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Research Article

On Hesitant QC- Fuzzy Soft Ring

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Abstract

The purpose of this paper is to extend the Q-fuzzy soft ring to hesitant QC- fuzzy soft ring .In this paper, we define the hesitant QC- fuzzy soft ring and prove some of their properties and structural characteristics are investigated some related properties ,.then the definition of hesitant QC-fuzzy soft ring and the theorem of homomorphic images are given. In this paper ,we study hesitant QC-fuzzy soft ring .In section 2 we discuss some preliminaries .In section 3 studies notions of hesitant QC –fuzzy soft ring

Keywords: Soft set, Fuzzy soft set, Soft ring, Fuzzy soft ring, Soft homomorphism, Fuzzy Soft isomorphism, Q-fuzzy set , Q- fuzzy soft ring ,Hesitant QC- fuzzy soft ring .

1. INTRODUCTION

The theory of soft sets was introduced by Molodtsov [6] ,soft sets theory has been extensively studied by many authors. It is well known that the concept of fuzzy sets, introduced by Zadeh [17], has been extensively applied to many scientific fields. Alam [2] introduced fuzzy rings and anti fuzzy rings with operators Pazar Varol ,Aygunoglu and Aygun [8] introduced on fuzzy soft rings . Marudai and Rajendran [5] introduced fuzzy soft rings on fuzzy lattices .Sarala and suganya [10] introduced the Q- fuzzy soft ring .Torra and Narukawa[14] introduced on hesitant fuzzy sets and decision .

2. PRELIMINARIES

Definition 2.1:

Suppose that U is an initial universe set and E is a set of parameters, let $P(U)$ denotes the power set of U .A pair (F, E) is called a ***soft set*** over U where F is a mapping given by $F: E \rightarrow P(U)$.

Clearly, a soft set is a mapping from parameters to $P(U)$, and it is not a set, but a parameterized family of subsets of the Universe.

Definition 2.2:

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Let U be an initial Universe set and E be the set of parameters. Let A⊂ E. A pair (F, A) is called ***fuzzy soft set*** over U where F is a mapping given by F: A→I^U, where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3:

Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be a ***fuzzy soft group*** over X iff for each a ∈ A and x, y ∈ X,

(i) $f_a\ (x\ .\ y) \geq T\ (f_a\ (x), f_a\ (y))$

(ii) $f_a\ (x^{-1}) \geq f_a\ (x)$

That is, for each a ∈ A, f_a is a fuzzy subgroup in Rosenfeld's sense [12]

Definition 2.4:

Let (f, A) be a soft set over a ring R. Then (f, A) is said to be a ***soft ring*** over R if and only if $f(a)$ is sub ring of R for each a∈ A.

Definition 2.5:

Let (φ,ψ): X → Y is a fuzzy soft function, if φ is a homomorphism from x → y then (φ,ψ) is said to be ***fuzzy soft homomorphism***. if φ is a isomorphism from X → Y and ψ is 1-1 mapping from A on to B then (φ,ψ) is said to be ***fuzzy soft isomorphism***.

Definition 2.6:

Let R be a soft ring. A fuzzy set 'μ' in R is called Q- fuzzy soft ring in R if

(i) $\mu\ ((x+y)\ , q) \geq T\{\mu(x,q), \mu(y,q)\}$

(ii) $\mu\ (-x,q) \geq \mu\ (x,q)$ and

(iii) $\mu\ ((xy)\ , q) \geq T\{\mu(x,q), \mu(y,q)\}$, for all $x, y \in R.\ \&\ q \in Q$

Definition 2.7:

Let H̃ (U) be the set of all hesitant fuzzy sets in U, a pair $(\tilde{F}, A)$ is called a HFSS over U, where $\tilde{F}$ is defined by $\tilde{F}\ :\ A \rightarrow\ \tilde{H}(U)$.A hesitant fuzzy soft set is a mapping from parameters to H̃ (U). It is a parameterised family of hesitant fuzzy subsets of U.

