

Meshless approach for the bending analysis of elastically supported Sandwich plate under transverse line load

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Research Article

Meshless approach for the bending analysis of elastically supported Sandwich plate under transverse line load

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Abstract

The paper presents the five variables higher order shear deformation model for bending analysis of sandwich plate under uniformly line load using radial basis function (RBF) based mesh free method. The governing differential equations (GDEs) is obtained using energy principle. Multiquadric RBF (MQRBF) is used to discretize GDEs. The present results are compared with the available 3D elasticity solutions and the other results available in the literature. The effect of transverse loads with location of applied load, stiffness ratio 'R', and the core thickness to plate thickness ratio is studied. Some new results for sandwich plate considered which may serve as a benchmark for future investigations.

Keywords: Meshless method, Bending, RBF, Stress, Line load.

1. INTRODUCTION

As a result of good inclinations sandwich structures are by and large used in fields like aviation, marine designing, vehicle, common constructions. It have high solidness/high solidarity to weight proportion, better tailor capacity, magnificent consumption obstruction, and high weakness strength. But cross over load applied on the sandwich structure are exceptionally basic. Subsequently, it is important to comprehend their static practices under various sorts of cross over loads.

Various models developed in the writing to get to the bowing investigation of sandwich plate. In light of the kinematic field, the shear disfigurement hypothesis for the most part named comparable single layer (ESL) [1],[2], Layer-Wise [3],[4], and Zig-Zag speculations [5],[6]. Out of ESL, Classical Plate Theory (CPT) [7] is the easiest model disregarded the cross over shear impacts. After CPT, First request Shear Deformation Theory (FSDT) was proposed by Mindlin [8] in which shear rectification factor utilized for the cross over shear impacts [9],[10], [11]. After FSDT, Higher request Shear Deformation Theories (HSDT), [12],[13] were created in which the impacts of shear deformity or both shear and ordinary misshapenings is thought of. Pagano [14] explored 3D versatility answers for rectangular overlays with stuck edges. O'Connor [15] presented limited component techniques for the examination of sandwich plate. Lee and Fan, [16] explored statics and vibration reaction of composite sandwich plates utilizing limited component strategy.

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Thomsen, [17] explored statics examination in sandwich plates with orthotropic face layers exposed to restricted burdens. Grover et al., [18] proposed new HSDT models for the examination of composite and sandwich plate utilizing limited component and scientific strategy. Sahoo and Singh, [19] presented new inverse geometrical crisscross model for the twisting examination of composite and sandwich plate using limited component technique. Sahoo and Singh [20] proposed new reverse hyperbolic crisscross hypothesis for the twisting examination of sandwich and composite plates. There are numerous computational strategies accessible for settling the higher request half-way differential condition (PDE) and investigating the composite and sandwich structures. The age of a cross section in network based computational techniques is a troublesome errand for any construction so meshfree strategies are a choice to work based mathematical strategies. A meshfree strategy doesn't need a cross section to discretize the PDE. Some meshless strategies rely upon foundation networks that might uphold mathematical quadrature computations., Nayroles et al., [21] were first to propose another diffuse guess to introduce through moving least squares into a Galerkin conspiracy.. The outspread premise work strategy was first used by Hardy [22] and proposed a basic shape boundary $c = 0.815h$, where 'h' is the normal distance between nearest neighbors. Following 20 years Kansa[23], fostered the arrangement of PDEs. Franke [24] has positioned MQRBF as the best addition technique dependent on its exactness, execution time and simplicity of execution. A definite conversation of RBFs can be seen in Liu and Gu [25]-[26]. While using RBFs, a few shape boundaries still up in the air for better execution Chen et al., [27] researched free vibration of FGM plate utilizing a nearby normal neighbor insertion meshless technique dependent on FSDT.

In the present work, an attempt to analyse the MQRBF based meshfree method for the bending analysis of sandwich plate under uniformed line load with different location. By the authors knowledge, it is the first time, sandwich plate under line loads with different locations is studied.

