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Research Article

Fuzzy Inventory Modeling for Deteriorating Items with Varying Holding Cost Rate and Partially Backlogged Shortages

Paramjit Kaur[1],Daljeet kaur[1] , Madhuchanda Rakshit[1], Vinod kumar[2]

Abstract

This paper considers a two warehouse inventory model for deteriorating items with backorder in a fuzzy situation by employing triangular fuzzy number. A fully fuzzy model is developed where input cost parameters are fuzzified. Inventory cost(including holding cost and deterioration cost)in a rented warehouse is higher than cost in owned warehouse due to better preservation facilities in rented warehouse. The demand and holding cost, both are taken as linear function of cycle length. Shortages are allowed in the own warehouse only and a fraction of shortages inventory is backlogged during the next replenishment cycle .This paper mainly dealt with deteriorating items with time dependent demand and variable holding cost which is constant up to a fixed point of cycle length and after that it increases according to length of ordering cycle in rented warehouse only and remains constant owned warehouse. Transportation cost is taken to be negligible and goods are transported on the basis of bulk release pattern. A numerical example is presented to illustrate the model and sensitivity is performed for a parameter keeping rest unchanged.

Keywords

Two warehouses, Instantaneous deterioration, Time-dependent Demand, Variable holding cost and shortages, fuzzy cost parameters.

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[1] Department of Mathematics, Guru Kashi University, Talwandi Sabo, Bathinda, Punjab, India.
[2] Department of Mathematics, NIET,NIMS University ,Jaipur Rajasthan ,India.
Email addresses: alishadatra@gmail.com (Paramjit Kaur), daljitk053@gmail.com (Daljeet Kaur), drmrakshit@gmail.com (Madhuchanda Rakshit), vinod.k4bais@gmail.com (Vinod kumar)

Introduction

The classical inventories models are basically developed with the single ware house system .In the past, researchers have established a lot of research in the field of Inventory management and Inventory control system. Inventory management and control system basically deals with demand and supply chain problems and for this, production units(Producer of finished goods), vendors, suppliers and retailers need to store the raw materials, finished goods for future demand and to supplying the market and to the customers. In the traditional models it is assumed that the demand and holding cost are constant and goods are supplied instantly under infinite replenishment policy but as time passed away many researchers considered that demand may vary with time, due to uncertainty in the price and holding cost may vary with time due to better facilities provided by the owner of rented warehouse to minimize deterioration cost. Many models have been developed considering various time dependent demand with shortages and without shortage. All those models that consider demand variation in response to inventory level, assumes that the holding cost is constant for the entire inventory cycle. In the studies of inventory models, unlimited warehouse capacity is often assumed. However, in busy marketplaces, such as super markets, corporation markets etc. the storage area for items may be limited. Another case, of inadequate storage area, can occur when a procurement of a large amount of items is decided. That could be due to, an attractive price discount for bulk purchase which is available or, when the cost of procuring goods is higher than the other inventory related costs or, when demand for items is very high or, when the item under consideration is a seasonal product such as the yield of a harvest or, when there are some problems in frequent procurement. In these cases, these items cannot be accommodated in the existing store house (the own warehouse, abbreviated as OW). Hence, in order to store the excess items, an additional warehouse (the rented warehouse, abbreviated as RW),which may be located at a short distance from the OW or a little away from it, due to non-availability of ware-house nearby, is hired on a rental basis. Hartely [1976]discussed an inventory model with two storage facilities**.** Ghare and Schrader [1963] initially worked in this field and they extended Harris [1915]EOQ model with deterioration and shortages**.** Goyal and Giri [2001] gave a survey on recent trends in the inventory modelling of deteriorating items. Lee and Wu [2004] developed a note on EOQ model for items with mixtures of exponential distribution deterioration, shortages and time varying demand. It is generally assumed, that the holding cost in the RW is higher than that in the OW, due to the additional cost of maintenance, material handling, etc. To reduce the inventory costs, it will be cost-effective to consume the goods of the RW at the earliest.

Deterioration is a continuous phenomenon in which product losses its originality and after expiry of its life time period, it remains unusable. Assuming the deterioration in the both warehouses**,** Sarma (1987), extended his earlier model to the case of infinite replenishment rate with shortages. Pakkala and Achary (1992) extended the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages, taking time as discrete and continuous variable, respectively. In these models mentioned above the demand rate was assumed to be constant. Subsequently, the ideas of time varying demand and stock dependent demand considered by some authors, such as Goswami and Chaudhary (1998), Bhunia and Maiti(1998), Bankerouf (1997), Kar et al. (2001) and others. Bhunia and Maiti (1994) extended the model of Goswami and Chaudhary (1972), in that model they were not consider the deterioration and shortages were allowed and backlogged. Goel and Giri[2001] suggested a review of deteriorating inventory literature in which all the inventory models for deteriorating items assume that the deterioration occurs as soon as the retailer receive the commodities.

Holding cost is key factor of an inventory cost and has drawn attention of researchers. In traditional models holding cost is considered to be constant per item per unit of time but in reality holding cost will not remain always constant and thus it may vary with time. Chang (2004) developed an inventory models with stock dependent demand and nonlinear holding costs for deteriorating items. Ajanta Roy [2008] developed an inventory model for deteriorating items with time varying holding cost and price dependent demand. C.Sugapriya and K.Jeyaraman[2008] developed an EPQ model for deteriorating items in which holding cost varies with time. Maya Gyan and A.K.Pal[2009] developed a two ware house inventory model for deteriorating items with stock dependent demand rate and holding cost.Bindu Vaish and Garima Garg[2011] consider variable holding cost for development of Optimal Ordering and Transfer Policy for an Inventory System. Mukesh Kumar et.al. [2012] developed A Deterministic Inventory Model for Deteriorating Items with Price Dependent Demand and Time Varying Holding Cost under Trade credits .Yadav A.S. andSwami A.[2013] developed a two ware house inventory model for deteriorating items with exponential demand and variable holding cost. K.D.Rathor and P.H Bhathawala[2013]constructed a model with variable holding cost and inventory level dependent demand. R.P.Tripathi[2013] developed an Inventory model for varying demand and variable holding cost. Vinod Kumar Mishra et.al.[2013] developed an inventory model with variable holding cost and salvage value. Vipin Kumar et. el. [2013]developed an inventory model with selling price dependent demand and variable holding cost.Dr. Meghna Tyagi and Dr. S. R. Singh [2013] developed two-ware-house inventory model with time dependent demand and variable holding cost.