3. Hesitant QC-Fuzzy Soft Rings

Definition 3.1:

Let R be a soft ring. A fuzzy set '$\tilde{f}_{a_a}$' in R is called hesitant QC- fuzzy soft ring in R if

(i) $\tilde{f}_a\ \left((x+y), qe^{i\theta}\right) \geq min\{\tilde{f}_a\ \left(x, qe^{i\theta}\right), \tilde{f}_a\ \left(y, qe^{i\theta}\right)\}$

(ii) $\tilde{f}_a\ \left(-x, qe^{i\theta}\right) \quad \geq\ \tilde{f}_a\ \left(x, qe^{i\theta}\right)\ and$

(iii) $\tilde{f}_a\ \left((xy), qe^{i\theta}\right) \quad \geq$
$T\{\tilde{f}_a\ \left(x, qe^{i\theta}\right), \tilde{f}_a\left(y, qe^{i\theta}\right)\}\ , for\ all\ x, y \in R.\ \&\ q \in Q\ , e^{i\theta} \in \mathbb{C}$

Proposition 3. 1:

Every imaginable hesitant QC- fuzzy soft ring $\tilde{f}_a$ is a hesitant QC-fuzzy soft ring of R.

Proof:

Assume that $\tilde{f}_a$is imaginable hesitant QC- fuzzy soft ring of R, then we have

$$\tilde{f}_a((x+y)\ , qe^{i\theta})\ \geq min\{\tilde{f}_{a_a}(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}$$
$$\tilde{f}_a\ (-x, qe^{i\theta})\ \geq\ \tilde{f}_a\ (x, qe^{i\theta})\ \ and$$
$$\tilde{f}_a\left((xy), qe^{i\theta}\right) \geq min\{\tilde{f}_a\left(x, qe^{i\theta}\right), \tilde{f}_a\left(y, qe^{i\theta}\right)\}\ ,$$

$$for\ all\ x, y \in R. \&\ q \in Q, e^{i\theta} \in \mathrm{C}$$

Since f_a is imaginable, we have

$$min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}$$
$$= min\{min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}, min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}\}$$
$$\le min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}$$
$$\le min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}$$

and so

$$min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\} = min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}$$

It follows that

$$\tilde{f}_a((x+y), qe^{i\theta}) \ge min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\}$$
$$= min\{\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})\} for\ all\ x, y \in R, q \in Q, e^{i\theta} \in \mathrm{C}$$

Hence $\tilde{f}_a$ is a hesitant QC-fuzzy soft ring of R.

Proposition 3.2:

If $\tilde{f}_a$ is hesitant QC-fuzzy soft ring R and φ is an endomorphism of R, then $\tilde{f}_a\ [\varphi]$ is a Hesitant QC- Fuzzy soft ring of R

Proof:

For any $x, y \in R$, $q \in Q, e^{i\theta} \in \mathrm{C}$ we have

(HQCFSR1)

(i) $\tilde{f}_a\ [\varphi]((x+y), qe^{i\theta})) = \tilde{f}_a\ (\varphi((x+y), qe^{i\theta}))$
$= \tilde{f}_a\ (\varphi(x, qe^{i\theta}), \varphi(y, qe^{i\theta}))$
$\ge min\{\tilde{f}_a(\varphi(x, qe^{i\theta})), \tilde{f}_a\ (\varphi(y, qe^{i\theta}))\}$
$\ge min\{\tilde{f}_a\ [\varphi]\ (x, qe^{i\theta}), \tilde{f}_a[\varphi](y, qe^{i\theta})\}$

(**HQCFSR2**)

(ii) $\tilde{f}_a\ [\varphi](-x, qe^{i\theta}) = \tilde{f}_a\ (\varphi(-x, qe^{i\theta}))$
$\ge \tilde{f}_a(\varphi(x, qe^{i\theta}))$
$\ge \tilde{f}_a\ [\varphi](x, qe^{i\theta})$