2. MATHEMATICAL FORMULATION

A three-layer sandwich plate (Figure 1) with dimensions ($a \times b \times h$) in the Cartesian co-ordinate system (x - y - z) is considered for analysis. Mid-plane of the plate is considered as the reference plane.

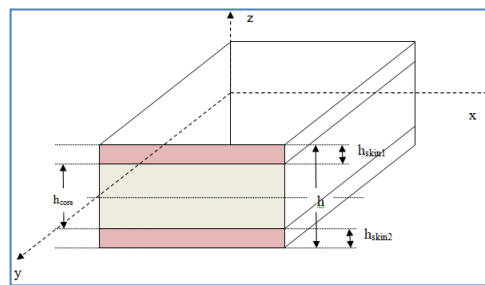


Figure 1 Configurations of the three layers sandwich plate

The displacement field at any point in the sandwich plate made up of perfectly bonded layers of uniform thickness is expressed as [1] :

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$$\begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} u_0(x, y) - z \frac{\partial w(x, y)}{\partial x} + f(z)\psi_x(x, y) \\ v_0(x, y) - z \frac{\partial w(x, y)}{\partial y} + f(z)\psi_y(x, y) \\ w_0(x, y) \end{Bmatrix} \quad (1)$$

Where, $f(z) = \left(\frac{9}{10} \left(\frac{z}{h} \right)^3 - \frac{27}{40h} z \right)$

U, V and W are the in-plane and transverse displacements of the plate at any point (x, y, z) in x, y and z directions, respectively. u_0 , v_0 and w_0 are the displacements at mid plane of the plate at any point (x, y) in x, y and z directions, respectively. The functions ψ_x and ψ_y are the higher order rotations of the normal to the mid plane due to shear deformation about y and x axes, respectively.

Using Generalized Hooke's law, the linear constitutive relations are expressed as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}_k \quad (2)$$

The governing differential equations of plate are obtained using energy principle and expressed as :

$$\begin{aligned} \delta u_0 : \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0 \\ \delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} &= 0 \\ \delta w_0 : \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} - q_z &= 0 \\ \delta \psi_x : \frac{\partial M_{xx}^f}{\partial x} + \frac{\partial M_{xy}^f}{\partial y} - Q_x^f &= 0 \\ \delta \psi_y : \frac{\partial M_{xy}^f}{\partial x} + \frac{\partial M_{yy}^f}{\partial y} - Q_y^f &= 0 \end{aligned} \quad (3)$$

The force and moment resultants in the plate are expressed as:

$$N_{ij}, M_{ij}, M_{ij}^f = \int_{-h/2}^{+h/2} (\sigma_{ij}, z\sigma_{ij}, f(z)\sigma_{ij}) dz \quad (4)$$

$$Q_x^f, Q_y^f = \int_{-h/2}^{+h/2} (\sigma_{xz}, \sigma_{yz}) \left(\frac{\partial f(z)}{\partial z} \right) dz \quad (5)$$

The stiffness coefficients of plate are taken as :

$$\begin{aligned} A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \\ \int_{-h/2}^{h/2} \left\{ (1, z, z^2, f(z), z f(z), f^2(z)) \right\} dz \\ \text{for } i, j = 1, 2, 6 \end{aligned} \quad (6)$$

$$A_{ij} = \int_{-h/2}^{h/2} \left\{ \left(\frac{\partial f(z)}{\partial z} \right)^2 \right\} dz \quad (7)$$

for $i, j = 4, 5$

The boundary condition for simply supported edges is:

$$\begin{aligned} x = 0, a : v = 0; \psi_y = 0; w = 0; M_x = 0; N_x = 0 \\ y = 0, b : u = 0; \psi_x = 0; w = 0; M_y = 0; N_y = 0 \end{aligned} \quad (8)$$

3. SOLUTION METHODOLOGY

Governing differential equations (eq.3) and boundary conditions (eq.8) are discretized using MQ-RBF. MQRBF based meshless formulation performed on the principle of interpolation of scattered data over whole domain. For present analysis the MQRBF function used is as follows:

$$g = (r^2 + c^2)^{1/2}, c = \alpha \sqrt{\left(\frac{a}{n_x}\right)^2 + \left(\frac{b}{n_y}\right)^2} \quad (9)$$

