

## **Ranking of Football Players by DEA-Super Efficiency Model: An Evidence From English Premier League 2016/17 Season**

**Dr. Zahoor ul Haq Bhat<sup>1</sup>, Dr. Naseer Ahmad Bhat<sup>1</sup>, Dr. Qaiser Farooq Dar<sup>2</sup>**

<sup>1</sup>Lecturer, Department of Physical Education & Sports, University of Kashmir, Hazratbal Srinagar-190006, Email: [haqzahoorbhat@gmail.com](mailto:haqzahoorbhat@gmail.com)

<sup>2</sup>PDF, Incheon National University, Incheon South Korea. Email: [qaiserdea@gmail.com](mailto:qaiserdea@gmail.com)

### **Abstract**

In the professional and competitive sport ranking and rating of players and teams in response to their performance is vital. There is no universal method of ranking which can be applied to all sports. In this study we suggest Data Envelopment Analysis super efficiency technique for ranking of football players in English Premier League 2016/17 season. Performance of 46 forwards and 31 midfielders was evaluated and subsequently ranked who had played a minimum of 1000 minutes and had scored at least 3 goals during the season based on their input and output statistics. According to the CCR DEA model, only twelve players reached the performance of 100%. This means that only 26.086% of the sample reached the efficiency frontier. The average CCR performance of the forward players during the 2016/17 was 0.85%. The results showed that midfielders were more efficient than forwards but strikers were more successful in scoring goals. The use of super efficiency analysis sets out to rank the DMUs which allows an unbiased assessment of player performance. Such an efficiency measuring system would provide useful information to management of football clubs, their coaches and to football players themselves.

**Key Words:** Data Envelopment Analysis, Forwards, Midfielders, Super efficiency, Football.

### **1. Introduction**

Manchester United is efficient or Chelsea. If you have Lionel Messi or Cristiano Ronaldo, who would you choose. The question “who is efficient or who is better” is the one that everyone associated with sport seeks to find. Abstract and subjective elements have a great deal of bearing on this discourse; and mystique, intensity, specialized quality, class and sportsmanship all raise a player's prestige in the supporters' eyes Santin (2014). For example, in football, basketball, baseball etc. the national and international prizes won by the players, as well as an ever expanding array of statistics support in evaluating the efficiency and performance of players and additionally are essential factors for their rating and ranking. Furthermore, different players can only be compared taking into account how many seasons they played for the same team or their performance across a particular season. For instance, it is unjustifiable to reason that player  $x$  who won four cups in four seasons was less efficient than player  $y$  who earned six cups across 13 seasons.

Efficiency analysis in football is a well established line of research making use of different methodological approaches such as parametric and non parametric methods. Parametric methods use

## Ranking of Football Players by DEA-Super Efficiency Model: An Evidence From English Premier League 2016/17 Season

the tools of deterministic correlation and regression analysis in order to identify relationship between the variables and the functional dependence Kulikova and Goshunova (2014). On the other hand non parametric methods focus on the overall assessment of the efficiency, based on the analysis of a set of inputs and outputs Kulikova and Goshunova (2014). The main tool used in our paper is the non parametric Data Envelopment Analysis (DEA) method of player's efficiency measurement. Also we used the Super Efficiency ranking method introduced by Andersen and Petersen (1993) to discriminate the performance of efficient Decision Making Units (DMUs). The aim of this paper is to analyze the technical efficiency and super efficiency of forward playing and midfielders who participated in the English Premier League during 2016/17 season. The article is organized as follows. In section 2 a brief review of literature is presented. Section 3 illustrates DEA and super efficiency model used in this paper. Section 4 describes the data and the variables employed for analysis. Results of analysis are reported in section 5 and finally, section 6 concludes the paper.