(**HQCFSR3**)

(iii) $\tilde{f}_a\ [\varphi]((xy), qe^{i\theta})) = \tilde{f}_a\ (\varphi((xy), qe^{i\theta}))$
$= \tilde{f}_a((\varphi x, qe^{i\theta}), (\varphi y, qe^{i\theta}))$
$\ge min\{\tilde{f}_a\ (\varphi x, qe^{i\theta}), \tilde{f}_a\ (\varphi y, qe^{i\theta})\}$
$\ge min\{\tilde{f}_a\ \varphi(x, qe^{i\theta}), \tilde{f}_a\ \varphi(y, qe^{i\theta})\}$
$\ge min\{\tilde{f}_a\ [\varphi]\ (x, qe^{i\theta}), \tilde{f}_a\ [\varphi](y, qe^{i\theta})\}$

Hence $\tilde{f}_a\ [\varphi]$ is a hesitant QC-fuzzy soft ring of R.

Proposition 3.3:

Let R and R' be two rings and φ: $R \rightarrow R'$ be a soft homomorphism. If $\tilde{f}_a$ and μ is a hesitant QC -fuzzy soft ring of R then the pre-image $\varphi^{-1}(\tilde{f}_a)$ hesitant QC-fuzzy soft ring of R.

Proof:-

Assume that μ is a hesitant QC-fuzzy soft ring of R'. Let $x, y \in R$ & q $\in$Q, $e^{i\theta} \in \mathrm{C}$

(i) $\tilde{f}_{a\ \varphi^{-1}(\mu)}((x+y), qe^{i\theta})) = \tilde{f}_{a\ \mu}(\varphi(x+y), qe^{i\theta}))$

$= \tilde{f}_{a\ \mu}((\varphi x, qe^{i\theta}), (\varphi y, qe^{i\theta}))$
$\geq min\{\tilde{f}_{a\ \mu}\ (\varphi(x, qe^{i\theta})), \tilde{f}_{a\ \mu}\ (\varphi(y, qe^{i\theta}))\}$
$\geq min\{\tilde{f}_{a\ \varphi^{-1}(\mu)}\ (x, qe^{i\theta}),\ \tilde{f}_{a\ \varphi^{-1}(\mu)}\ (y, qe^{i\theta})\}$

(ii) $\tilde{f}_{a_{\varphi^{-1}(\mu)}}(-x, qe^{i\theta})$ $= \tilde{f}_{a_\mu}((\varphi(-x, qe^{i\theta}))$
$\geq \tilde{f}_{a_\mu}(\varphi(x, qe^{i\theta}))$
$\geq \tilde{f}_{a_{\varphi^{-1}(\mu)}}(x, qe^{i\theta})$

(iii) $\tilde{f}_{a_{\varphi^{-1}(\mu)}}(\ (xy), qe^{i\theta}))$ $= \tilde{f}_{a_\mu}\ (\varphi\ (xy), qe^{i\theta}))$
$= \tilde{f}_{a_\mu}((\varphi x, qe^{i\theta}), (\varphi y, qe^{i\theta}))$
$\geq \min\{\tilde{f}_{a_\mu}((\varphi\ (x, qe^{i\theta})), \tilde{f}_{a_\mu}(\varphi(y, qe^{i\theta}))\}$
$\geq \min\ \{\tilde{f}_{a_{\varphi^{-1}(\mu)}}\ (x, qe^{i\theta}),\ \tilde{f}_{a_{\varphi^{-1}(\mu)}}\ (y, qe^{i\theta})\}$

Hence $\varphi^{-1}(\mu)$ hesitant QC-fuzzy soft ring of R.

Proposition 3. 4

Let φ: $R \to R'$ be an epimorphism and μ be fuzzy soft set in R' . If $\varphi[\mu]$ is hesitant QC-fuzzy soft ring of R' then μ is hesitant QC- fuzzy soft ring of R.

