day is 2.22 customers per minute (cpm) while the service rate is 2.24 cpm. The average number of customers in the workshop is 122 and the utilization period is 0.991. We conclude the paper by discussing the benefits of performing queuing analysis to a busy workshop.

Keywords: Queue; Little's Theorem; Workshop; Waiting Lines

INTRODUCTION

There are several determining factors for a workshop to be considered a good or a bad one. Service, paintwork repairs to scratches, scuffs dents to vehicle damage the workshop layout and settings are some of the most important factors. These factors, when managed carefully, will be able to attract plenty of customers. However, there is also another factor that needs to be considered especially when the workshop has already succeeded in attracting customers. This factor is the customers queuing time.

Queuing theory is the study of queue or waiting lines. Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average time in the system, the expected queue length, the expected number of customers served at one time, the probability of balking customers, as well as the probability of the system to be in certain states, such as empty or full.

Waiting lines are a common sight in workshop especially in noon and evening time. Hence, queuing theory is suitable to be applied in a workshop setting since it has an associated queue or waiting line where customers who cannot be served immediately have to queue (wait) for service. Researchers have previously used queuing theory to model the workshop operation reduce cycle time in a busy workshop as well as to increase throughput and efficiency. This paper uses queuing theory to study the waiting lines in Ford workshop in Agra. The workshop provides three workers per customer. There are 8 to 9 worker working at any one time. on a daily basis, it serves over 400 customers during weekdays, and over 1000 customers during weekends. This paper seeks to illustrate the usefulness of applying queuing theory in a real case simulation.

QUEUING THEORY

In 1908, Copenhagen Telephone Company requested Agner K. Erlang to work on the holdingtimes in a telephone switch. He identified that the number of telephone conversations and telephone holding time fit into Poisson distribution and exponentially distributed. This was the beginning of the study of queuing theory. In this section, we will discuss two common concepts in queuing theory.

LITTLE THEOREM

Little's theorem describes the relationship between throughput rate (i.e. arrival and service rate), cycle time and work in process (i.e. number of customers/jobs in the system). This relationship has been shown to be valid for a wide class of queuing models. The theorem states that the expected number of customers (N) for a system in steady state can be determined using the following equation:

$$L = \lambda T \qquad (1)$$

Here, λ is the average customer arrival rate and T is the average service time for a customer. Consider the example of a workshop where the customer's arrival rate (λ) doubles but the customers still spend the same amount of time in the workshop (*T*). These facts will double the number of customers in the workshop (*L*). By the same logic, if the customer arrival rate (λ) remains the same but the customers service time doubles this will also double the total number of customers in the workshop. This indicates that in order to control the three variables, managerial decisions are only required for any two of the three variables.
Three fundamental relationships can be derived from Little's theorem:

- L increases if λ or T increases
- λ increases if L increases or T decreases
- T increases if L increases or λ decreases

Rust said that the Little's theorem can be useful in quantifying the maximum achievable operational improvements and also to estimate the performance change when the system is modified

QUEUING MODELS AND KENDALL'S NOTATION

In most cases, queuing models can be characterized by the following factors:

- *Arrival time distribution:* Inter-arrival times most commonly fall into one of the following distribution patterns: a Poisson distribution, a Deterministic distribution, or a General distribution. However, inter-arrival times are most often assumed to be independent and memory less, which is the attributes of a Poisson distribution.
- *Service time distribution*: The service time distribution can be constant, exponential, hyper-exponential, hypo-exponential or general. The service time is independent of the inter-arrival time.
- *Number of servers:* The queuing calculations change depends on whether there is a single server or multiple servers for the queue. A single server queue has one server for the queue. This is the

situation normally found in a grocery store where there is a line for each cashier. A multiple server queue corresponds to the situation in a bank in which a single line waits for the first of several tellers to become available.

- *Queue Lengths (optional):* The queue in a system can be modeled as having infinite or finite queue length.
- *System capacity (optional):* The maximum number of customers in a system can be from 1 up to infinity. This includes the customers waiting in the queue.
- *Queuing discipline (optional):* There are several possibilities in terms of the sequence of customers to be served such as FIFO (First In First Out, i.e. in order of arrival), random order, LIFO (Last In First Out, i.e. the last one to come will be the first to be served), or priorities.

