

Reduction Attributes By Using Pretopological Space Based on Similarity Matrix

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Abstract

Attribute reduction and function selection have end up one of the essential steps for pattern recognition and machine getting to know responsibilities. The purpose of attribute reduction is to discover a minimal attribute subset that satisfies a few particular criteria, even as the minimal attribute subset is referred to as attribute reduce. . In this paper a pretopological space in an information system is to be constructed, as well as defining some essential operations for the pretopological space and a brief study of its properties will be presented. Moreover, we use similarity method by using pretopological space' matrix to reduce the attribute. We will compare the pretopologies resulting after the removing of the attributes with the original pretopology resulting from the information system.

Keywords

Pretopological Space; Closed Set; Open Set; Closure Set; Interior Set; Information System.

1. Introduction

Attribute reduction is one of the most critical research issues troubles inside the pretopological space. Initially introduced by Marcel Brissaud, Pretopology was established in 1975 as a branch of Topology [7]. Despite naming Brissaud as the father of Pretopology, for giving the first definition of a pretopological space, Belmandt was the one who developed those concepts and published the first book in 1993[16]; followed by a second book which was published in 2011[15]. And since, pretopology became an important tool that deals with mathematical models to measure the concepts of proximity and distance between groups and points; and started to play an important role in many different applications such as: economical modeling [5], image analysis [11], data analysis [12], graph theory generalization [8], complex networks modeling [13], modeling the impact of geographic proximity on scientific collaborations [2], analyzing the evolution of a communicable disease [1], Data Structure [14]., and Fuzziness and Soft Computing [4].

The aim of this paper is to introduce a new methodology for preprocessing the information system by constructing its pretopology space.

The remaining of this paper is structured as follows: In section 2, the fundamental definitions from the Pretopological space is introduced. While, section 3 is dedicated to generate a pretopological space from

an information system. And hence, the method to reduce attributes from information system by using pretopological space is presented in section 4. In section 5 is presented an illustrative example to generate a pretopological space from an information system and show the results of the reduction of attributes. To conclude, section 6 gives a brief conclusion and a plan for future work.

The structure of the paper is as follows:

2. Basic concepts

2.1 Pretopological space

The concept of pretopology [8] comes from the concept of topology, as the concept of topology can be defined via Open sets, Neighborhoods and A Kuratowski Closure Operator [7]. For any set X , any given mapping $a: P(X) \rightarrow P(X)$ is said to be a pseudo Closure Operator, if it satisfies the following axioms:

- i. $a(\emptyset) = \emptyset$.
- ii. $A \subseteq a(A), \forall A \subseteq X$.

2.2 Interior function

For any set (X, a) , any interior function $i: P(X) \rightarrow P(X)$ defined as;

$$\text{int}(A) = \text{co}(a(\text{co}A)) = \overline{a(\overline{A})}.$$

Where $\text{co}A$ is complement of A , \overline{A} is also complement of A .

2.3 Knowledge base

A knowledge base is an ordered pair (X, R) , where X is a nonempty set of objects and R is an equivalence relation over X , a family $K = \{U_1, U_2, U_3, \dots, U_n\}$ where $U_i \subseteq X, U_i \neq \emptyset, U_i \cap U_j = \emptyset$ for $i \neq j, i, j = 1, 2, 3, \dots, n$ and $\cup U_i = X$, in this case, the class K is said to be knowledge base of (X, R) .

2.4 Information System

An information system (IS, for short) [1, 2] is an ordered pair $IS = (U, Q, V)$, such that U is a nonempty finite set of objects called universe where $U = \{x_1, x_2, x_3, \dots, x_n\}$. (students, toy blocks,), Q is a nonempty set of attributes (colors, characteristic,), and V is an attribute value set where $V = \{v_0, v_1, v_3, \dots, v_n\}$.

3. New Definitions of Pretopological Spaces used in Information System (Similarity Matrix)

In our work, we aim to develop a new type of a pretopological space used in information system by using similarity method.