Proof :-

Let $x, y \in R$, Then there exist $a, b \in R$ such that $\varphi(a) = x$, $\varphi(b) = y$. It follows that

(HQC-FSR1)

(i) $\tilde{f}_{a_{\varphi[\mu]}}(\ (x+y), qe^{i\theta}))$ $= \tilde{f}_{a_\mu}(\varphi\ (x+y), qe^{i\theta}))$
$= \tilde{f}_{a_\mu}((\varphi\ x, qe^{i\theta}), (\varphi\ y, qe^{i\theta}))$
$\geq \min\ \{\tilde{f}_{a_\mu}\ (\varphi\ (x, qe^{i\theta})), \tilde{f}_{a_\mu}(\varphi\ (y, qe^{i\theta}))\}$
$\geq \min\ \{\tilde{f}_{a_{\varphi[\mu]}}(x, qe^{i\theta}),\ \tilde{f}_{a_{\varphi[\mu]}}(y, qe^{i\theta})\}$

(HQC-FSR2)

(ii) $\tilde{f}_{a_{\varphi[\mu]}}(-x, qe^{i\theta})$ $= \tilde{f}_{a_\mu}(\varphi\ (-x, qe^{i\theta}))$
$\geq \tilde{f}_{a_\mu}(\varphi\ (x, qe^{i\theta}))$
$\geq \tilde{f}_{a_{\varphi[\mu]}}(x, qe^{i\theta})$

(HQC-FSR3)

(iii) $\tilde{f}_{a_{\varphi[\mu]}}(\ (xy), qe^{i\theta}))$ $= \tilde{f}_{a_\mu}(\varphi\ (xy), qe^{i\theta}))$
$= \tilde{f}_{a_\mu}((\varphi\ x, qe^{i\theta}), (\varphi\ y, qe^{i\theta}))$
$\geq \min\ \{\tilde{f}_{a_\mu}\ (\varphi\ (x, qe^{i\theta})), \mu\ (\varphi\ (y, qe^{i\theta}))\}$
$\geq \min\{\tilde{f}_{a_{\varphi[\mu]}}(x, qe^{i\theta}),\ \tilde{f}_{a_{\varphi[\mu]}}(y, qe^{i\theta})\}$

Hence $\varphi[\mu]$ hesitant QC-fuzzy soft ring of R.

Proposition 3.5 :

Onto homomorphic image of a hesitant QC-fuzzy soft ring with the **sup** property is hesitant QC -fuzzy soft ring of R.

Proof:

Let φ: R→R' be an onto homomorphism of hesitant QC –fuzzy soft rings and let f_a be a **sup** property of hesitant QC-fuzzy soft ring of R.

Let x', y' ∈ R^1, and $x_0 \in \varphi^{-1}(x^1)$, $y_0 \in \varphi^{-1}(y')$ be such that

$\tilde{f}_a(x_0, qe^{i\theta}) = \sup_{(h, qe^{i\theta}) \in \varphi^{-1}(x')} \tilde{f}_a(h, qe^{i\theta})$, and $\tilde{f}_a(y_0, qe^{i\theta}) = \sup_{(h, qe^{i\theta}) \in \varphi^{-1}(y')} \tilde{f}_a(h, qe^{i\theta})$

Respectively, then we can deduce that

(HQC-FSR1)

$$\text{(i) } \tilde{f}_a^{\varphi}((x'+y'), qe^{i\theta}) = \sup_{(z, qe^{i\theta}) \in \varphi^{-1}((x'+y'), qe^{i\theta})} \tilde{f}_a(z, qe^{i\theta})$$
$$\geq \min\{\tilde{f}_a(x_0, qe^{i\theta}), \tilde{f}_a(y_0, qe^{i\theta})$$
$$= \min\{\sup_{(h, qe^{i\theta}) \in \varphi^{-1}(x', qe^{i\theta})} \tilde{f}_a(h, qe^{i\theta}), \sup_{(h, qe^{i\theta}) \in \varphi^{-1}(y', qe^{i\theta})} \tilde{f}_a(h, qe^{i\theta})\}$$
$$= \min\{\tilde{f}_a^{\varphi}(x', qe^{i\theta}), \tilde{f}_a^{\varphi}(y', qe^{i\theta})\}$$