Where,

$$r = \|X - X_j\| = \sqrt{(x - x_j)^2 + (y - y_j)^2} \quad (10)$$

The field variables (displacements) in terms of radial basis function are expressed as:

$$\begin{aligned} u &= \sum_{j=1}^N \alpha_j^u g(\|X - X_j\|, c), v = \sum_{j=1}^N \alpha_j^v g(\|X - X_j\|, c), \\ w &= \sum_{j=1}^N \alpha_j^w g(\|X - X_j\|, c), \psi_x = \sum_{j=1}^N \alpha_j^{\psi_x} g(\|X - X_j\|, c), \\ \psi_y &= \sum_{j=1}^N \alpha_j^{\psi_y} g(\|X - X_j\|, c) \end{aligned}$$

Where, $g(\|X - X_j\|, c)$ is radial basis function, α_j^u is unknown coefficient. $\|X - X_j\|$ is the radial distance between two nodes.

The static problem in terms of MQRBF can be expressed as:

$$\begin{bmatrix} L \\ B \end{bmatrix}_{N \times N} \begin{Bmatrix} \alpha^u \\ \alpha^v \\ \alpha^w \\ \alpha^{\psi_x} \\ \alpha^{\psi_y} \end{Bmatrix}_{N \times 1} = \begin{Bmatrix} F \\ G \end{Bmatrix}_{N \times 1} \quad (11)$$

Where,

$$L = [L_D g(\|X_i - X_j\|, c)]_{N \times N}, B = [L_B g(\|X_i - X_j\|, c)]_{N_B \times N} \quad (12)$$

4. RESULT AND DISCUSSION

In this section, the developed solution methodology for sandwich plates in the framework of HSDT is solved for various transverse loads.

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A simply supported square sandwich plate is considered throughout the study. The material properties of the skin stiffness matrix are calculated by multiplying core stiffness matrix by an integers $R(\text{stiffness ratio})$. The material properties of the core are given in the stiffness matrix as

$$\bar{Q}_{core} = \begin{bmatrix} 0.999781 & 0.231192 & 0 & 0 & 0 \\ 0.231192 & 0.524886 & 0 & 0 & 0 \\ 0 & 0 & 0.262931 & 0 & 0 \\ 0 & 0 & 0 & 0.266810 & 0 \\ 0 & 0 & 0 & 0 & 0.159914 \end{bmatrix} \quad (9)$$

The material properties of the skin's stiffness matrix are given by

$$\bar{Q}_{skin} = R\bar{Q}_{core} \quad (10)$$

Here stiffness ratio 'R' =R1 and R2 have been taken for top and bottom skin stiffness matrix. The normalized stresses and transverse displacements of simply supported three layered sandwich plate are taken for the bending analysis subjected to maximum intensity of pressure at the center of the plate (q_0) for different values of R and different types of loads are presented in *Error! Reference source not found.* The normalized deflection and stresses are as follows:

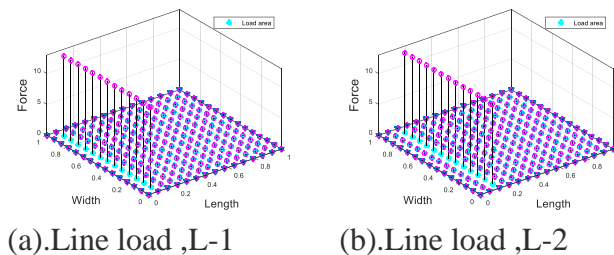
$$\bar{w} = \frac{0.999781 \times w(a/2, a/2, 0)}{q_0 h}, \bar{\sigma}_{xz}^1 = \frac{\sigma_{xz}^1(a/2, a/2, 0)}{q_0}$$