3.1 Definition

Let $U = \{x_1, x_2, x_3, \dots, x_n\}$ be a set of objects, let $x \in U$ be a set on U and let η_a is a relation between x_i, x_j , its value is determined by δ .

Then;

$$\eta_a = \{x \in U / \mu(x_i, x_j \geq \delta), 1 \geq \delta \geq 0, i, j \in [1, n], n = |U|\}$$

Where η_a is called a pretopological space used in information system.

Note: The cardinality of a set, which is basically the size of the set. The cardinality of a set is denoted by $|U|$.

3.2 Lemma

When choosing the values for μ , the conditions of pretopology space denoted in [8] must be considered.

3.3 Proposition

For any $x \in U$, we can get the following properties of the pretopology and interior approximations;

1. $x \subseteq \eta_a\{x\}$

2. $\text{int}_{\eta_a}\{x\} \subseteq x$
3. $\eta_a U = \text{int}_{\eta_a} U = U$
4. $\eta_a \emptyset = \text{int} \emptyset = \emptyset$
5. $\eta_a(x \cup y) = \eta_a x \cup \eta_a y$
6. $\eta_a(x \cap y) = \eta_a x \cap \eta_a y$
7. $\text{co} \eta_a\{x\} \subset \eta_a\{\text{co}x\}$
8. $\text{int}_{\eta_a}\{\text{co}x\} \subset \text{co} \text{int}_{\eta_a}\{x\}$
9. $\eta_a(\eta_a x) = \eta_a(x)$
10. $\text{int}_{\eta_a}(\text{int}_{\eta_a} x) = \text{int}_{\eta_a}(x)$

3.4 Definition

Given information system (IS), defines a matrix M_A , called a similarity matrix, each entry $M_Q(x, y) \subseteq Q$ consists of a set of attributes that can be discern between objects $x, y \in U$, and we can be described a matrix M_A is an $|U| \times |U|$ matrix, and has the form:-

$$M_{ij}(x, y) = \{a \in V / M_{ij} = \frac{\sum_{i=1}^n \sum_{j=1}^n |\beta|}{\sum_{k=1}^m |Q|}\}$$

where $\beta = \{a(x_i) = a(y_i)\}$ and $|Q|$ is a cardinal number of a set of attributes Q .

3.5 Lemma

When constructing a dissimilarity matrix, we have to take into account that main diagonal elements are equal to correct one (this element referred to achieve that $(x_i = x_j)$)

That is $b_{ii} = 1$ where b_{ii} is called the main diagonal, b_{ij} is an element in a matrix M_{ij} .

3.6 Definition

For any set (X, a) , any interior function $i: P(X) \rightarrow P(X)$ defined as;

$$\text{int}_{\eta_a}(A) = \text{co}(\eta_a(\text{co}A)) = \overline{\eta_a(\overline{A})}.$$

Where $\text{co}A$ is complement of A , \overline{A} is also complement of A .

3.7 Definition

Let (X, a) be a pretopological space, $\forall A \subseteq X$; A is said to be a closed subset of X if and only if

$$\eta_a(x) = x.$$

3.8 Definition

Let (X, a) be a pretopological space, $\forall A \subseteq X$; A is said to be an open subset of X if and only if

$$x = \text{int}_{\eta_a}(x).$$

4. Reduction of Attributes

The aim of this section, is to experiment the effect on the constructed pretopology when removing some attribute from the original information system. For that purpose, we will remove one attribute at a time.

4.1 The New Proposed Set of Reduction of Attributes

The following method shows how remove one attribute from original pretopology:-

Let Q is a set of attribute, $V \subseteq Q$, V is one attribute in Q , let V is unnecessary attribute in Q if:

$$\eta_a = \eta_a(Q - \{V\})$$

then we find open and closed set, when removing one attribute, then follow the following method:

1. $\eta_a = \eta_a(Q - \{V\})$
2. $\eta_a\{x\} = x$ (condition of closed set)
3. $\text{int}\{x\} = x$ (condition of open set)

and when achieve the above three properties, this shows that this attribute is unnecessary, and but when these properties not achieve, this shows that this attribute is necessary and so this attribute is called the core.