All the research papers discussed above are crisp inventory model in which parameters consider crisp value. In the present scenario of market and under uncertain situation decision variables affecting inventory cost cannot be considered to be fixed. To deal with uncertain situation Zadeh A.[1965] introduced fuzziness system which described the vague nature of situation. Zadeh et. al.[1970] proposed some strategies for decision making in fuzzy environment. Kacpryzk et. al. [1982] discussed some long term inventory policy making through fuzzy decision making model. Various inventory models are considered by researchers in last two three decays considering fuzziness but with single storage facility.

A.Kaufmanm et. al.[1985] wrote a book titled introduction to fuzzy arithmetic: theory and applications. A few researchers are developing inventory model to deal with fuzziness considering two warehouse storage facilities such as Debdulal Panda et.al.[2014] developed a fuzzy mixture two ware house inventory model involving fuzzy random variable lead tie demand and fuzzy total demand considering fixed penalty cost and lead time as decision variable. Wasim et. al.[2015] considered power demand with fuzzy pentagonal number and defuzzified the system using graded mean integration method. Recently Indrajeet singha et. al. [2109] developed a fuzzy two warehouse model for single deteriorating item with selling price dependent demand rate under partially backlogged condition for shortages and solved defuzzified the fuzziness using signed distance and centroid method for fuzzy triplets i.e. fuzzy triangular numbers. Although above discussed models provide some general understanding of the behaviour of inventory under different assumptions, they are not adept of representing real life situations. Further, using these models require inventory managers to have some flexibility when deciding parameters values when modelling the inventory system to reduce the cost of uncertainty. Hence applying fuzzy set theory to solve inventory problems, instead of traditional probability theory, more accurate results can be obtained.

In the present paper a fuzzy inventory model for deteriorating items with two level of storage system and time dependent demand with partial backlogged shortages is developed. Stock is transferred RW to OW under bulk release pattern. The deterioration rates in both the warehouses are constant but different due to the different preservation procedures. Holding cost is considered to be constant up to a definite time and is increases with respect to time of cycle length. The numerical example is presented to demonstrate the development of model and to validate it. Sensitivity analysis is performed separately for each parameter in case of crisp model and in the same way sensitivity of the fuzzy model may be analysed.

Definition and Preliminaries

For the development of fuzzy inventory model we need the following definitions:

1. A fuzzy set $\tilde{S}$ on a given universal set X is denoted and defined by

$$\{(x, \lambda_{\tilde{S}}(x)) : x \,\varepsilon\, X\}$$

Where $\lambda_{\tilde{S}}$: X→ [0,1], is called the membership function and $\lambda_{\tilde{S}}$(x) = degree of x in $\tilde{S}$.

2. A triangular fuzzy number is specified by the triplet (a, b, c) where $a < b < c$ and defined by its continuous membership function $\lambda_{\tilde{S}}$: X→ [0,1]as follows:

$$\lambda_{\tilde{S}}(x) = \begin{cases} \frac{x-a_1}{c-a_1} & if\ a_1 \le x \le b_1 \\ \frac{b_1-x}{c_1-b_1} & if\ b_1 \le x \le c_1 \\ 0 & otherwise \end{cases}$$

3. Let $\tilde{S}$ be the fuzzy set defined on the R (set of real numbers), then the signed distance of $\tilde{S}$ is defined as

$$D\,(\tilde{S},0) = \frac{1}{2}\int_0^1 [S_L(\alpha) + S_R(\alpha)] d_\alpha$$

Where $A_\alpha = [S_L(\alpha) + S_R(\alpha)] a\ \varepsilon\ [0,1]$ is a a-cut of a fuzzy set $\tilde{S}$.

Notations and Assumption

Mathematical model of two warehouse inventory system for deteriorating items is based on the following notation and assumptions:

Notations:

A: Ordering cost per order.

W:Storage capacity of OW.

R:Storage capacity of RW.

T: Cycle length for repenishment.

Q: Maximum Inventory level per cycle to be ordered.

μ: The time up to which inventory vanishes in RW.

ν: The time at which inventory level reaches to zero in OW and shortages begins.

λ: Definite time up to which holding cost is constant in RW.

h_w: Holding cost rate per unit time in OW.

h_r: Holding cost rate per unit time in RW.

C_s: Shortages cost rate per unit per unit time.

C_L: Opportunity cost rate per unit per time.

$I_r(t)$: Inventory level in RW at epoch t.

$I_i(t)$: The Inventory level in OW at epoch t in different time interval, where $i = 1,2$.

$I_s(t)$: Determine the inventory level at time t in which the product has shortages.

α:Deterioration rate in RW Such that$0 < \alpha < 1$;

β: Deterioration rate in OW such that $0 < \beta < 1$;

C_p: Purchase cost per unit of items.

B_I: Amount of shortages inventory backlogged.

L_I: Amount of inventory lost as fraction of shortages is backlogged.

PC: Total present worth cost of purchase.

SC: Total present worth cost of shortages.

LC: Total present worth cost of lost sale.

HC: Total present worth cost of holding inventory in both ware houses.

$IC(\mu\ T)$: The total relevant inventory cost per unit time of inventory system.

$\tilde{h}_w$: Fuzzy holding cost per unit time in OW.

$\tilde{h}_r$: Fuzzy holding cost per unit time in RW.

$\tilde{C}_s$: Fuzzy shortages cost per unit per unit time.

$\tilde{C}_L$: Fuzzy opportunity cost per unit per time

$\tilde{C}_p$: Fuzzy Purchase cost per unit of items.

$\tilde{SC}$: The fuzzy present worth cost of shortages

$\tilde{L}C$: Fuzzy present worth cost of lost sale

$\tilde{H}C$: Fuzzy present worth cost of holding inventory

$\tilde{T}C(\mu\ T)$: Fuzzy total relevant inventory cost per unit time of inventory system.