(HQC-FSR2)

$$\text{(ii) } \tilde{f}_a^{\varphi}(-x', qe^{i\theta}) = \sup_{(z, qe^{i\theta}) \in \varphi^{-1}(-x', qe^{i\theta})} \tilde{f}_a(z, qe^{i\theta})$$
$$\geq \tilde{f}_a(x_0, qe^{i\theta})$$
$$\geq \sup_{(h, qe^{i\theta}) \in \varphi^{-1}(x', qe^{i\theta})} \tilde{f}_a(h, qe^{i\theta})$$
$$= \tilde{f}_a^{\varphi}(x', qe^{i\theta})$$

(HQC-FSR3)

$$\text{(i) } \tilde{f}_a^{\varphi}((x'y'), qe^{i\theta}) = \sup_{(z, qe^{i\theta}) \in \varphi^{-1}((x'y'), qe^{i\theta})} \tilde{f}_a(z, qe^{i\theta})$$
$$\geq \min\{\tilde{f}_a(x_0, qe^{i\theta}), \tilde{f}_a(y_0, qe^{i\theta})\}$$
$$= \min\{\sup_{(h, qe^{i\theta}) \in \varphi^{-1}(x', qe^{i\theta})} \tilde{f}_a(h, qe^{i\theta}), \sup_{(h, qe^{i\theta}) \in \varphi^{-1}(y', qe^{i\theta})} \tilde{f}_a(h, qe^{i\theta})\}$$
$$= \min\{\tilde{f}_a^{\varphi}(x', qe^{i\theta}), \tilde{f}_a^{\varphi}(y', qe^{i\theta})\}$$

Hence $\tilde{f}_a^{\varphi}$ is a hesitant QC-fuzzy soft ring of R'

Proposition 3. 6:

Let T be a continuous t-norm and let μ be a soft homomorphism on R. If $\tilde{f}_a$ is hesitant QC-fuzzy soft ring of R, then $\tilde{f}_a^{\mu}$ is hesitant QC-fuzzy soft ring of $f_a(R)$.

Proof:

Let $A_1 = \mu^{-1}(y_1, qe^{i\theta})$, $A_2 = \mu^{-1}(y_2, qe^{i\theta})$ and $A_{12} = \mu^{-1}((y_1+y_2), qe^{i\theta})$
where $y_1, y_2 \in \tilde{f}_a(R)$, $q \in Q$,
Consider the set

$$A_1 + A_2 = \{x \in R \,/\, (x, qe^{i\theta}) = (a_1, qe^{i\theta}) + (a_2, qe^{i\theta})\}$$

for some $(a_1, qe^{i\theta}) \in A_1$ and $(a_2, qe^{i\theta}) \in A_2$.

If $(x, qe^{i\theta}) \in A_1+A_2$, then $(x, qe^{i\theta}) = (x_1, qe^{i\theta}) + (x_2, qe^{i\theta})$
for some $(x_1, qe^{i\theta}) \in A_1$ and $(x_2, qe^{i\theta}) \in A_2$
so that we have

$$\mu(x, qe^{i\theta}) = \mu(x_1, qe^{i\theta}) + \mu(x_2, qe^{i\theta}) = y_1 + y_2$$

Since $(x, qe^{i\theta}) \in \mu^{-1}((y_1, qe^{i\theta}) + (y_2, qe^{i\theta})) = A_{12}$
Thus $A_1+A_2 \in A_{12}$
It follows that