### 2. Literature review

Several authors have estimated efficiency in sports and particularly football. To mention a few Haas (2003a) analyzed the efficiency of English Premier League clubs by DEA. Haas (2004b) evaluated efficiency of German Bundesliga football teams employing DEA procedure. The DEA methodology was also applied to estimate the efficiency of 20 soccer teams that participated in the Spanish First Division during 1998/99 to 2000/01 seasons by Espitia and Cebrian (2004). The performance of English Premier League football clubs during 1998/99 to 2002/03 seasons was estimated by Barros and Leach (2006) to find out if wealth affects sports performance. Alp (2006) analyzed goalkeeper's super efficiency during 2002 FIFA World Cup. Garcia-Sanchez (2007) determined the efficiency of the offense and defense, athletic efficacy and social productiveness of 20 football teams that participated in First Division of the Spanish league for 2004/05. Using mathematical optimization DEA method Bosca et al. (2009) focused on an analysis of correspondence and dissimilarity between Spanish and Italian football league by estimating their technical efficiency in three seasons from 2000 to 2003. Santin (2014) proposed super-efficiency DEA model to measure the performance of Real Madrid's football players as per their position in the field: forwards, midfielders, defenders, and goalkeepers. Other games have also been scrutinized by DEA to evaluate the efficiency such as baseball (Anderson and Sharp, 1997), basketball (Cooper et al. 2009), cricket (Singh, 2011), Olympics (Lozano et al. 2002) and tennis (Ramon et al. 2012).

The empirical application reported in this paper analyzes the technical efficiency and super efficiency of forwards and midfielders of the English Premier League 2016/17 season in order to figure out how efficiency tools can be used to point out the top performing players in the considered season.

### 3. DEA and Super Efficiency

The technique of frontier analysis was first suggested by Farrell (1957) and the mathematical framework was developed by Charnes, Cooper and Rhodes (1979). DEA is a nonparametric linear programming method which forms a linear function that envelops the best practicing DMUs so that all units lie on or below the frontier Doble (1995). Efficiency in DEA is the ratio of the weighted sum of outputs to the weighted sum of inputs of a DMU. In sports, a DMU can be an athlete, a player, manager of a team, coach, sports federation/sports club, or a game. The utility of DEA lies in that it provides information about an inefficient DMU by determining the value of inputs and outputs.

Figure 1 gives the graphical interpretation of super-efficiency model. An illustration is based on seven *DMU*'s as in the figure 1 for the sample example. The efficient frontier comprises of the line segment connecting *DMU*'s *B\**, *C\** and *F\**. If *DMU C* is excluded from the reference set, the frontier shifts and new frontier is consisting of *B\**, *C\** and *F\**. The super efficiency of *DMU C\** becomes  $OC^*/OC \geq 1$ . This infers that *DMU C\** could increase both inputs and still stays efficient.

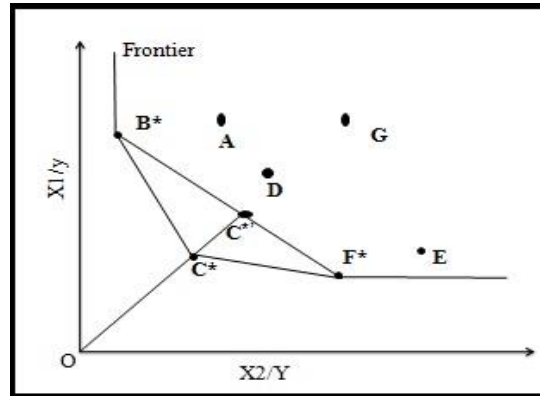


Figure 1

Andersen and Petersen (1993) introduced the super efficiency model as a ranking technique to discriminate the performance of efficient decision making units (DMUs). A DMU is any entity which is under observation and is responsible for transforming inputs into outputs. The formulation of the super-efficiency model is sensibly clear, whereby the column pertaining to the *DMU* being scored is excluded from the DEA envelopment linear program (LP) technology matrix. This generates super-efficiency scores for each *DMU*. The production possibility set  $P(x_{i0}, y_{r0})$  spanned by  $(X, Y)$  of super-efficiency for *DMU0* is defined as given below.

$$P = \left[ \begin{array}{l} (x_{i0}, y_{r0}) \text{ s/t } x_{i0} \geq \sum_{\substack{j=1 \\ j \neq 0}}^n \sum_{i=1}^m \lambda_j x_{ij} \\ \text{and } y_{r0} \leq \sum_{\substack{j=1 \\ j \neq 0}}^n \sum_{r=1}^s \lambda_j y_{rj} \quad \forall \lambda_j \geq 0 \end{array} \right]$$

Under assumption  $X \geq 0, Y \geq 0$  and  $P(x_{i0}, y_{r0})$  is non-empty set.