$$\bar{\sigma}_{xx}^1 = \frac{\sigma_{xx}^1(a/2, a/2, h/2)}{q_0}, \bar{\sigma}_{xx}^2 = \frac{\sigma_{xx}^2(a/2, a/2, 2h/5)}{q_0}, \bar{\sigma}_{xx}^3 = \frac{\sigma_{xx}^3(a/2, a/2, 2h/5)}{q_0}$$

$$\bar{\sigma}_{yy}^1 = \frac{\sigma_{yy}^1(a/2, a/2, h/2)}{q_0}, \bar{\sigma}_{yy}^2 = \frac{\sigma_{yy}^2(a/2, a/2, 2h/5)}{q_0}, \bar{\sigma}_{yy}^3 = \frac{\sigma_{yy}^3(a/2, a/2, 2h/5)}{q_0}$$

4.1 Convergence and validation study

The point of this segment is to represent the exactness, steadiness and proficiency of the current strategy. Union and comparision investigation of just upheld sandwich plate ($a/h=10, R1=R2=10, h_{core}/h=0.8$) on standardized redirection and stresses under uniormly distrubuted load, L-8 is examined and introduced in Table 1 . It is seen that by expanding the hubs size from 9×9 to 15×15 as displayed in Table 1 the intermingling study is completed. It tends to be seen that the redirection and stresses are merged from 7×7 to 17×17 hubs size. Nonetheless, a hubs size of (15×15) is adequate (regard to exactness and time cost) to predicts all the more nearer results with the specific arrangement of the Srinivas [28] utilized as benchmark results, and furthermore contrasted and RBF technique for, and meshfree strategy for Xiang et al., [29] . Figure 3 shows the assembly investigation of sandwich plate ($a/h=10, h_{core}/h=0.8$) under L-8 load. It is appears to be that by expanding the quantity of hub, standardized avoidance become nearer to Srinivas [28] results and after 13×13 hubs , the union % is with in 1 %. Therefore, (15×15) hubs size is considered for all mathematical examination to get exact and stable outcomes for removals and stresses.



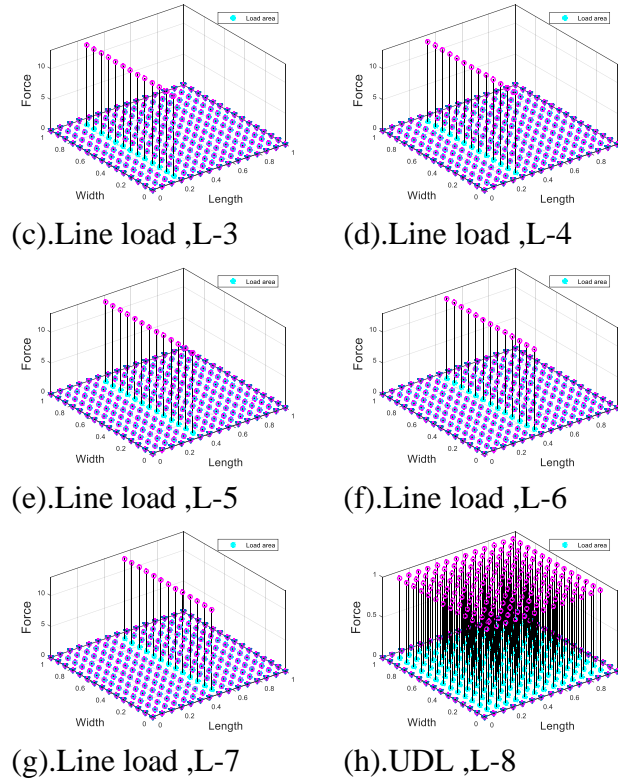


Figure 2 various types of transverse load use in investigation

Table 1 Convergence and comparison study on normalized deflection and stresses of three layered sandwich plate .