Assumption

1. Replenishment rate is infinite and lead time is negligible.
2. The time horizon is infinite.
3. Demands are fulfilled from RW first and thereafter from OW to reduce holding cost in RW.
4. OW has limited capacity of storage and RW unlimited.
5. Demand is function of time and varies linearly, represented as
   $f(t) = \begin{cases} a\text{ if} & t = 0 \\ a + bt \text{ if} & t > 0 \end{cases}$; Where $a > 0$ and $b > 0$;
6. A fraction of stocked inventory deteriorates per unit time in both the warehouse with different constant rate of deterioration.
7. Shortages are allowed and a fraction of demand backlogged and supplied to customers at the beginning of next replenishment.
8. The unit inventory cost (Holding cost) in RW>OW.
9. Holding cost is fixed till a definite point of cycle length in RW and increases according to a fraction of ordering cycle length. So for holding cost (h_i),λ a time moment up to which holding cost is constant.
   a. $h_r = \begin{cases} h_r & \text{if } t \leq \lambda \\ h_r t & \text{if } t > \lambda \end{cases}$ $\left. in\ RW \quad and\ h_w\ in\ OW\ through\ cycle\ length \right\}$

Mathematical formulation of model and analysis

In the beginning of business, at $t = 0$ a lot size of Q units of inventory enters into the system in which $(Q - B_I)$backlogged units are cleared and remaining units is kept into two storage as W units in OW and R units in RW.(See Figure-1)

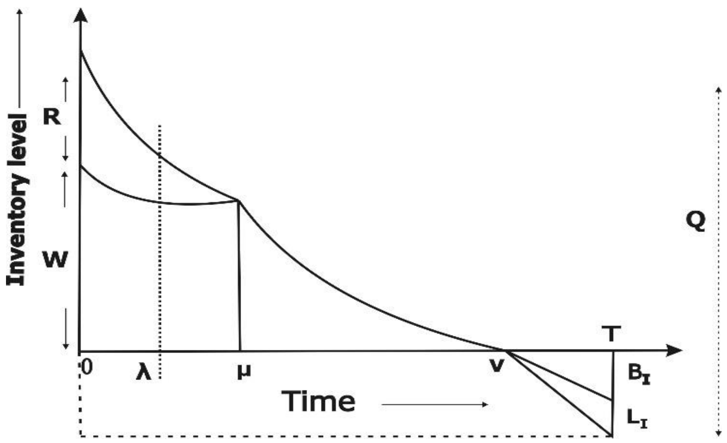

Figure-1: Representing inventory levels in warehouses

During the time interval $[0,\ \mu]$ the inventory in RW decrease due to the demand and deterioration and is governed by the following differential equation:

$$dI_r(t)/dt = -(a\ + bt) - \alpha I_r(t); \qquad 0 \leq t \leq \mu \quad (1)$$

In the time interval $[0 \quad \mu]$the inventory level decreases in OW decreases due to deterioration only and is governed by differential equation

$$dI_1(t)/dt = -\beta I_1(t); \qquad 0 \le t \le \mu \quad (2)$$

During time interval $[\mu \ \ \nu]$the inventory level in OW is decreases due to demand and deterioration both and is governed by the following differential equation

$$dI_2(t)/dt = -(a + bt) - \beta I_2(t); \quad \mu \le t \le \nu (3)$$

Now at $t = \nu$the inventory level vanishes and the shortages occurs in the interval $[\nu, \ T]$a fraction f of the total shortages is backlogged and the shortages quantity supplied to the customers at the beginning of the next replenishment cycle. The shortages is governed by the differential equation

$$dI_s(t)/dt = -f_1(a + bt); \qquad \nu \le t \le T \qquad (4)$$

Now inventory level at different time intervals is given by solving the above differential equations (1) to (4) under boundary conditions

$I_r(\mu) = 0;I_1(0) = W; I_2(\nu) = 0;I_s(\nu) = 0;$

$$I_r(t) = \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp\{\alpha(\mu - t)\} - \{\frac{a}{\alpha} + \frac{b}{\alpha^2}\alpha\, t - 1)\}; \qquad (5)$$

$$I_1(t) = W \exp\{\alpha(\mu - t)\} \quad ; \qquad (6)$$

$$I_2(t) = \{\frac{a}{\beta} + \frac{b}{\beta^2}\{(\beta\nu - 1)\}\exp\{\beta(\nu - t)\} - \{\frac{a}{\beta} + \frac{b}{\beta^2}(\beta t - 1)\}; \qquad (7)$$

$$I_s(t) = f_1\{a(\nu - t) + \frac{b}{2}(\nu^2 - t^2\}; \qquad (8)$$

Now at $t = 0$; $I_r(0) = R$ therefore equation (5) yield

$$R = \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp\{-\alpha\mu\}\}; \qquad (9)$$

Amount of inventory backlogged during shortages period$t = T$ is given by

$$B_I = -I_s(t)$$

$$= f_1\{a(T - t) + \frac{b}{2}(T^2 - t^2)\}; \qquad (10)$$

Amount of inventory lost during shortages period

$$L_I = 1 - B_I$$

$$= 1 - f_1\left\{a(T - t) + \frac{b}{2}(T^2 - t^2)\right\} \qquad (11)$$

The maximum Inventory to be ordered is given as

$$Q = W + I_r(0) + B_I$$

$$= W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T - t) + \frac{b}{2}(T^2 - t^2)\}; \qquad (12)$$

Now continuity at $t = \mu$shows that $I_1(\mu) = I_2(\mu)$ therefore from eq. (6) & (7) it is observed that

$$b\beta^2\nu^2 - a\beta^2\nu - (\beta^2(W_1 + Z) + (b - a\beta)) = 0 \qquad (13)$$

Where $Z = \{\frac{a}{\beta} + \frac{b}{\beta^2}(\beta\mu - 1)\}\exp(-\beta\mu)$

Which is quadratic in ν and further can be solved for ν in terms of μ i.e.

$$\nu = fun(\mu) \qquad (14)$$

Therefore $\nu = \frac{-a^2\beta^4 \pm \sqrt{D}}{2b\beta^2}$

Where $D = a^2\beta^4 + 4b\,\beta^2(b - a\beta) + \beta^2\{(W + (\frac{a}{\beta} + \frac{b}{\beta^2})(\beta\mu - 1)\exp(-\beta\mu)\}$

Next, the total relevant inventory cost per cycle includes following costs:

Ordering cost per cycle= A

The present worth purchase cost per cycle = $C_p * Q$

The present worth holding cost= HC

Depending upon the position of λ two cases arises

Case-1: When $\lambda < T$ and $0 \leq \lambda < \mu$ in RW then

$$HC = \int_0^{\lambda} h_r I_r(t)dt + \int_{\lambda}^{\mu} h_r\, t\, I_r(t)dt + \int_0^{\mu} h_w I_1(t)dt + \int_{\mu}^{\nu} h_w\, I_2(t)dt$$