(i) $$\tilde{f}_a{}^{\mu}((y_1+y_2), qe^{i\theta}) = \textbf{sup}\ \{\tilde{f}_a(x, qe^{i\theta}) / (x, qe^{i\theta}) \in f^{-1}(y_1+y_2, qe^{i\theta})\}$$
$$= \textbf{sup}\ \{\tilde{f}_a(x, qe^{i\theta}) / (x, qe^{i\theta}) \in A_{12}\}$$
$$\geq \textbf{sup}\ \{\tilde{f}_a(x, qe^{i\theta}) / (x, qe^{i\theta}) \in A_1+A_2\}$$
$$\geq \textbf{sup}\ \{\tilde{f}_a((x_1, qe^{i\theta}) + (x_2, qe^{i\theta})) / (x_1, qe^{i\theta}) \in A_1 \text{ and } (x_2, qe^{i\theta}) \in A_2\}$$
$$\geq \textbf{sup}\ \{(\tilde{f}_a(x_1, qe^{i\theta}), \tilde{f}_a(x_2, qe^{i\theta})) / (x_1, qe^{i\theta}) \in A_1 \text{ and } (x_2, qe^{i\theta}) \in A_2\}$$

Since T is continuous. For every ε > 0 , we see that if

$$\textbf{sup}\ \{\tilde{f}_a(x_1, qe^{i\theta}) / (x_1, qe^{i\theta}) \in A_1\} + (x_1*, qe^{i\theta})\} \leq \delta \text{ and}$$
$$\textbf{sup}\ \{\tilde{f}_a(x_2, qe^{i\theta}) / (x_2, qe^{i\theta}) \in A_2\} + (x_2*, qe^{i\theta})\} \leq \delta$$

$T\{\textbf{sup}\{\tilde{f}_a(x_1, qe^{i\theta}) / (x_1, qe^{i\theta}) \in A_1\}$, $\textbf{sup}\ \{\tilde{f}_a(x_2, qe^{i\theta}) / (x_2, qe^{i\theta}) \in A_2\}$ $+T((x_1*, qe^{i\theta}), (x_2*, qe^{i\theta}) \leq \varepsilon$

Choose $(a_1, qe^{i\theta}) \in A_1$ and $(a_2, qe^{i\theta}) \in A_2$ such that

$$\textbf{sup}\{\tilde{f}_a(x_1, qe^{i\theta}) / (x_1, qe^{i\theta}) \in A_1\} + \tilde{f}_a(a_1, qe^{i\theta}) \leq \delta \text{ and}$$
$$\textbf{sup}\{\tilde{f}_a(x_2, qe^{i\theta}) / (x_2, qe^{i\theta}) \in A_2\} + \tilde{f}_a(a_2, qe^{i\theta}) \leq \delta.$$

Then we have

$$T\{\textbf{sup}\{\tilde{f}_a(x_1, qe^{i\theta}) / (x_1, qe^{i\theta}) \in A_1\}, \textbf{sup}\{\tilde{f}_a(x_2, qe^{i\theta}) / (x_2, qe^{i\theta}) \in A_2\} + T(\tilde{f}_a(a_1, qe^{i\theta}), \tilde{f}_a(a_2, qe^{i\theta}) \leq \varepsilon$$

Consequently, we have

$$\tilde{f}_a{}^{\mu}((y_1+y_2), qe^{i\theta}) \geq \textbf{sup}\{T(\tilde{f}_a(x_1, qe^{i\theta}), \tilde{f}_a(x_2, qe^{i\theta})) / (x_1, qe^{i\theta}) \in A_1, (x_2, qe^{i\theta}) \in A_2\}$$
$$\geq T(\textbf{sup}\{\tilde{f}_a(x_1, qe^{i\theta}) / (x_1, qe^{i\theta}) \in A_1\}, \textbf{sup}\{\tilde{f}_a(x_2, qe^{i\theta}) / (x_2, qe^{i\theta}) \in A_2\}$$
$$\geq T\{(\tilde{f}_a{}^{\mu}(y_1, qe^{i\theta}), \tilde{f}_a{}^{\mu}(y_2, qe^{i\theta})\}$$