The super-efficiency for  $n - DMUs$  using  $m$ -input and  $s$ -output can be defined as, let  $x_{ij}$  and  $y_{rj}$  denotes  $i^{th}$  input,  $i = 1, 2, 3, \dots, m$  and  $r^{th}$  output,  $r = 1, 2, 3, \dots, s$  respectively of the  $j^{th}$  *DMU*;  $j = 1, 2, 3, \dots, n$ . The super -efficiency can be calculating by using mathematical model as given below, under the assumption that *DMU0* should be efficient;

Ranking of Football Players by DEA-Super Efficiency Model: An Evidence From English Premier League 2016/17 Season

$$\text{Min } \rho_0 = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_{i0}^-}{x_{i0}}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_{r0}^+}{y_{r0}}}$$

Subject to

$$\sum_{\substack{j=1 \\ j \neq 0}}^n \lambda_j x_{ij} + s_i^- = x_{i0} ; i = 1, 2, 3, \dots, m$$

$$\sum_{\substack{j=1 \\ j \neq 0}}^n \lambda_j y_{rj} - s_r^+ = y_{r0} ; r = 1, 2, 3, \dots, s$$

$$\rho_0, \lambda_j (j \neq 0) \geq 0$$

$$s_i^- \geq 0, s_r^+ \geq 0 \quad \forall j = 1, 2, 3, \dots, n$$

Where  $s_{i0}^-$  and  $s_{r0}^+$  are the slacks of  $r^{\text{th}}$  output and  $i^{\text{th}}$  input for evaluating an efficient  $DMU_0$ , and ( $\rho \geq 0$ ) is a super-efficiency value.

$$\text{Min } \rho_0 = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_{i0}^-}{x_{i0}}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_{r0}^+}{y_{r0}}}$$

Subject to

$$\sum_{\substack{j=1 \\ j \neq 0}}^n \lambda_j x_{ij} + s_i^- = x_{i0} ; i = 1, 2, 3, \dots, m$$

$$\sum_{\substack{j=1 \\ j \neq 0}}^n \lambda_j y_{rj} - s_r^+ = y_{r0} ; r = 1, 2, 3, \dots, s$$

$$\rho_0, \lambda_j (j \neq 0) \geq 0$$

$$s_i^- \geq 0, s_r^+ \geq 0 \quad \forall j = 1, 2, 3, \dots, n$$

Where  $s_{i0}^-$  and  $s_{r0}^+$  are the slacks of  $r^{\text{th}}$  output and  $i^{\text{th}}$  input for evaluating an efficient  $DMU_0$ , and ( $\rho \geq 0$ ) is a super-efficiency value.

#### 4. Data and variables

The data used for the empirical analysis has been taken and compiled from various authentic and reliable sources like (<https://www.premierleague.com/stats> and <https://www.whoscored.com>). Only those players have been included who have played a minimum of 1000 minutes and have scored at least 3 goals during the season. These constraints reduce the data set of forward playing players to 46 and midfielders to 31.

The selection of variables for this study primarily follows previous studies in the literature. Availability of data was also a factor in determining the list of input/output variables. The two inputs employed are the number of matches and number of minutes played by each player during the season. On the other hand, we include four outputs which are considered important for both playing positions. The first output is the number of goals scored by the player. In several sports including football goal is the sole method of scoring and determines the offensive ability of a player. The second output is number of assists contributed by a player which helps to score a goal. It also represents a player's offensive power. The third output is the ball winning ability or tackle which is an act or move to deprive an opponent of the ball, to stop the player from gaining ground towards goal or to stop them from carrying out what they intend. In other words it simply describes one or more methods of contesting for possession of the ball. It can therefore be used as both a defensive or attacking move. Finally, the fourth output is the pass success percentage which is determined by dividing the accurate passes by the total passes attempted multiplied by hundred. It is a measure of player's contribution to effective play development. Table 1 displays the raw data of inputs and outs considered for forward playing players and Table 2 shows data of midfielders.