Case-2: When $\lambda > T$

$$HC = \int_0^{\mu} h_r I_r(t)dt + \int_0^{\mu} h_w I_1(t)dt + \int_{\mu}^{\nu} h_w\, I_2(t)dt$$

On simplification holding cost for two cases are obtained as under Holding cost for Case -1

$$HC = h_r(a\mu\lambda + b\mu^2\lambda - \frac{b\lambda^2}{2\alpha} - \frac{b\mu^2}{3\alpha} + a\mu^3 + b\mu^4 - \frac{a\,k}{\alpha} - b\mu k^2 - a\mu\lambda^2 - b\mu^2\lambda^2 + \frac{b\,\mu\lambda^2}{\alpha} + \frac{a\,\lambda^2}{\alpha} + \frac{b\,\lambda^3}{3\alpha})$$
$$+ h_w(W\mu + \frac{b\nu^2}{\beta} - \frac{b\mu\nu}{\beta} + \frac{b\mu^2}{2\beta} - \frac{b\nu^2}{2\beta}) \qquad (15)$$

Holding cost for Case -2

$$HC = h_r(a\mu^2 + b\mu^3 + \frac{b\,\mu^2}{\alpha} - \frac{b\,\mu^2}{2\alpha}) + h_w(W\mu + \frac{b\nu^2}{\beta} - \frac{b\mu\nu}{\beta} + \frac{b\mu^2}{2\beta} - \frac{b\nu^2}{2\beta}) \qquad (16)$$

The present worth of shortages cost

$$CS = C_s f_1(\frac{aT^2}{2} - \frac{a\nu^2}{2} + \frac{bT^3}{6} - \frac{b\nu^3}{6} - a\mu T + a\nu^2 - \frac{b\nu^2\,T}{2} + \frac{b\nu^3}{2}) \qquad (17)$$

The present worth opportunity cost/Lost sale cost

$$CL = C_L\left(1 - f_1\,(\frac{aT^2}{2} - \frac{a\nu^2}{2} + \frac{bT^3}{6} - \frac{b\nu^3}{6} - a\mu T + a\nu^2 - \frac{b\nu^2\,T}{2} + \frac{b\nu^3}{2})\right) \qquad (18)$$

Present worth purchase cost

$$CP = C_p\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T - t) + \frac{b}{2}(T^2 - t^2)\}\} \qquad (19)$$

Therefore Total relevant inventory cost per unit per unit of time is denoted and given as for

Case-1

$$IC(\mu\ T) = \frac{1}{T}[A + CP + CS + CL + HC]$$

$$= \frac{1}{T}[A + C_p\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\} + C_s f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_L\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_r(a\mu\lambda + b\mu^2\lambda - \frac{b\lambda^2}{2\alpha} - \frac{b\mu^2}{3\alpha} + a\mu^3 + b\mu^4 - \frac{a\,k}{\alpha} - b\mu k^2 - a\mu\lambda^2 - b\mu^2\lambda^2 + \frac{b\,\mu\lambda^2}{\alpha} + \frac{a\,\lambda^2}{\alpha} + \frac{b\,\lambda^3}{3\alpha}) + h_w(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})] \quad (20)$$

Case-2

$$IC(\mu\ T) = \frac{1}{T}[A + CP + CS + CL + HC]$$

$$= \frac{1}{T}[A + C_p\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\} + C_s f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_L\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_r(a\mu^2 + b\mu^3 + \frac{b\,\mu^2}{\alpha} - \frac{b\,\mu^2}{2\alpha}) + h_w(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta}) \quad (21)$$

The total relevant inventory cost is minimum if

$$\frac{\partial IC(\mu\ T)}{\partial \mu} = 0 \text{ and } \frac{\partial IC(\mu\ T)}{\partial T} = 0$$

subject to the satisfaction of following:-

$$\left(\frac{\partial^2 IC(\mu\ T)}{\partial \mu^2}\right)\left(\frac{\partial^2 IC(\mu\ T)}{\partial T^2}\right) - \frac{\partial^2 IC(\mu\ T)}{\partial \mu\,\partial T} > 0$$

Fuzzy Model:

In the crisp model developed in the previous section the following parameters are considered to be fuzzy in nature and are represented by triangular fuzzy number.

$$\tilde{A} = (A_1, A_2, A_3)$$

$$\tilde{h}_w = (h_{w1}, h_{w2}, h_{w3})$$

$$\tilde{h}_r = h_{r1}, h_{r2}, h_{r3})$$

$$\tilde{C}_s = (C_{s1}, C_{s2}, C_{s3})$$

$$\tilde{C}_L = (C_{L1}, C_{L2}, C_{L3})$$

$$\tilde{C}_p = (C_{p1}, C_{p2}, C_{p3})$$

Using fuzzy parameters inventory models for two cases of crisp nature are converted into fuzzy model as under:

Case-1:

$$\tilde{I}C(\mu\ T) = \frac{1}{T}\left[(\tilde{I}C_1(\mu\ T)), (\tilde{I}C_2(\mu\ T)), (\tilde{I}C_3(\mu\ T))\right] \quad (22)$$

Where

$$\daleth C_1(\mu\ T) = \frac{1}{T}[A_1+C_{p1}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\}+ C_{s1}f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_{L1}\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_{r1}(a\mu\lambda + b\mu^2\lambda - \frac{b\lambda^2}{2\alpha} - \frac{b\mu^2}{3\alpha} + a\mu^3 + b\mu^4 - \frac{a\,k}{\alpha} - b\mu k^2 - a\mu\lambda^2 - b\mu^2\lambda^2 + \frac{b\,\mu\lambda^2}{\alpha} + \frac{a\,\lambda^2}{\alpha} + \frac{b\,\lambda^3}{3\alpha}) + h_{w1}(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})]$$

$$\daleth C_2(\mu\ T) = \frac{1}{T}[A_2+C_{p2}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\}+ C_2 f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_{L2}\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_{r2}(a\mu\lambda + b\mu^2\lambda - \frac{b\lambda^2}{2\alpha} - \frac{b\mu^2}{3\alpha} + a\mu^3 + b\mu^4 - \frac{a\,k}{\alpha} - b\mu k^2 - a\mu\lambda^2 - b\mu^2\lambda^2 + \frac{b\,\mu\lambda^2}{\alpha} + \frac{a\,\lambda^2}{\alpha} + \frac{b\,\lambda^3}{3\alpha}) + h_{w2}(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})]$$