Kendall, in 1953, proposed a notation system to represent the six characteristics discussed above. The notation of a queue is written as:

A/B/P/Q/R/Z

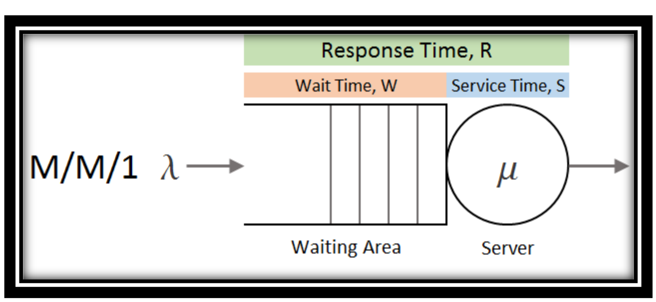
where A, B, P, Q, R and Z describe the queuing system properties.

- A describes the distribution type of the inter arrival times.
- B describes the distribution type of the service times.
- P describes the number of servers in the system
- Q (optional) describes the maximum length of the queue.
- R (optional) describes the size of the system population
- Z(optional) describes the queuing discipline.

FORD WORKSHOP QUEUING MODEL

The data were obtained through interview with the workshop manager as well as data collections through observations at the workshop. The daily number of visitors was obtained from the itself. The workshop has been recording the data as part of its end of day routine. We also interviewed the workshop manager to find out about the capacity of the workshop, the number of workers as well as the number of services in the workshop. Based on the interview with the workshop manager, we concluded that the queuing model that best illustrate the operation of Ford workshop M/M/1. This means that the arrival and service time are exponentially distributed (Poisson process). The workshop system consists of only one server. In our observation the workshop has several workers but in the actual waiting queue, they only have one manager to serve all of the customers.

Figure 1 illustrates the M/M/1 queuing model.



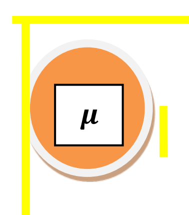


Figure 1. M/M/1 Queuing Model

For the analysis of the Ford workshop M/M/1 queuing model, the following variables will be investigated

[6]:

- λ : The mean customers arrival rate
- μ : The mean service rate
- ρ : λ/μ : utilization factor
- probability of zero customers in the workshop
  $P_0 = 1 - \rho$ (2)
- P_n : The probability of having n customers in the workshop.
  $P_n = P_0 \rho^n = (1-\rho)\rho$ (3)
- L: average number of customers dining in the workshop
  $L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$ (4)
- L_q: average number in the queue.

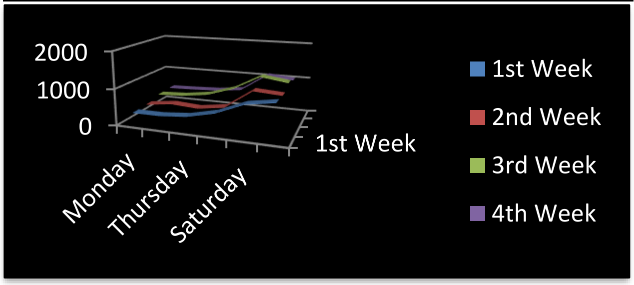
  $L_q = L \times \rho = \frac{\rho^2}{1-\rho} = \frac{\rho\lambda}{\mu-\lambda}$ (5)

- W: average time spent in Ford workshop, including the waiting time.
  $W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda}$ (6)
- Wq : average waiting time in the queue.
  $W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu-\lambda}$ (7)

RESULT AND DISCUSSION

The one month daily customer data were shared by the workshop as shown in Table

	Mon	Wed	Thurs	Fri	Sat	Sun
1st Week	327	329	392	563	873	992
2nd Week	345	436	389	497	1015	966
3rd Week	441	477	579	818	1219	1112
4th Week	459	481	513	597	1088	1013



MONTHLY CUSTOMER COUNTS

As can be seen in Figure 2, the number of customers on Saturdays and Sundays are double the number of customers during weekdays. The busiest period for the workshop on weekend is found to be during morning time. Hence, we will focus our analysis in this time window

CALCULATION

Our teams conducted the research at morning time. There are on average 300 people are coming to the workshop in 135 minutes window of morning time. From this we can derive the arrival rate as

$\lambda = \frac{3\,00}{135} = 2.22$ customers/minute (cpm).