**Table 1: Input/output data of forward players**

Player Name	Matches	Min played	Goals	Assists	Tackles	Pass Success %
Abel Hernández	24	1428	4	3	8	72.2
Alexis Sánchez	38	3223	24	10	53	73.6
Álvaro Negredo	36	2877	9	4	18	66.9
André Ayew	25	1440	6	3	17	79.1
Andre Gray	32	2267	9	3	10	79.1
Anthony Martial	25	1559	4	6	17	81.3
Ashley Barnes	28	1775	6	2	21	55
Benik Afobe	31	1454	6	3	9	78.8
Callum Wilson	20	1363	6	0	6	72.8
Christian Benteke	36	3167	15	2	9	59.6
Cristhian Stuani	23	1361	4	0	35	68
Diego Costa	35	3089	20	7	18	75.2
Divock Origi	34	1459	7	3	11	76.2
Dusan Tadic	33	2426	3	5	19	79.3
Eden Hazard	36	3002	16	5	18	84.2
Fernando Llorente	33	2453	15	1	18	66.4
Harry Kane	30	2531	29	7	22	71.7
Islam Slimani	23	1279	7	4	12	70.7
Jamie Vardy	35	2808	13	5	15	62.5
Jermain Defoe	37	3322	15	2	20	79.3
Jonathan Walters	23	1296	4	2	8	57.1
Joshua King	36	2719	16	2	49	79.6
Kevin Mirallas	35	2080	4	6	18	82.9
Leroy Sané	26	1786	5	3	35	81.3
Marcus Rashford	32	1701	5	1	16	78.7
Marko Arnautovic	32	2724	6	5	32	75.4
Olivier Giroud	29	1194	12	3	14	71.5
Pedro	35	2151	9	9	48	82.7

Ranking of Football Players by DEA-Super Efficiency Model: An Evidence From English Premier  
League 2016/17 Season

Peter Crouch	27	1335	7	2	4	59.1
Raheem Sterling	33	2515	7	6	24	80.4
Roberto Firmino	35	3068	11	7	58	79.4
Romelu Lukaku	37	3267	25	6	9	65.5
Sadio Mané	27	2240	13	5	23	77.7
Salomón Rondón	38	2895	8	2	18	66.1
Sam Vokes	37	2055	10	3	7	50.4
Sergio Agüero	31	2403	20	3	18	82.4
Shane Long	32	1248	3	1	12	69.9
Shinji Okazaki	30	1572	3	1	30	74.9
Son Heung-Min	34	2068	14	6	19	81.2
Theo Walcott	28	1925	10	2	36	75.3
Troy Deeney	37	2941	10	4	20	58.5
Wayne Rooney	25	1539	5	5	21	84.2
Wilfried Zaha	35	3025	7	9	76	79.9
Willian	34	1529	8	2	19	83.5
Xherdan Shaqiri	21	1707	4	2	11	78.7
Zlatan Ibrahimovic	28	2437	17	5	10	73.6

**Table 2: Input/output data of midfielders**

<b>Player Name</b>	<b>Matches</b>	<b>Min played</b>	<b>Goals</b>	<b>Assists</b>	<b>Tackles</b>	<b>Pass Success %</b>
Adam Lallana	31	2346	8	7	51	84.3
Andros Townsend	36	2526	3	4	77	75.2
Chris Brunt	31	2478	3	4	36	73.3
Christian Eriksen	36	3164	8	15	45	80.2
Coutinho	31	2243	13	7	35	83.9
David Silva	34	2766	4	7	43	87.2
Dele Alli	37	3043	18	7	52	77.8
Emre Can	32	2368	5	2	66	81.6
E Georginio	37	3212	7	1	61	82.9
Wijnaldum	36	2975	6	9	46	87.3
Gylfi Sigurdsson	38	3328	9	13	45	77
James Milner	36	3159	7	3	94	79.7
James Morrison	31	1738	5	2	26	80.8
James Ward-Prowse	30	1884	4	4	29	83.1
Joe Allen	36	2936	6	2	86	81.3
Kevin De Bruyne	36	2883	6	18	50	82
Leroy Fer	34	2403	6	2	55	82.2
Manuel Lanzini	35	2711	8	2	44	87.3
Mark Noble	30	2398	3	0	49	86.1
Marten de Roon	33	2777	4	0	73	79.2