$$\daleth C_3(\mu\ T) = \frac{1}{T}[A_3+C_{p3}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\}+ C_{s3}f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_{L3}\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_{r3}(a\mu\lambda + b\mu^2\lambda - \frac{b\lambda^2}{2\alpha} - \frac{b\mu^2}{3\alpha} + a\mu^3 + b\mu^4 - \frac{a\,k}{\alpha} - b\mu k^2 - a\mu\lambda^2 - b\mu^2\lambda^2 + \frac{b\,\mu\lambda^2}{\alpha} + \frac{a\,\lambda^2}{\alpha} + \frac{b\,\lambda^3}{3\alpha}) + h_{w3}(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})]$$

Case-2:

$$\daleth C(\mu\ T) = \frac{1}{T}\left[\left(\daleth C_1(\mu\ T)\right), \left(\daleth C_2(\mu\ T)\right), \left(\daleth C_3(\mu\ T)\right)\right]$$

$$\daleth C_1(\mu\ T) = \frac{1}{T}[A_1+C_{p1}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\}+ C_{s1}f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_{L1}\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_{r1}(a\mu^2 + b\mu^3 + \frac{b\,\mu^2}{\alpha} - \frac{b\,\mu^2}{2\alpha}) + h_{w1}(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})]$$

$$\daleth C_2(\mu\ T) = \frac{1}{T}[A_2+C_{p3}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\}+ C_{s2}f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_{L2}\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_{r2}(a\mu^2 + b\mu^3 + \frac{b\,\mu^2}{\alpha} - \frac{b\,\mu^2}{2\alpha}) + h_{w2}(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})]$$

$$\daleth C_3(\mu\ T) = \frac{1}{T}[A_3+C_{p3}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\}+ C_{s3}f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_{L3}\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_{r3}(a\mu^2 + b\mu^3 + \frac{b\,\mu^2}{\alpha} - \frac{b\,\mu^2}{2\alpha}) + h_{w3}(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})]$$

Signed distance method is applied to de fuzzify the fuzziness of the fuzzy models at equations 22 & 23 and represented as

Case-1:

$$\daleth C(\mu\ T) = \frac{1}{4T}[[A_1+C_{p1}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(\alpha\mu - 1)\exp(\alpha\mu)\} + f_1\{a(T-t) + \frac{b}{2}(T^2 - t^2)\}\}+ C_{s1}f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2}) + C_{L1}\left(1 - f_1 (\frac{aT^2}{2} - \frac{av^2}{2} + \frac{bT^3}{6} - \frac{bv^3}{6} - a\mu T + av^2 - \frac{bv^2 T}{2} + \frac{bv^3}{2})\right) + h_{r1}(a\mu\lambda + b\mu^2\lambda - \frac{b\lambda^2}{2\alpha} - \frac{b\mu^2}{3\alpha} + a\mu^3 + b\mu^4 - \frac{a\,k}{\alpha} - b\mu k^2 - a\mu\lambda^2 - b\mu^2\lambda^2 + \frac{b\,\mu\lambda^2}{\alpha} + \frac{a\,\lambda^2}{\alpha} + \frac{b\,\lambda^3}{3\alpha}) + h_{w1}(W\mu + \frac{bv^2}{\beta} - \frac{b\mu v}{\beta} + \frac{b\mu^2}{2\beta} - \frac{bv^2}{2\beta})] + 2\ [A_2+C_{p2}\{W + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} +$$

$\frac{b}{\alpha^2}(\alpha\mu-1)\exp(\alpha\mu)\}\ +f_1\{a(T-t)\ +\frac{b}{2}\ (T^2-t^2)\}\}+\ C_2 f_1\ (\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})+C_{L2}\left(1-f_1\,(\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})\right)+h_{r2}(a\mu\lambda+b\mu^2\lambda-\frac{b\lambda^2}{2\alpha}-\frac{b\mu^2}{3\alpha}+a\mu^3+b\mu^4-\frac{a\,k}{\alpha}-b\mu k^2-a\mu\lambda^2-b\mu^2\lambda^2+\frac{b\,\mu\lambda^2}{\alpha}+\frac{a\,\lambda^2}{\alpha}+\frac{b\,\lambda^3}{3\alpha})\ \ +h_{w2}(W\mu+\frac{bv^2}{\beta}-\frac{b\mu v}{\beta}+\frac{b\mu^2}{2\beta}-\frac{bv^2}{2\beta})]+A_3+C_{p3}\{W+\{(\frac{b}{\alpha^2}-\frac{a}{\alpha})+\{\frac{a}{\alpha}+\frac{b}{\alpha^2}(\alpha\mu-1)\exp(\alpha\mu)\}\ +f_1\{a(T-t)\ +\frac{b}{2}(T^2-t^2)\}\}+C_{s3}f_1\ (\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})+C_{L3}\left(1-f_1\,(\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})\right)+h_{r3}(a\mu^2+b\mu^3+\frac{b\,\mu^2}{\alpha}-\frac{b\,\mu^2}{2\alpha})+h_{w3}(W\mu+\frac{bv^2}{\beta}-\frac{b\mu v}{\beta}+\frac{b\mu^2}{2\beta}-\frac{bv^2}{2\beta})]]$ (24)

Case-2:

$\daleth C(\mu\ T)=\frac{1}{4\mathrm{T}}[[A_1+C_{p1}\{W+\ \{(\frac{b}{\alpha^2}-\frac{a}{\alpha})\ +\ \{\frac{a}{\alpha}+\frac{b}{\alpha^2}(\alpha\mu-1)\exp(\alpha\mu)\}\ +f_1\{a(T-t)\ +\frac{b}{2}\ (T^2-t^2)\}\}+C_{s1}f_1\ \ (\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})+C_{L1}\left(1-f_1\,(\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})\right)+h_{r1}(a\mu^2+b\mu^3+\frac{b\,\mu^2}{\alpha}-\frac{b\,\mu^2}{2\alpha})+h_{w1}(W\mu+\frac{bv^2}{\beta}-\frac{b\mu v}{\beta}+\frac{b\mu^2}{2\beta}-\frac{bv^2}{2\beta})]+2\frac{1}{\mathrm{T}}[A_2+C_{p3}\{W+\ \{(\frac{b}{\alpha^2}-\frac{a}{\alpha})\ +\ \{\frac{a}{\alpha}+\frac{b}{\alpha^2}(\alpha\mu-1)\exp(\alpha\mu)\}\ +f_1\{a(T-t)\ +\frac{b}{2}\ (T^2-t^2)\}\}+\ C_{s2}f_1\ (\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})+C_{L2}\left(1-f_1\,(\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})\right)+h_{r2}(a\mu^2+b\mu^3+\frac{b\,\mu^2}{\alpha}-\frac{b\,\mu^2}{2\alpha})+h_{w2}(W\mu+\frac{bv^2}{\beta}-\frac{b\mu v}{\beta}+\frac{b\mu^2}{2\beta}-\frac{bv^2}{2\beta})]+[A_3+C_{p3}\{W+\ \{(\frac{b}{\alpha^2}-\frac{a}{\alpha})\ +\ \{\frac{a}{\alpha}+\frac{b}{\alpha^2}(\alpha\mu-1)\exp(\alpha\mu)\}\ +f_1\{a(T-t)\ +\frac{b}{2}\ (T^2-t^2)\}\}+\ C_{s3}f_1\ (\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})+C_{L3}\left(1-f_1\,(\frac{aT^2}{2}-\frac{av^2}{2}+\frac{bT^3}{6}-\frac{bv^3}{6}-a\mu T+av^2-\frac{bv^2\,T}{2}+\frac{bv^3}{2})\right)+h_{r3}(a\mu^2+b\mu^3+\frac{b\,\mu^2}{\alpha}-\frac{b\,\mu^2}{2\alpha})+h_{w3}(W\mu+\frac{bv^2}{\beta}-\frac{b\mu v}{\beta}+\frac{b\mu^2}{2\beta}-\frac{bv^2}{2\beta})]]$ (25)

To obtain the values of decision variables to minimize the total relevant inventory cost for fuzzy models equation 24 & 25 are solved under the conditions given below

$\frac{\partial\,\daleth C(\mu\ T)}{\partial\mu}=0$ and $\frac{\partial\,\daleth C(\mu\ T)}{\partial \mathrm{T}}=0$

subject to the satisfaction of following:-

$\left(\frac{\partial^2\daleth C(\mu\ T)}{\partial\mu^2}\right)\left(\frac{\partial^2\daleth C(\mu\ T)}{\partial T^2}\right)-\frac{\partial^2\daleth C(\mu\ T)}{\partial\mu\,\partial\mathrm{T}}>0$

Numerical Analysis

In this section, numerical examples are presented to illustrate the behaviour of the crisp model as well as fuzzy model developed in earlier section and corresponding values of decision variables(μ^* , $\boldsymbol{v}^*\boldsymbol{T}$) for both models are obtained.

1. Considering an inventory situation with crisp parameters having the following values$a=500$, A =1500,W =2000,$b=0.50$, h_w=60, h_r=75,C_p=1500,$\alpha=0.013$, $\beta=0.014$, C_s=250, $\lambda=1.61$, f_1=0.06 and C_L=100.The values of decision variables are computed for two cases separately. The computational optimal solutions of the models are shown in Table-1 and convexity of the model for two cases have shown in Figure-2 and Figure-3.

Table-1:

Case	Cost function	μ^*	$\boldsymbol{\nu}^*$	$\boldsymbol{T}$*	Total Relevant Cost
1	$IC(\mu^*\ \ T^*)$	2.47477	62.7053	74.2487	135249
2	$IC(\mu^*\ \ T^*)$	7.31247	35.7460	37.9896	61042.2

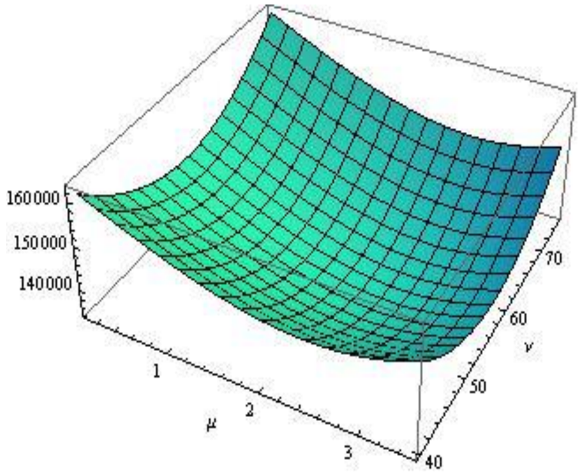

Figure-2: 3D Convexity of the inventory model (case-1) is represented graphically w.r.t. μ^* , $\boldsymbol{\nu}^*$ and Inventory cost (When $\boldsymbol{T}^* = 74.2487$)

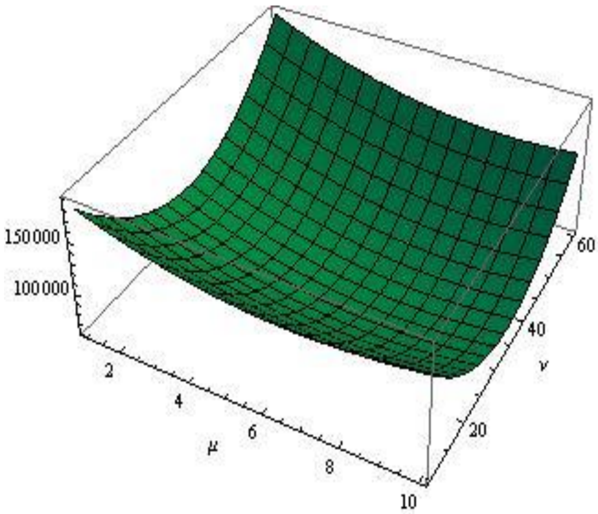

Figure-3: 3D Convexity of the inventory model (case-2) is represented graphically w.r.t. $\mu^*, \boldsymbol{\nu}^*$ and Inventory cost (When$\boldsymbol{T}^* = 37.9896$)

2. Considering an inventory situation with cost parameters which are of fuzzy nature and represented by triangular fuzzy numbers having the following values $a = 500$, A =(1400 1500 1600), W =2000, $b = 0.50$, h_w=(55 60 65), h_r=(70 75 80),C_p=(1400 1500 1600), $\alpha = 0.013$, $\beta = 0.014$, C_s=(240 250 260), $\lambda = 1.61$, f_1=0.06 and C_L=(90 100 110).The values of decision variables are computed for two cases separately. The computational optimal solutions of the models are shown in Table-1 and convexity of the model for two cases has shown in Figure-4 and Figure-5.