Methods	\bar{w}	$\bar{\sigma}_{xx}^1$	$\bar{\sigma}_{xx}^2$	$\bar{\sigma}_{xx}^3$	$\bar{\sigma}_{yy}^1$	$\bar{\sigma}_{yy}^2$	$\bar{\sigma}_{yy}^3$	$\bar{\sigma}_{xz}^1$
Srinivas, [28]	159.38	65.332	48.857	4.903	43.566	33.413	3.5	4.096
Xiang et al., [29]	152.66	65.008	49.684	4.968	42.945	33.394	3.339	3.45
(7x7) Nodes	145.05	64.15	49.60	4.96	42.20	33.23	3.32	2.54
(9x9) Nodes	149.48	64.73	49.77	4.98	42.63	33.19	3.32	2.85
(11x11) Nodes	152.21	66.00	50.10	5.01	43.40	33.59	3.36	3.07
(13x13) Nodes	153.25	65.81	50.35	5.04	43.24	33.71	3.37	3.20
(15x15) Nodes	153.82	65.74	50.38	5.04	43.30	33.71	3.37	3.29
(17x17) Nodes	154.17	65.88	50.34	5.03	43.39	33.72	3.37	3.35

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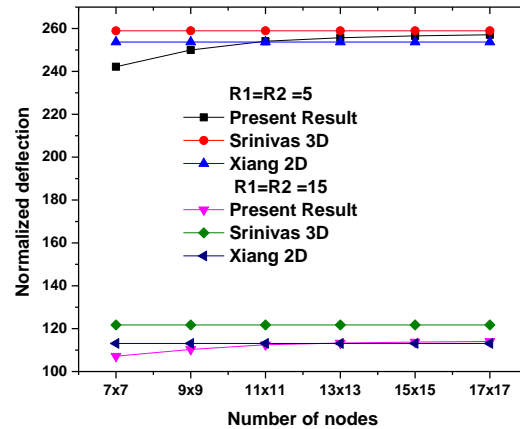


Figure 3 Convergence study on normalized deflection of three-layered sandwich plate.

4.2 Parametric study

In this part, the impact of cross over loads, solidness proportion, and center thickness proportion on standardized diversion and stresses have been examined. The impact of cross over load on standardized diversion of sandwich plate (R1=R2=5, a/h=10) is contemplated and introduced in Table 2. Seven kinds of cross over formally dressed line load loads are examined. The area of line load moved from limit to focus of the plate. It appears to be that L-7 foresee the most noteworthy standardized diversion of sandwich plate predicts and its area is at focus of the plate and L-1 predicts least standardized redirection and its area is the nearest to limit. And it is likewise seen that by moving the area of cross over load, standardized avoidance increments and same with R1=10, R2=10 and R1=5, R2=10 is portrayed in

Table 2 Influence of transverse load on normalized deflection and stresses of three-layered sandwich plate.

Loads	\bar{w}	$\bar{\sigma}_{xx}^1$	$\bar{\sigma}_{xx}^2$	$\bar{\sigma}_{xx}^3$	$\bar{\sigma}_{yy}^1$	$\bar{\sigma}_{yy}^2$	$\bar{\sigma}_{yy}^3$	$\bar{\sigma}_{xz}^1$
L-1	48.74	65.14	43.26	4.33	19.10	13.24	1.32	5.23
L-2	116.79	116.26	62.63	6.26	40.40	27.10	2.71	8.81
L-3	162.50	138.06	80.74	8.07	53.49	37.24	3.72	7.59
L-4	199.06	148.31	90.51	9.05	64.30	44.68	4.47	6.83
L-5	228.71	155.61	95.82	9.58	72.90	51.09	5.11	5.90
L-6	247.42	158.74	98.51	9.85	77.59	54.84	5.48	5.48
L-7	253.48	159.42	99.31	9.93	79.05	56.05	5.61	5.06