4.2 An Algorithm for the new proposed set

Algorithm Reduction Attributes Algorithm based on Similarity Matrix Method.

Input: An information system $IS = (U, Q, V)$, where $U = \{x_1, x_2, x_3, \dots, x_n\}$, Q is a set of attributes and V is an attribute value set where $V = \{v_0, v_1, v_3, \dots, v_n\}$.
 // v_0 denotes the first attribute.

Output: An attribute reduction of this information system.

Step 1 Create a similarity matrix $|U| \times |U|$.

Step 2 Calculate a matrix elements using,

$$M_{ij}(x, y) = \{a \in V / M_{ij} = \frac{\sum_{i=1}^n \sum_{j=1}^n |\beta|}{\sum_{k=1}^m |Q|}\}$$

where $\beta = \{a(x_i) = a(y_i)\}$ and $|Q|$ is a cardinal number of a set of attributes Q .

Step 3 If $(x_i = x_j)$, then put $b_{ii} = 1, i \in [1, n]$.

Step 4 Calculate

$$\eta_a = \{X \in U / \mu(x_i, x_j \geq \delta), 1 \geq \delta \geq 0, i, j \in [1, n], n = |U|\}$$

Step 6 Calculate $int_{\eta_a}(x_i) = co(\eta_a(cox_i)), i \in [1, n]$.

Step 7 For each product pretopological space, Calculate closed set $\eta_a\{x_i\} = \{x_i\}$ and open set $int_{\eta_a}\{x_i\} = \{x_i\}$.

Step 8 For each attribute $V \in Q$, Update the attribute $Q = Q - \{v_0\}$, and then go to Step 1.

Step 9 If $(\eta_a \neq \eta_a(Q - \{V\}))$, $(\eta_a\{x_i\} \neq \{x_i\})$ and $(int_{\eta_a}\{x_i\} \neq \{x_i\})$, then this attribute is core, then the algorithm terminates.

5. Generating pretopological space from information system:

In this section, we give an example to demonstrate how to construct the pretopology from some information system; how to use the properties of the pretopology for the purpose studying the attributes and the relationship between the different pretopologies construct from information system.

5.1 Example of Information System

In this subsection, we give an example from information system, which consists of eight electrical transformers; $E = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}$ and four attributes; transmission lines A and B, self-generation C and self-load D;

The following (table 1) shows the information system:

	Trans-Line A	Trans-Line B	Self-Gen C	Self-Load D
E₁	medium	medium	medium	medium
E₂	medium	low	low	medium
E₃	medium	low	low	medium
E₄	low	low	low	high
E₅	low	medium	medium	medium
E₆	medium	medium	low	high
E₇	medium	medium	low	low
E₈	medium	medium	low	high

Table 1

5.1.1 Create a pretopology from an information system using a matrix

For the example given in section 3.1, we propose the following methodology:

Firstly, five 8 * 8 matrices representing five pretopologies which are deduced from the given information system, is to be constructed.

Secondly, use the following relation to find a pretopology for the attributes; denoted by α :

$$\eta_{\alpha} E = \{E: \mu(E_i, E_j) = 1 \quad i, j = 1, 2, 3, \dots, 8\} \quad \text{Eq. (1)}$$

Thirdly, the interior of all the attributes; denoted by int ; Then, is to be found using the relation:

$$\text{int } E = \overline{\alpha\{E\}} \quad \text{Eq. (2)}$$

Fourthly, use Definition 2.6 in order to find the following closed sets:

$$\eta_{\alpha}\{E\} = E \quad \text{Eq. (3)}$$

Finally, use Definition 2.7 in order to find the following open sets:

$$\text{int}\{E\} = E \quad \text{Eq. (4)}$$

5.1.2 Constructing a pretopology from information system:

To commence, pretopologies resulting from the four attributes are to be obtained as follows: for each pretopology an 8*8 matrix is to be constructed where each element along the main diagonal is equal one; while each element of the remaining elements of the matrix is to be equal to the number of the similar attributes divided by the total number of the attributes.