We also found out from observation and discussion with manager that each customer spends 55 minutes on average in the workshop, the queue length is around 36 people (L_q) on average and the waiting time is around 15 minutes.

It can be shown using (7) that the observed actual waiting time does not differ by much when compared to the theoretical waiting time as shown below.

$$W_q = \frac{36\ customers}{2.22\ cpm} = 16.22\ minutes$$

Next, we will calculate the average number of people in the workshop using (1)

$$L = 2.22\ cpm\ \times 55\ mins = 122.1\ customers$$

Having calculated the average number of customers in the restaurant, we can also derive the utilization rate and the service rate and the service rate using (4)

$$\mu = \frac{\lambda(1+L)}{L} = \frac{2.22(1+122.1)}{122.1} = 2.24\ cpm$$

very small as can be derived using (2)

Hence $\quad P_0 = 1 - \rho = 1 - 0.991 = 0.009$

The generic formula that can be used to calculate the probability of having n customers in the worshop is as follows:

$$P_n = P_0\rho^n = (1-\rho)\rho^n = (1-0.991)(0.991)^n = 0.019(0.991)^n$$

We assume that potential customers will start to balk when they see more than 10 people are already queuing for the workshop. We also assume that the maximum queue length that a potential customer can tolerate is 40 people. As the capacity of the workshop when fully occupied is 120 people, we can calculate the probability of 10 people in the queue as the probability when there are 130 people in the system (i.e. 120 in the workshop and 10 or more queuing). Probability of customers going away = P (more than 15 people in the queue) (more than 130) people in the workshop

$$P_{131-160} = \sum_{n=131}^{160} P_n = 0.1534 = 15.34\%$$

EVALUATION

- The utilization is directly proportional with the mean number of customers. It means that the mean number of customers will increase as the utilization increases.
- The utilization rate at the workshop is very high at 0.991. This, however, is only the utilization rate morning and evening time on Saturdays and Sundays. On weekday, the utilization rate is almost half of it. This is because the number of visitors on weekdays is only half of the number of visitors on weekends. In addition, the number of workers remains the same regardless whether it is peak hours or off-peak hours.
- In case the customers waiting time is lower or in other words we waited for less than 15 minutes, the

number of customers that are able to be served per minute will increase. When the service rate is higher the utilization will be lower, which makes the probability of the customers going away decreases.

BENIFITS

- This research can help Ford workshop to increase their QOS (Quality Of Service), by anticipating if there are many customers in the queue.
- The result of this paper work may become the reference to analyze the current system and improve the next system. Because the workshop can now estimate of how many customers will wait in the queue and the number of customers that will go away each day.
- By anticipating the huge number of customers coming and going in a day, the workshop can set a target profit that should be achieved daily.
- The formulas that were used during the completion of th0e research is applicable for future research and also could be used to develop more complex theories.
- The formulas provide mechanism to model the workshop queue that is simpler than the creation of simulation model in [9,4].

CONCLUSION

This research paper has discussed the application of queuing theory of Ford workshop. ++From the result we have obtained that the rate at which customers arrive in the queuing system is 2.22 customers per minute and the service rate is 2.24 customers per minute. The probability of buffer flow if there are 10 or more customers in the queue is 15 out of 100 potential customers. The probability of buffer overflow is the probability that customers will run away, because may be they are impatient to wait in the queue. This theory is also applicable for the workshop if they want to calculate all the data daily. It can be concluded that the arrival rate will be lesser and the service rate will be greater if it is on weekdays since the average number of customers is less as compared to those on weekends. The constraints that were faced for the completion of this research were the inaccuracy of result since some of the data that we use was just based on assumption or approximation. We hope that this research can contribute to the betterment of Ford workshop in terms of its way of dealing with customers.

As our future works, we will develop a simulation model for the workshop. By developing a simulation model we will be able to confirm the results of the analytical model that we develop in this paper. In addition, a simulation model allows us to add more complexity so that the model can mirror the actual operation of the workshop more closely.

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