Similarly we can show $\tilde{f}_a{}^{\mu}(-x, qe^{i\theta}) \geq \tilde{f}_a{}^{\mu}(x, qe^{i\theta})$ and $\tilde{f}_a{}^{\mu}(xy, qe^{i\theta}) \geq T\{(\tilde{f}_a{}^{\mu}(x, qe^{i\theta}), \tilde{f}_a{}^{\mu}(y, qe^{i\theta})\}$

Hence $\tilde{f}_a{}^{\mu}$ is hesitant QC-fuzzy soft ring of $\tilde{f}_a(R)$.

Proposition3.7:

Let $\tilde{f}_a$ be a hesitant QC-fuzzy soft ring R and let $\tilde{f}_a$* be a hesitant QC-fuzzy set in N defined by $\tilde{f}_a*(x, qe^{i\theta})$ = $\tilde{f}_a$ $(x, qe^{i\theta})$ +1-$\tilde{f}_a(0, qe^{i\theta})$ for all $x \in N$. Then $\tilde{f}_a*$ is a normal a hesitant QC-fuzzy subgroup of R

Proof :

(HQCFSR1)

$$\begin{aligned}\tilde{f}_a*((x+y), qe^{i\theta}) &= \tilde{f}_a((x+y), qe^{i\theta})) + 1 - \tilde{f}_a(0, qe^{i\theta})\\ &\geq \mathrm{T}(\tilde{f}_a(x, qe^{i\theta}), \tilde{f}_a(y, qe^{i\theta})) + 1 - \tilde{f}_a(0, qe^{i\theta}))\\ &\geq \mathrm{T}(\tilde{f}_a(x, qe^{i\theta}) + 1 - \tilde{f}_a(0, qe^{i\theta})), (\tilde{f}_a(y, qe^{i\theta}) + 1 - \tilde{f}_a(0, qe^{i\theta}))\\ &= \mathrm{T}(\tilde{f}_a*(mx, qe^{i\theta})), \tilde{f}_a*(my, qe^{i\theta})).\end{aligned}$$

(HQCFSR2)

$$\begin{aligned}\tilde{f}_a*(-x, qe^{i\theta}) &= \tilde{f}_a(-x, qe^{i\theta}) + 1 - \tilde{f}_a(0, qe^{i\theta})\\ &\geq \tilde{f}_a(x, qe^{i\theta})) + 1 - \tilde{f}_a(0, qe^{i\theta})\\ &= \tilde{f}_a(x, qe^{i\theta})\end{aligned}$$

(HQCFSR3)

$$\begin{aligned}\tilde{f}_a*((xy), qe^{i\theta}) &= \tilde{f}_a((xy), qe^{i\theta})) + 1 - \tilde{f}_a(0, qe^{i\theta}))\\ &\geq \mathrm{T}(\tilde{f}_a(x, qe^{i\theta})), \tilde{f}_a(y, qe^{i\theta})) + 1 - \tilde{f}_a(0, qe^{i\theta}))\\ &\geq \mathrm{T}(\tilde{f}_a(x, qe^{i\theta})) + 1 - \tilde{f}_a(0, qe^{i\theta})), (\tilde{f}_a(y, qe^{i\theta})) + 1 - \tilde{f}_a(0, qe^{i\theta}))\\ &= \mathrm{T}(\tilde{f}_a*(mx, qe^{i\theta})), \tilde{f}_a*(my, qe^{i\theta})).\end{aligned}$$

Conclusion

In this paper we investigate the notion of hesitant QC-fuzzy soft ring .This work focused on hesitant QC-fuzzy soft rings of Q-fuzzy soft ring. To extend this work one could study the some new properties of fuzzy soft sets in other algebraic structure .

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