Table-2:

Case	**Cost function**	t_1^*	$\boldsymbol{\nu}^*$	***T****	**Total relevant cost** $IC(\mu^*\ T^*)$
3	$\mathcal{T}C(\mu^*\ T^*)$	2.47477	62.7053	74.2487	140652
4	$\mathcal{T}C(\mu^*\ T^*)$	7.31247	35.7460	37.9896	61042.2

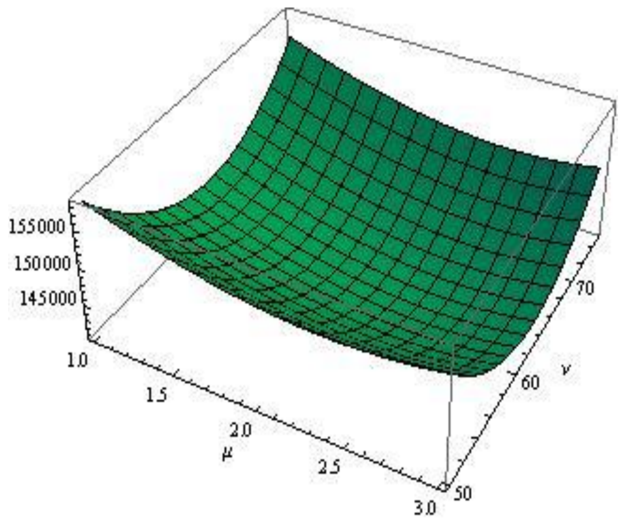

Figure-4: 3D Convexity of the fuzzy inventory model (case-21is represented graphically w.r.t. $\mu^*, \boldsymbol{\nu}^*$ and Inventory cost (When $\boldsymbol{T}^* = 74.2487$)

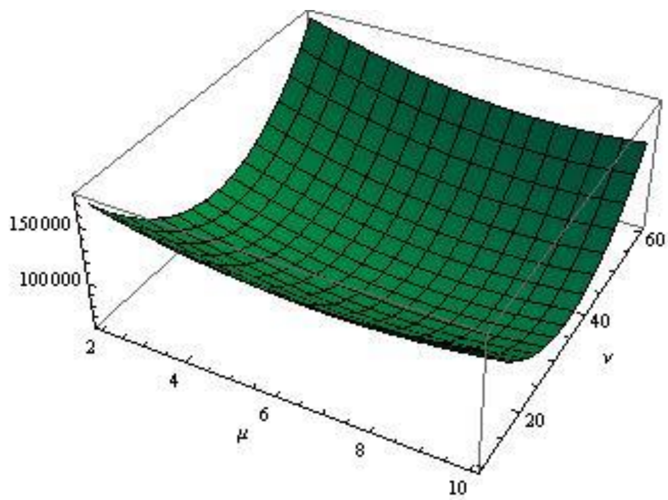

Figure-5: 3D Convexity of the fuzzy inventory model (case-2) is represented graphically w.r.t.$\mu^*, \boldsymbol{\nu}^*$and Inventory cost (When $\boldsymbol{T}^* = 37.9896$)

Sensitivity analysis

Sensitivity analysis performed on every parameter of the model. The analysis is carried out by changing the value of only one parameter at a time by increasing and decreasing by 10% , 20% and 50%, keeping the rest parameters at their initial values. The change in the values of decision variablesμ^*,$\boldsymbol{\nu}^*$,$T*$ and the percentage change in the value of $IC(\mu^*\ T^*)$ is taken as a measure of sensitivity with respect to the changes in the value of the parameter and the result is shown in Table-4 to Table-13.

The following observation is made from the tables given below.

(1) The total average inventory cost $IC(\mu^*\ T^*)$ is directly proportional to the values of$a$,b $(demand\ parameters)$, λ, C_p,$h_p$and$h_r$and highly sensitive to these parametershowever the value of $IC(\mu^*\ T^*)$ changes slightly with respect to change in the values of ordering cost, shortages cost and opportunity cost.

(2) The total relevant inventory cost is indirectly proportional to the deterioration rate in the both ware-houses and moderately sensitive to these parameters.

(3) If value of parameters λ, C_p ,h_r ,βincreases, the length of order cycle increases and hence total inventory cost also increases with respect to increased value of λ, C_p ,$h_r$and decreases with respect to increased value of β.

Table-3: Sensitivity analysis with respect to constant demand rate

A	μ^*	$\boldsymbol{\nu}^*$	$\boldsymbol{T}^*$	$IC(\mu^*\ T^*)$	**% change in** $IC(\mu^*\ T^*)$
550	2.49618	64.1910	74.6198	140800	4.10
600	2.51352	65.5790	75.0507	143858	6.37
750	2.55002	69.3147	76.5811	151940	12.34
450	2.44771	61.0971	73.4559	134125	-0.83
400	2.41251	59.3314	73.7677	130524	-3.49
250	2.1942	52.4114	74.3116	116654	-13.75

Table-4: Sensitivity analysis with respect to variable demand rate

B	μ^*	$\boldsymbol{\nu}^*$	$\boldsymbol{T}^*$	$IC(\mu^*\ T^*)$	**% change in** $IC(\mu^*\ T^*)$
0.55	2.46488	59.4764	71.6827	143436	6.05
0.60	2.45425	56.6482	69.4710	148950	10.13
0.75	2.41861	49.8863	64.3341	163695	21.03
0.45	2.48385	66.4413	77.2679	131372	-2.87
0.40	2.49201	70.8351	80.8821	124634	-7.85
0.25	2.50921	90.9246	98.1062	100468	-25.72

Table-5: Sensitivity analysis with respect to ordering cost

C_A	μ^*	$\boldsymbol{\nu}^*$	$\boldsymbol{T}^*$	$IC(\mu^*\ T^*)$	**% change** $IC(\mu^*\ T^*)$
1650	2.47477	62.7062	74.2498	137624	1.76
1800	2.47478	62.7071	74.2709	137626	1.76
2250	2.47479	62.7098	74.2542	137632	1.76
1350	2.47477	62.7045	74.2475	137620	1.76
1200	2.47475	62.7045	74.2475	137620	1.76
750	2.47475	62.7009	74.2431	137612	1.76