Mesut Özil	33	2847	8	9	26	86.6
Nacer Chadli	31	2147	5	5	29	80.4
Nathan Redmond	37	2907	7	1	39	75.1
Paul Pogba	30	2608	5	4	53	85.1
Riyad Mahrez	36	2835	6	3	36	77.6
Robert Snodgrass	35	2550	7	5	38	82.9
Ross Barkley	36	2905	5	8	30	82.5
Sam Clucas	37	3188	3	1	89	83.2
Victor Moses	34	2494	3	2	47	79.2
Victor Wanyama	36	3015	4	1	90	87.5
Yohan Cabaye	32	2132	4	3	60	80.3

**Table 3: Descriptive statistics of forwards and midfielders**

Variable	Forward (N = 46)		Midfielder (N = 31)	
	Mean	SD	Mean	SD
Matches	31.11	5.03	34.09	2.53
Min played	2167.45	678.43	2676.32	400.69
Goals	10.02	6.3	6.12	3.11
Assists	3.84	2.38	4.86	4.37
Tackles	21.54	14.94	51.61	19.59
Pass success%	73.51	8.61	81.7	3.8

## 5. Results

The technical efficiency and super efficiency was determined by using Efficiency measurement System (EMS) DEA software. Its advantage is that it not only measures normal efficiency, but also super efficiency for players having efficiency score more than 1. The results for forward player's efficiency are shown in Table 4.

**Table 4: Forwards' efficiency results**

DMU	CCR	Super efficiency	Final rank
Abel Hernández	0.92	0.918	22
Alexis Sánchez	1.00	1.072	8
Álvaro Negredo	0.60	0.599	42
André Ayew	0.99	0.991	13
Andre Gray	0.74	0.744	35
Anthony Martial	1.00	1.085	7
Ashley Barnes	0.66	0.662	40
Benik Afobe	0.93	0.935	18
Callum Wilson	1.00	1.086	6
Christian Benteke	0.58	0.580	43
Cristhian Stuani	1.00	1.209	5
Diego Costa	0.83	0.835	27
Divock Origi	0.88	0.881	26

Ranking of Football Players by DEA-Super Efficiency Model: An Evidence From English Premier  
League 2016/17 Season

Dusan Tadic	0.72	0.720	37
Eden Hazard	0.77	0.774	32
Fernando Llorente	0.71	0.715	39
Harry Kane	1.00	1.468	1
Islam Slimani	1.00	1.018	12
Jamie Vardy	0.63	0.630	41
Jermain Defoe	0.72	0.716	38
Jonathan Walters	0.79	0.791	30
Joshua King	0.95	0.949	16
Kevin Mirallas	0.76	0.762	34
Leroy Sané	1.00	1.023	11
Marcus Rashford	0.82	0.821	28
Marko Arnautovic	0.79	0.786	31
Olivier Giroud	1.00	1.316	3
Pedro	1.00	1.250	4
Peter Crouch	0.77	0.773	33
Raheem Sterling	0.79	0.793	29
Roberto Firmino	0.93	0.931	20
Romelu Lukaku	0.72	0.723	36
Sadio Mané	0.97	0.971	14
Salomón Rondón	0.55	0.547	45
Sam Vokes	0.52	0.523	46
Sergio Agüero	0.93	0.931	21
Shane Long	0.94	0.935	19
Shinji Okazaki	0.90	0.898	25
Son Heung-Min	0.90	0.899	24
Theo Walcott	0.96	0.959	15
Troy Deeney	0.55	0.549	44
Wayne Rooney	1.00	1.057	10
Wilfried Zaha	1.00	1.317	2
Willian	0.94	0.939	17
Xherdan Shaqiri	1.00	1.065	9
Zlatan Ibrahimovic	0.92	0.917	23

According to the CCR DEA model, only twelve players reached the performance of 100% Table 4. This means that only 26.086% of the sample reached this performance. André Ayew (0.99) and Sadio Mané (0.97) were both close to the efficiency frontier. On the other hand, the lowest CCR performance scores reached Troy Deeney (0.55), Salomón Rondón (0.55) and Sam Vokes (0.52) see Table 4. The average CCR performance of the forward players during the 2016/17 was 0.85. Six forward playing players (Diego Costa, Eden Hazard, Raheem Sterling, Sergio Agüero, Wayne Rooney and Zlatan Ibrahimovic ) figured among the top ten highest paid players for 2016/17 playing season. Among these Wayne Rooney was the sole player to be efficient as he had played less matches and spent less time on ground as compared to others.