The following (table 2), shows the pretopology obtained from the information system.

	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8
E_1	1	$\frac{2}{4}$	$\frac{2}{4}$	0	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$
E_2	$\frac{2}{4}$	1	1	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$
E_3	$\frac{2}{4}$	1	1	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$
E_4	0	$\frac{2}{4}$	$\frac{2}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$
E_5	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
E_6	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{3}{4}$	1
E_7	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$
E_8	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	1	$\frac{3}{4}$	1

Table 2
Therefore, the of matrix by using Eq.

$$\eta_{\alpha_1}\{E_1\} = \{E_1\}$$

$$\eta_{\alpha_1}\{E_2\} = \{E_2, E_3\}$$

$$\eta_{\alpha_1}\{E_3\} = \{E_2, E_3\}$$

$$\eta_{\alpha_1}\{E_4\} = \{E_4\}$$

pretopology component (1)

$$\begin{aligned} \eta_{a_1}\{E_5\} &= \{E_5\} \\ \eta_{a_1}\{E_6\} &= \{E_6, E_8\} \\ \eta_{a_1}\{E_7\} &= \{E_7\} \\ \eta_{a_1}\{E_8\} &= \{E_6, E_8\} \end{aligned}$$

Therefore, the interior component of matrix by using Eq. (2)

$$\begin{aligned} \text{int}_{\eta_{a_1}}\{E_1\} &= \overline{\eta_{a_1}\{E_1\}} = \overline{\eta_{a_1}\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \{E_1\} \\ \text{int}_{\eta_{a_1}}\{E_2\} &= \overline{\eta_{a_1}\{E_2\}} = \overline{\eta_{a_1}\{E_1, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset \\ \text{int}_{\eta_{a_1}}\{E_3\} &= \overline{\eta_{a_1}\{E_3\}} = \overline{\eta_{a_1}\{E_1, E_2, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset \\ \text{int}_{\eta_{a_1}}\{E_4\} &= \overline{\eta_{a_1}\{E_4\}} = \overline{\eta_{a_1}\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \{E_4\} \\ \text{int}_{\eta_{a_1}}\{E_5\} &= \overline{\eta_{a_1}\{E_5\}} = \overline{\eta_{a_1}\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \{E_5\} \\ \text{int}_{\eta_{a_1}}\{E_6\} &= \overline{\eta_{a_1}\{E_6\}} = \overline{\eta_{a_1}\{E_1, E_2, E_3, E_4, E_5, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset \\ \text{int}_{\eta_{a_1}}\{E_7\} &= \overline{\eta_{a_1}\{E_7\}} = \overline{\eta_{a_1}\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \{E_7\} \\ \text{int}_{\eta_{a_1}}\{E_8\} &= \overline{\eta_{a_1}\{E_8\}} = \overline{\eta_{a_1}\{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset \end{aligned}$$

Therefore, we obtain closed sets by using **Eq. (3)**

$$\begin{aligned} \eta_{a_1}\{E_1\} &= \{E_1\} \\ \eta_{a_1}\{E_4\} &= \{E_4\} \\ \eta_{a_1}\{E_5\} &= \{E_5\} \\ \eta_{a_1}\{E_7\} &= \{E_7\} \end{aligned}$$

Therefore, we obtain open sets by using **Eq. (4)**

$$\begin{aligned} \text{int}_{\eta_{a_1}}\{E_1\} &= \{E_1\} \\ \text{int}_{\eta_{a_1}}\{E_4\} &= \{E_4\} \\ \text{int}_{\eta_{a_1}}\{E_5\} &= \{E_5\} \\ \text{int}_{\eta_{a_1}}\{E_7\} &= \{E_7\} \end{aligned}$$

5.1.3 Further pretopologies from the original information system:

The aim of this section is to experiment the effect on the constructed pretopology, when removing some attributes from the original information system. For that purpose we will remove on attribute at a time, hence we deduce the interior, the open and closed sets each time.