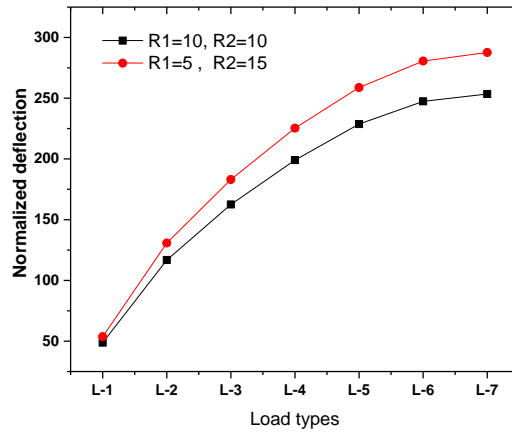


Figure 4 Influence of transverse load on normalized deflection of three layered sandwich plate

Now the effect of transverse load on normalized stress σ_{xx} , and σ_{xz} across thickness of the sandwich plate are shown in Figure 5 and Figure 6. It is seems from Figure 6 that the the L1 predicts minimum stress σ_{xx} and L8 predicts maximum stress σ_{xx} on the top and bottom of the plate. From Figure 7, it is observed that all the loads satisfy the the zero shear stress conditions a the bottom and top of the plate for σ_{xz} .

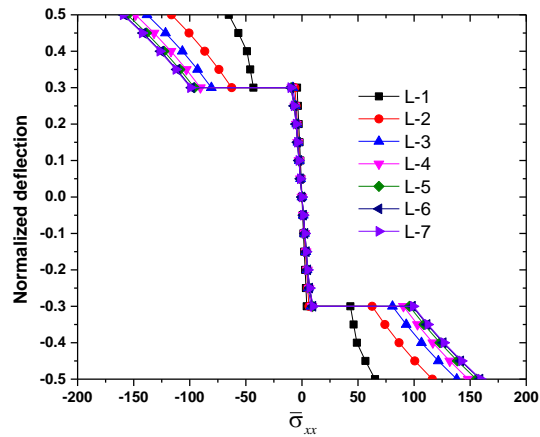


Figure 5 Influence of transverse loads on σ_{xx} of sandwich plate

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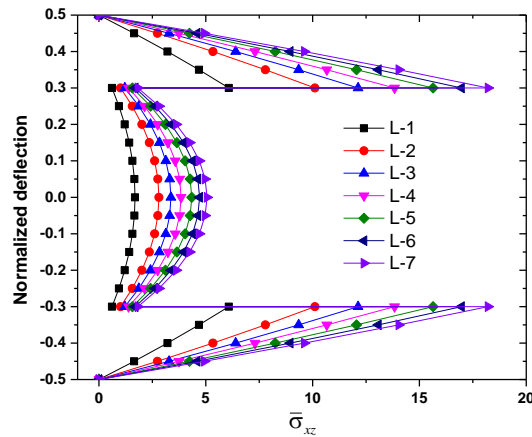


Figure 6 Influence of transverse loads on σ_{xz} of sandwich plate

The next example is the effect of stiffness ratio ‘R’ on normalized deflection and stresses for the sandwich plate (a/h=10,L-7).It seems that by increasing the stiffness ratio, the normalized deflection decreases.

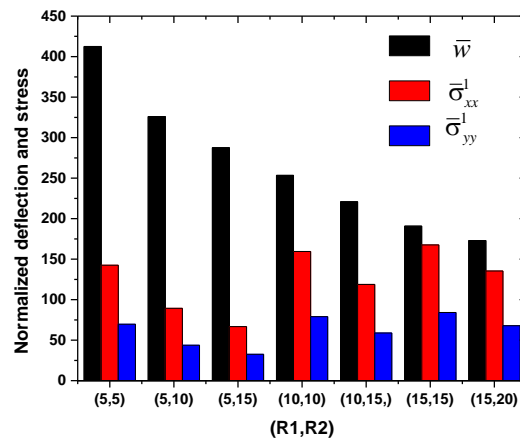


Figure 7 Influence of stiffness ratio ‘R’ on normalized deflection and stresses of three layered sandwich plate (L-7)