The analysis reveals that there are a number of CCR efficient units, which restricts one to rank them. For inefficient units one can directly rank them according to their respective scores, while it is not possible for efficient ones, as all are having the same efficiency score as one, in order to discriminate between them super efficiency model was used which gives the score as greater than one. On the basis of super efficiency scores ranking of all DMUs was done and their ranks are displayed in “Final rank” column. Harry Kane was the most super efficient forward playing player as per our analysis which can be ascertained by his bagging of golden boot and player of month for four times during 2016/17 season.

**Table 5: Midfielders' efficiency results**

<b>DMU</b>	<b>CCR</b>	<b>Super efficiency</b>	<b>Final rank</b>
Adam Lallana	1.00	1.053	7
Andros Townsend	1.00	1.043	9
Chris Brunt	0.84	0.845	29
Christian Eriksen	0.96	0.961	16
Coutinho	1.00	1.220	2
David Silva	0.94	0.940	20
Dele Alli	1.00	1.185	3
Emre Can	1.00	1.016	13
Etienne Capoue	0.86	0.857	26
Georginio Wijnaldum	0.92	0.917	22
Gylfi Sigurdsson	0.87	0.874	24
James Milner	1.00	1.120	4
James Morrison	1.00	1.079	5
James Ward-Prowse	1.00	1.046	8
Joe Allen	0.99	0.992	15
Kevin De Bruyne	1.00	1.317	1
Leroy Fer	0.93	0.927	21
Manuel Lanzini	0.89	0.894	23
Mark Noble	1.00	1.026	12
Marten de Roon	0.95	0.952	17
Mesut Özil	0.99	0.994	14
Nacer Chadli	0.94	0.942	19
Nathan Redmond	0.73	0.728	31
Paul Pogba	1.00	1.032	11
Riyad Mahrez	0.77	0.768	30
Robert Snodgrass	0.86	0.856	27
Ross Barkley	0.86	0.859	25
Sam Clucas	0.95	0.950	18
Victor Moses	0.85	0.852	28
Victor Wanyama	1.00	1.037	10
Yohan Cabaye	1.00	1.073	6

Table 5 indicates the efficiency scores of midfielders with Kevin De Bruyne being the most efficient and Nathan Redmond being the least efficient midfielder. Paul Pogba was the highest paid

Ranking of Football Players by DEA-Super Efficiency Model: An Evidence From English Premier League 2016/17 Season

player with an income of £290,000 per week during the 2016/17 playing season. Although he was CCR efficient but attained eleventh rank as per super efficiency score. The average CCR performance of the midfielders during the 2016/17 was 0.94.

**Table 6: Efficient football players & their teams (*in parentheses*) in the 2016/17 playing season**

<b>Forward</b>	<b>Midfield</b>
Harry Kane ( <i>Tottenham Hotspur</i> )	Kevin De Bruyne ( <i>Manchester City</i> )
Wilfried Zaha ( <i>Crystal Palace</i> )	Coutinho ( <i>Tottenham Hotspur</i> )
Olivier Giroud ( <i>Chelsea</i> )	Dele Alli ( <i>Tottenham Hotspur</i> )
Pedro ( <i>Chelsea</i> )	James Milner ( <i>Liverpool</i> )
Cristhian Stuani ( <i>Middlesbrough</i> )	James Morrison ( <i>W B Albion</i> )
Callum Wilson ( <i>AFC Bournemouth</i> )	Yohan Cabaye ( <i>Manchester United</i> )
Anthony Martial ( <i>Manchester United</i> )	Adam Lallana ( <i>Liverpool</i> )
Alexis Sánchez ( <i>Tottenham Hotspur</i> )	James Ward-Prowse ( <i>Southampton</i> )
Xherdan Shaqiri ( <i>Manchester United</i> )	Andros Townsend ( <i>Crystal Palace</i> )
Wayne Rooney ( <i>Manchester United</i> )	Victor Wanyama ( <i>Tottenham Hotspur</i> )
Leroy Sané ( <i>Manchester City</i> )	Paul Pogba ( <i>Manchester United</i> )
Islam Slimani ( <i>Manchester City</i> )	Mark Noble ( <i>West Ham United</i> )
	Emre Can ( <i>Liverpool</i> )