5.1.3.1 Constructing a pretopology when removing the attribute trans-line A:

The following (table 3), shows the pretopology obtained when removing the attribute trans-line A.

	E ₁	E ₂	E ₃	E ₄	E ₅	E ₆	E ₇	E ₈
E ₁	1	$\frac{1}{3}$	$\frac{1}{3}$	0	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E ₂	$\frac{1}{3}$	1	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

E_3	$\frac{1}{3}$	1	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E_4	0	$\frac{2}{3}$	$\frac{2}{3}$	1	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
E_5	1	$\frac{1}{3}$	$\frac{1}{3}$	0	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E_6	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	1
E_7	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{2}{3}$
E_8	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	1

Table 3

Therefore, the pretopology component of matrix by using Eq. (1)

$$\eta_{a_2}\{E_1\} = \{E_1, E_5\}$$

$$\eta_{a_2}\{E_2\} = \{E_2, E_3\}$$

$$\eta_{a_2}\{E_3\} = \{E_2, E_3\}$$

$$\eta_{a_2}\{E_4\} = \{E_4\}$$

$$\eta_{a_2}\{E_5\} = \{E_1, E_5\}$$

$$\eta_{a_2}\{E_6\} = \{E_6, E_8\}$$

$$\eta_{a_2}\{E_7\} = \{E_7\}$$

$$\eta_{a_2}\{E_8\} = \{E_6, E_8\}$$

Therefore, the interior component of matrix by using Eq. (2)

$$\text{int}_{\eta_{a_2}}\{E_1\} = \overline{\eta_{a_2}\{E_1\}} = \overline{\eta_{a_2}\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_2}}\{E_2\} = \overline{\eta_{a_2}\{E_2\}} = \overline{\eta_{a_2}\{E_1, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_2}}\{E_3\} = \overline{\eta_{a_2}\{E_3\}} = \overline{\eta_{a_2}\{E_1, E_2, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_2}}\{E_4\} = \overline{\eta_{a_2}\{E_4\}} = \overline{\eta_{a_2}\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \{E_4\}$$

$$\text{int}_{\eta_{a_2}}\{E_5\} = \overline{\eta_{a_2}\{E_5\}} = \overline{\eta_{a_2}\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_2}}\{E_6\} = \overline{\eta_{a_2}\{E_6\}} = \overline{\eta_{a_2}\{E_1, E_2, E_3, E_4, E_5, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_2}}\{E_7\} = \overline{\eta_{a_2}\{E_7\}} = \overline{\eta_{a_2}\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \{E_7\}$$

$$\text{int}_{\eta_{a_2}}\{E_8\} = \overline{\eta_{a_2}\{E_8\}} = \overline{\eta_{a_2}\{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

Therefore, we obtain closed sets by using Eq. (3)

$$\eta_{a_2}\{E_4\} = \{E_4\}$$

$$\eta_{a_2}\{E_7\} = \{E_7\}$$

Therefore, we obtain open sets by using Eq. (4)

$$\text{int}_{\eta_{a_2}}\{E_7\} = \{E_7\}$$

5.1.3.2 Constructing a pretopology when removing the attribute trans-line B:

The following (table 4), shows the pretopology obtained when removing the attribute trans-line B.