Table-6: Sensitivity analysis with respect shortages cost

C_s	μ^*	$\boldsymbol{\nu}^*$	$\boldsymbol{T}^*$	$IC(\mu^*\ T^*)$	**% change in** $IC(\mu^*\ T^*)$
275	2.47605	63.0333	73.6067	138282	2.24
300	2.47715	63.3126	73.0670	138844	2.66

375	2.47963	63.9473	71.8635	140123	3.60
225	2.47324	62.3148	75.0249	136835	3.60
200	2.47138	61.8417	75.9826	135.883	0.47
125	2.46235	59.5406	80.9334	131257	-2.95

Table-7: Sensitivity analysis with respect to opportunity cost

L_c	μ^*	ν^*	T^*	$IC(\mu^*\ T^*)$	% change in $IC(\mu^*\ T^*)$
110	2.47468	62.6829	74.2602	137572	1.72
120	2.47459	62.6604	74.2717	137522	1.68
150	2.47433	62.5924	74.3056	137371	1.57
90	2.47486	62.7278	74.2371	137671	1.79
80	2.47494	62.7501	74.2254	137721	1.83
50	2.47521	62.8169	74.1899	137868	1.94

Table-8: Sensitivity analysis with respect to time up to which holding cost remain constant

Λ	μ^*	ν^*	T^*	$IC(\mu^*\ T^*)$	% change in $IC(\mu^*\ T^*)$)
1.771	2.52073	68.6183	81.7013	151730	12.19
1.932	2.56961	74.4808	89.3418	166340	22.99
2.415	2.73079	92.8567	112.8710	212288	56.96
1.449	2.43217	57.1093	67.0702	124162	-8.20
1.288	2.39351	51.8280	60.2908	111562	-17.51
0.805	2.31021	39.5857	44.5368	82565.8	-38.95

Table-9: Sensitivity analysis with respect to purchase cost

C_p	μ^*	ν^*	T^*	$IC(\mu^*\ T^*)$	% change in $IC(\mu^*\ T^*)$
1650	2.59282	63.9583	75.2051	140218	3.67
1800	2.70547	65.1431	76.0761	142667	5.48
2250	3.01070	68.3322	78.2298	149574	10.59
1350	2.35049	61.3789	73.1999	134868	-2.82
1200	2.2189	59.9723	72.0506	131945	-2.44
750	1.76310	55.1799	67.8856	121981	-9.81

Table-10: Sensitivity analysis with respect to holding cost in OW

h_w	μ^*	ν^*	T^*	$IC(\mu^*\ T^*)$	% change in $IC(\mu^*\ T^*)$
66	2.46324	59.6488	71.9610	143077	5.79
72	2.45096	56.9546	69.9022	148202	9.58
90	2.41038	50.4533	65.2274	161955	19.75
54	2.48546	66.2148	76.9845	131789	-2.57
48	2.49519	70.3033	80.2418	125519	-7.19
30	2.51653	88.4028	95.3834	103172	-23.72

Table-11: Sensitivity analysis with respect to holding cost in RW

h_r	μ^*	ν^*	T^*	$IC(\mu^*\ T^*)$	% change in $IC(\mu^*\ T^*)$
82.5	2.37132	64.3002	76.3952	142302	5.21
90.0	2.28116	65.8104	78.4338	146798	8.54
112.5	2.06749	69.9336	84.0350	159432	17.88
67.5	2.5951	61.0109	71.9746	132718	-1.87
60.0	2.73743	59.1963	69.5458	127538	-5.70
37.5	3.39647	52.6439	60.8134	109302	-19.18

Table-12: Sensitivity analysis with respect to deterioration rate in RW

A	μ^*	$\boldsymbol{\nu}^*$	$\boldsymbol{T}^*$	$IC(\mu^*\ T^*)$	**% change in** $IC(\mu^*\ T^*)$
0.0143	2.48562	61.0270	72.1171	133742	-1.11
0.0156	2.49444	59.5881	70.2887	130418	-3.57
0.0195	2.51304	56.2776	66.0783	122777	-9.22
0.0117	2.46111	64.6911	76.7690	142216	5.15
0.0104	2.44345	67.0818	79.8010	147752	9.24
0.0065	2.33759	78.6239	94.4048	174561	29.07

Table-13: Sensitivity analysis with respect to deterioration rate in OW

$\boldsymbol{\beta}$	μ^*	$\boldsymbol{\nu}^*$	$\boldsymbol{T}^*$	$IC(\mu^*\ T^*)$	**% change in** $IC(\mu^*\ T^*)$
0.0154	2.46512	66.0232	76.9065	132683	-1.90
0.0168	2.45643	69.1559	79.4509	128287	-5.15
0.0210	2.43479	77.6484	86.5026	117490	-13.13
0.0126	2.48558	59.1742	71.4646	143201	5.88
0.0112	2.49781	55.3941	68.5391	149575	10.60
0.007	2.54720	42.0103	58.7006	175946	30.09

Conclusions

In this paper a fuzzy two-ware house inventory model for deteriorating items with linear time-dependent demand and varying holding cost with respect to ordering cycle length with the objective of minimizing the total inventory cost has developed. Shortages are allowed and a fraction of demand is backlogged. Two different cases of crisp inventory model has been discussed one with variable holding cost in RW during the cycle period and other with constant holding cost during total cycle length in both warehouses and it is observed that during variable holding cost the total inventory cost is much more than the other case. The corresponding inventory model of crisp model is developed and fuzzy values of parameters are defuzzified with signed distance method and values of decision variables are obtained. It is observed from result that values of decision variables and inventory cost are identical in case of both models crisp as well as fuzzy except in case-1,when holding cost varies in RW inventory cost increases as compared to crisp model. The fuzzy model has flexibility to choose range of values around a parameter which may minimizes total inventory cost after choosing a suitable range of values of parameter. Furthermore the proposed model is very useful for the items which are highly deteriorating, since as the deterioration rate increases in both ware-houses the total inventory cost decreases. This model can be further extended by incorporation with other generalised deterioration rate, probabilistic demand pattern and fuzzy demand as decision variable.

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