## 6. Conclusion

In this paper we proposed the use of output oriented CRS and super efficiency model to determine the efficiency of strikers (forwards and midfielders) in English Premier League for 2016/17 playing season. Regarding the analysis of football players Manchester United and Tottenham Hotspur players have ruled the roost. Some players which strongly dominate in the championship and at the same time being most expensive players like Zlatan Ibrahimovic, Eden Hazard, Raheem Sterling and Diego Costa are not most the most efficient players because of the short falls in outputs.

The use of super efficiency analysis sets out to rank the DMUUs which allows an impartial evaluation of player performance. Such an efficiency measuring system would provide useful information to management of football clubs, their coaches and to football players themselves.

## References

1. Farrell, M. and E. Pearson, *SERIES A (GENERAL)*. Journal of the Royal Statistical Society. Series A (General), 1957. **120**(3): p. 253-29.
2. Charnes, A., W.W. Cooper, and E. Rhodes, *Measuring the efficiency of decision-making units*. European Journal of Operational Research, 1979. **3**(4): p. 339-338.
3. Doble, M., *Measuring and improving technical efficiency in UK post office counters using data envelopment analysis*. Annals of Public and Cooperative Economics, 1995. **66**(1): p. 31-64.
4. Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management science*, 39(10), 1261-1264.
5. Anderson, T. R. and Sharp, G. P. (1997) 'A new measure of baseball batters using DEA', *Annals of Operations Research*, 73, pp. 141–155.
6. Barros, C. P. and Leach, S. (2006) 'Performance evaluation of the English Premier Football

League with data envelopment analysis', *Applied Economics*, 38(12), pp. 1449–1458.

7. Boscá, J. E. *et al.* (2009) 'Increasing offensive or defensive efficiency? An analysis of Italian and Spanish football', *Omega*, 37(1), pp. 63–78.
  8. Cooper, W. W., Ruiz, L. and Sirvent, I. (2009) 'Selecting Non-zero Weights to Evaluate Effectiveness of Basketball Players With DEA', *European Journal of Operational Research*, 195(2), pp. 563–574.
  9. Espitia-Escuer, M. and García-Cebrián, L. I. (2004) 'Measuring the efficiency of Spanish first-division soccer teams', *Journal of Sports Economics*, 5(4), pp. 329–346.
  10. García-Sánchez, I. M. (2007) 'Efficiency and effectiveness of Spanish football teams: a three-stage-DEA approach', *Central European Journal of Operations Research*, 15(1), pp. 21–45.
  11. Haas, D. J. (2003a) 'Productive efficiency of English football teams—a data envelopment analysis approach', *Managerial and Decision Economics*, 24(5), pp. 403–410.
  12. Haas, D. J. (2003b) 'Technical efficiency in the major league soccer', *Journal of Sports Economics*, 4(3), pp. 203–215.
  13. İhsan, A. L. P. (2006). Performance evaluation of goalkeepers of the world cup. *Gazi University journal of science*, 19(2), 119-125.
  14. Kulikova, L. I. and Goshunova, A. V. (2014) 'Efficiency measurement of professional football clubs: a non-parametric approach', *Life Science Journal*, 11(11), pp. 117–122.
  15. Lozano, S. *et al.* (2002) 'Measuring the performance of nations at the Summer Olympics using data envelopment analysis', *Journal of the Operational Research Society*, 53(5), pp. 501–511.
  16. Ramón, N., Ruiz, J. L. and Sirvent, I. (2012) 'Common sets of weights as summaries of DEA profiles of weights: With an application to the ranking of professional tennis players', *Expert Systems with Applications*, 39(5), pp. 4882–4889.
  17. Santín, D. (2014) 'Measuring the technical efficiency of football legends: who were Real Madrid's all-time most efficient players?', *International Transactions in Operational Research*, 21(3), pp. 439–452.
- Singh, S. (2011) 'Measuring the performance of teams in the Indian premier league', *American Journal of Operations Research*, 1(3), pp. 180–184.