	E ₁	E ₂	E ₃	E ₄	E ₅	E ₆	E ₇	E ₈
E ₁	1	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E ₂	$\frac{2}{3}$	1	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
E ₃	$\frac{2}{3}$	1	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
E ₄	0	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
E ₅	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	0	0	0
E ₆	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	1	$\frac{2}{3}$	1
E ₇	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	1	$\frac{2}{3}$
E ₈	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	1	$\frac{2}{3}$	1

Table 4
Therefore, the of matrix by using Eq. (1)

pretopology component

- $\eta_{a_3}\{E_1\} = \{E_1\}$
- $\eta_{a_3}\{E_2\} = \{E_2, E_3\}$
- $\eta_{a_3}\{E_3\} = \{E_2, E_3\}$
- $\eta_{a_3}\{E_4\} = \{E_4\}$
- $\eta_{a_3}\{E_5\} = \{E_5\}$
- $\eta_{a_3}\{E_6\} = \{E_6, E_8\}$
- $\eta_{a_3}\{E_7\} = \{E_7\}$
- $\eta_{a_3}\{E_8\} = \{E_6, E_8\}$

Therefore, the interior component of matrix by using Eq. (2)

- $\text{int}_{\eta_{a_3}}\{E_1\} = \overline{\eta_{a_3}\{E_1\}} = \overline{\eta_{a_3}\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \{E_2, E_3, E_4, E_5, E_6, E_7, E_8\} = \{E_1\}$
- $\text{int}_{\eta_{a_3}}\{E_2\} = \overline{\eta_{a_3}\{E_2\}} = \overline{\eta_{a_3}\{E_1, E_3, E_4, E_5, E_6, E_7, E_8\}} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\} = \emptyset$
- $\text{int}_{\eta_{a_3}}\{E_3\} = \overline{\eta_{a_3}\{E_3\}} = \overline{\eta_{a_3}\{E_1, E_2, E_4, E_5, E_6, E_7, E_8\}} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\} = \emptyset$
- $\text{int}_{\eta_{a_3}}\{E_4\} = \overline{\eta_{a_3}\{E_4\}} = \overline{\eta_{a_3}\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \{E_1, E_2, E_3, E_5, E_6, E_7, E_8\} = \{E_4\}$
- $\text{int}_{\eta_{a_3}}\{E_5\} = \overline{\eta_{a_3}\{E_5\}} = \overline{\eta_{a_3}\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \{E_1, E_2, E_3, E_4, E_6, E_7, E_8\} = \{E_5\}$
- $\text{int}_{\eta_{a_3}}\{E_6\} = \overline{\eta_{a_3}\{E_6\}} = \overline{\eta_{a_3}\{E_1, E_2, E_3, E_4, E_5, E_7, E_8\}} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\} = \emptyset$
- $\text{int}_{\eta_{a_3}}\{E_7\} = \overline{\eta_{a_3}\{E_7\}} = \overline{\eta_{a_3}\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_8\} = \{E_7\}$
- $\text{int}_{\eta_{a_3}}\{E_8\} = \overline{\eta_{a_3}\{E_8\}} = \overline{\eta_{a_3}\{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\} = \emptyset$

Therefore, we obtain closed sets by using Eq. (3)

$$\eta_{a_3}\{E_1\} = \{E_1\}$$

$$\eta_{a_3}\{E_4\} = \{E_4\}$$

$$\eta_{a_3}\{E_5\} = \{E_5\}$$

$$\eta_{a_3}\{E_7\} = \{E_7\}$$

Therefore, we obtain open sets by using **Eq. (4)**

$$\text{int}_{\eta_{a_3}}\{E_1\} = \{E_1\}$$

$$\text{int}_{\eta_{a_3}}\{E_4\} = \{E_4\}$$

$$\text{int}_{\eta_{a_3}}\{E_5\} = \{E_5\}$$

$$\text{int}_{\eta_{a_3}}\{E_7\} = \{E_7\}$$

5.1.3.3 Constructing a pretopology when removing the attribute self-gene C:

The following (table 5), shows the pretopology obtained when removing the attribute self-gene C.