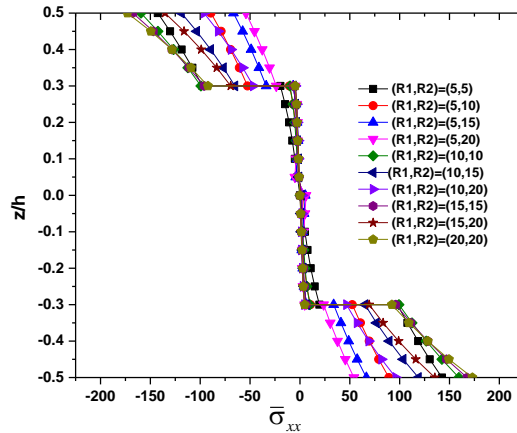


Figure 8 Influence of transverse loads on σ_{xx} of sandwich plate

Now the effect of stiffness ratio on normalized stress σ_{xx} , and σ_{xz} across thickness of the sandwich plate are shown in Figure 8 and Figure 9. It is seen from Figure 8 that all the stiffness ratio predicts maximum stress σ_{xx} on the top and bottom of the plate. From Figure 9, it is observed that all the stiffness ratio satisfy the zero shear stress conditions at the bottom and top of the plate for σ_{xz} .

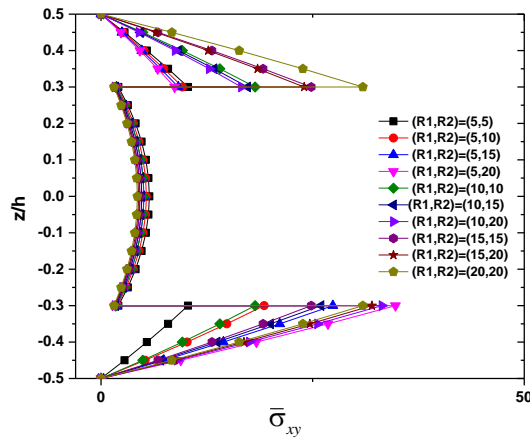


Figure 9 Influence of stiffness ratio on σ_{xz} of sandwich plate

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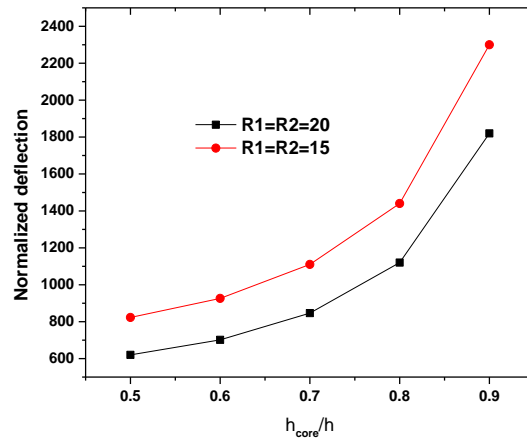


Figure 10 Influence of h_{core}/h on normalized deflection of three layered sandwich plate.

Figure 10 shows the effect of thickness ratio on normalized deflection of sandwich plate ($a/h=20$, $L=8$). It seems that by increasing the core thickness ratio, the normalized deflection increases for both ($R1=R2=20$ and $R1=R2=15$). The nature for both the stiffness ratio follows same patterns.

Conclusions

In this article, a meshless strategy dependent on the MQ-RBF method was created for the twisting examination of sandwich plate under different sorts of cross over loads. The GDEs of plate is gotten by energy rule. MQ-RBF is utilized to discretized the GDEs. Through the mathematical models and correlation of the current outcomes with those found in the writing (3-D precise arrangement and with different techniques), it was reasoned that the outcomes got by present strategy is acceptable in concurrence with the 3-D accurate arrangement and with different strategies in this show the appropriateness of the current arrangement procedure for the static examination of sandwich plate. The arrangement philosophy is likewise helpful for anticipating the static investigation of sandwich plates under formally dressed line loads. It inferred that the heap applied close to the middle predicts most extreme avoidance and burden close to the limit predicts low deflection. It additionally presumed that by expanding the stiffness proportion, standardized redirection decline. It likewise tracked down that The thickness of center additionally impacts the diversion, by builds the thickness proportion, standardized redirection increments.

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