Table 5
Therefore, the of matrix by using Eq. (1)

$$\eta_{a_4}\{E_1\} = \{E_1\}$$

$$\eta_{a_4}\{E_2\} = \{E_2, E_3\}$$

$$\eta_{a_4}\{E_3\} = \{E_2, E_3\}$$

$$\eta_{a_4}\{E_4\} = \{E_4\}$$

$$\eta_{a_4}\{E_5\} = \{E_5\}$$

$$\eta_{a_4}\{E_6\} = \{E_6, E_8\}$$

$$\eta_{a_4}\{E_7\} = \{E_7\}$$

$$\eta_{a_4}\{E_8\} = \{E_6, E_8\}$$

Therefore, the interior using Eq. (2)

	E₁	E₂	E₃	E₄	E₅	E₆	E₇	E₈
E₁	1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
E₂	$\frac{2}{3}$	1	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E₃	$\frac{2}{3}$	1	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E₄	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$
E₅	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E₆	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	1
E₇	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{2}{3}$
E₈	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$	1

pretopology component

component of matrix by

$$\text{int}_{\eta_{a_4}}\{E_1\} = \overline{\eta_{a_4}\{\overline{E_1}\}} = \overline{\eta_{a_4}\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \{E_1\}$$

$$\text{int}_{\eta_{a_4}}\{E_2\} = \overline{\eta_{a_4}\{\overline{E_2}\}} = \overline{\eta_{a_4}\{E_1, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_4}}\{E_3\} = \overline{\eta_{a_4}\{\overline{E_3}\}} = \overline{\eta_{a_4}\{E_1, E_2, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_4}}\{E_4\} = \overline{\eta_{a_4}\{\overline{E_4}\}} = \overline{\eta_{a_4}\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \{E_4\}$$

$$\text{int}_{\eta_{a_4}}\{E_5\} = \overline{\eta_{a_4}\{\overline{E_5}\}} = \overline{\eta_{a_4}\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \{E_5\}$$

$$\text{int}_{\eta_{a_4}}\{E_6\} = \overline{\eta_{a_4}\{\overline{E_6}\}} = \overline{\eta_{a_4}\{E_1, E_2, E_3, E_4, E_5, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_4}}\{E_7\} = \overline{\eta_{a_4}\{\overline{E_7}\}} = \overline{\eta_{a_4}\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \{E_7\}$$

$$\text{int}_{\eta_{a_4}}\{E_8\} = \overline{\eta_{a_4}\{\overline{E_8}\}} = \overline{\eta_{a_4}\{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

Therefore, we obtain closed sets by using **Eq. (3)**

$$\eta_{a_4}\{E_1\} = \{E_1\}$$

$$\eta_{a_4}\{E_4\} = \{E_4\}$$

$$\eta_{a_4}\{E_5\} = \{E_5\}$$

$$\eta_{a_4}\{E_7\} = \{E_7\}$$

Therefore, we obtain open sets by using **Eq. (4)**

$$\text{int}_{\eta_{a_4}}\{E_1\} = \{E_1\}$$

$$\text{int}_{\eta_{a_4}}\{E_4\} = \{E_4\}$$

$$\text{int}_{\eta_{a_4}}\{E_5\} = \{E_5\}$$

$$\text{int}_{\eta_{a_4}}\{E_7\} = \{E_7\}$$

5.1.3.4 Constructing a pretopology when removing the attribute self-load D:

The following (table 6), shows the pretopology obtained when removing the attribute self-load **D**.

	E₁	E₂	E₃	E₄	E₅	E₆	E₇	E₈
E₁	1	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
E₂	$\frac{1}{3}$	1	1	$\frac{2}{3}$	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
E₃	$\frac{1}{3}$	1	1	$\frac{2}{3}$	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
E₄	0	$\frac{2}{3}$	$\frac{2}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E₅	$\frac{2}{3}$	0	0	$\frac{1}{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
E₆	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	1	1
E₇	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	1	1
E₈	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	1	1

Table 6
Therefore, the of matrix by using Eq.

$$\eta_{a_5}\{E_1\} = \{E_1\}$$

$$\eta_{a_5}\{E_2\} = \{E_2, E_3\}$$

$$\eta_{a_5}\{E_3\} = \{E_2, E_3\}$$

$$\eta_{a_5}\{E_4\} = \{E_4\}$$

$$\eta_{a_5}\{E_5\} = \{E_5\}$$

$$\eta_{a_5}\{E_6\} = \{E_6, E_7, E_8\}$$

$$\eta_{a_5}\{E_7\} = \{E_6, E_7, E_8\}$$

$$\eta_{a_5}\{E_8\} = \{E_6, E_7, E_8\}$$

pretopology component (1)

Therefore, the interior component of matrix by using Eq. (2)

$$\text{int}_{\eta_{a_5}}\{E_1\} = \overline{\eta_{a_5}\{\overline{E_1}\}} = \overline{\eta_{a_5}\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \{E_1\}$$

$$\text{int}_{\eta_{a_5}}\{E_2\} = \overline{\eta_{a_5}\{\overline{E_2}\}} = \overline{\eta_{a_5}\{E_1, E_3, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_5}}\{E_3\} = \overline{\eta_{a_5}\{\overline{E_3}\}} = \overline{\eta_{a_5}\{E_1, E_2, E_4, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_5}}\{E_4\} = \overline{\eta_{a_5}\{\overline{E_4}\}} = \overline{\eta_{a_5}\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_5, E_6, E_7, E_8\}} = \{E_4\}$$

$$\text{int}_{\eta_{a_5}}\{E_5\} = \overline{\eta_{a_5}\{\overline{E_5}\}} = \overline{\eta_{a_5}\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_6, E_7, E_8\}} = \{E_5\}$$

$$\text{int}_{\eta_{a_5}}\{E_6\} = \overline{\eta_{a_5}\{\overline{E_6}\}} = \overline{\eta_{a_5}\{E_1, E_2, E_3, E_4, E_5, E_7, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_5}}\{E_7\} = \overline{\eta_{a_5}\{\overline{E_7}\}} = \overline{\eta_{a_5}\{E_1, E_2, E_3, E_4, E_5, E_6, E_8\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

$$\text{int}_{\eta_{a_5}}\{E_8\} = \overline{\eta_{a_5}\{\overline{E_8}\}} = \overline{\eta_{a_5}\{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}} = \overline{\{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8\}} = \emptyset$$

Therefore, we obtain closed sets by using **Eq. (3)**

$$\eta_{a_5}\{E_1\} = \{E_1\}$$

$$\eta_{a_5}\{E_4\} = \{E_4\}$$

$$\eta_{a_5}\{E_5\} = \{E_5\}$$

Therefore, we obtain open sets by using **Eq. (4)**

$$\text{int}_{\eta_{a_5}}\{E_1\} = \{E_1\}$$

$$\text{int}_{\eta_{a_5}}\{E_4\} = \{E_4\}$$

$$\text{int}_{\eta_{a_5}}\{E_5\} = \{E_5\}$$

5.2 Results from theses tables

Remark 1: Closed set resulting of pretopologies from the four attributes, the attribute trans-line B, the attribute self-gene C have the same closed sets.

Remark 2: Open set of resulting of pretopologies from the four attributes, the attribute trans-line B, the attribute self-gene C have the same open sets.

Remark 3: We conclude that the attribute A and the attribute B are the key attributes which could cowl all feasible durations of every measure.

So the Core = {A, D}.

6. Conclusion and Future Work

The pretopology seems, therefore, as a theoretical frame flexible and adapted to model and deals with issues improving from domain names varied as the theory of the graphs, so distinct pretopological spaces had been produced from some information system. A condition become set which will be capable of create a matrix that represents the information system's pretopology and then reduction attributes which have been added to apply an information system with fewer attributes. In this paper, the authors plan to use the pretopological structure to reduce the attributes of any information system by using the method of weighted decision column.

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