Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 9, August 2021: 3018-3035

Effect of Magnetic Field on Thermal Instability of Couple Stress Rivlin Ericksen Ferromagnetic Fluid With Variable Gravity Field Through Porous Medium.

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Abstract

The influence of magnetic field on thermal instability of a couple stress Rivlin-Ericksen ferromagnetic fluid layer through porous material in presence of variable gravity field is studied. The dispersion relation was established and analytically solved using the normal mode method. Couple stress, magnetic field, and medium permeability are found to have both a stabilizing and a destabilizing influence for the stationary convention. Magnetisation always has a stabilizing effect, whether the value of λ is greater than zero or less than zero. It is also possible to see oscillatory modes and stability in the system introduced by the presence of a magnetic field for $\lambda< 0$. Oscillatory modes are not permitted in the absence of a magnetic field, and the principle of exchange of stability is satisfied.

Keywords:-Thermal instability, Rotation, Couple stress fluid, Rivlin-Ericksen fluid, Ferromagnetic fluid, Magnetisation.

1. Introduction

The usage of non-Newtonian fluids in research, technology, and industry is growing. As a result, research with non-Newtonian fluids is preferable. Couple stress fluid has became the subject of research in the class of non Newtonian fluids due to its major application mechanism of lubrication of synovial joints in a wide range of industrial, scientific, and technical applications. Stokes [1] introduces and formulates the couple stress fluid theory. Chandrasekhar [2] presented the hypothesis of thermal instability in a fluid layer heated from below. Sharma and Sharma [3] discussed a couple stress fluid heated from below in a porous medium. Sharma and Thakur [4] investigate the thermal instability of an electrically conducting couple stress fluid passing through a porous media in the presence of a uniform magnetic field. Sunil et al. [5] investigated the influence of a magnetic field and rotation when a couple stress fluid is heated from below in a porous material. Kumar and Kumar [6] studied the effect of dust particles in the presence of a magnetic field and rotation on a couple stress fluid heated from below and discovered that dust particles have a destabilising influence on the considered layer, whereas rotation has a stabilising effect. Banyal [7], Banyal and Singh [8] investigated the influence of rotation on the thermal instability of a couple stress fluid in a porous and non-porous medium. Kumar et al. [9, 10, 11] investigated the influence of rotation and magnetic field on heat convection in a porous media. Mishra and Kumar [12] investigated thermal instability on an anisotropic couple stress fluid saturated porous layer in the presence of cross diffusion, as well as the effect of rotation and magnetic field of the considered fluid.

Sharma and Kango [13] investigated the influence of a magnetic field on the thermal instability of a rivlin ericksen elastico viscous fluid in a porous media. Pradeep et al [14] investigated the thermal instability of rivlin ericksen viscoelastic fluid in the presence of a uniform vertical magnetic field and discovered that rotation has a stabilizing impact, whereas magnetic field has both a stabilizing and a destabilizing influence on the fluid. Vasanthakumari et al. [15] investigated the thermal instability of non-Newtonian fluids in non-rotating media. Soudjada and Subbulaxmi [16] expanded on the research and investigated the thermal instability of non Newtonian fluids in spinning media. The influence of suspended particles on thermal instability in porous and Darcy Brinkman porous media has been explored by Rana [17], Rana and Thakur [18]. Kango and Singh [19] investigated the influence of a magnetic field and rotation on the thermal instability of a rivlin ericksen elastico viscous fluid in a porous media. Singh and Kumar [20] investigate the influence of suspended particles and magnetic fields on the stability of stratified Rivlin-Ericksen fluid in porous media.

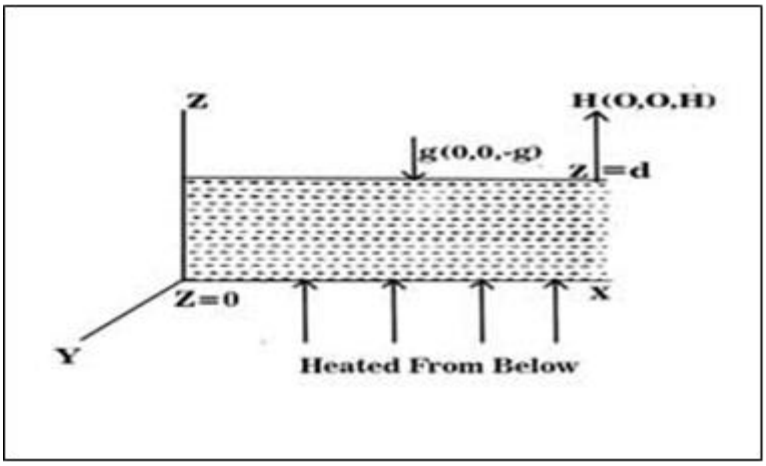
Polarization force and the body coupling are the two most important properties of ferromagnetic fluid. Ferromagnetic fluids are not naturally occurring fluids. These have been produced artificially. Ferromagnetic fluid is employed in a variety of scientific applications such as instrumentation, lubrication, vacuum technology, printing, vibration, and many more. On applying vertical magnetic field Convective instability when a ferromagnetic fluid layer is heated from below was studied by Finlayson [21]. Siddheswar [22, 23] has addressed Rayleigh – Benard convection in a ferromagnetic fluid in several contexts.The effect of rotation on the thermoconvective instability of a horizontal layer of ferrofluid have been analysed by Venkatasubramaniam and Kaloni [24]. Stiles and Kagan [25] have discussed effect of powerful vertical magnetic field on a horizontal ferromagnetic fluid. Sunil et al [26, 27] have examined the effect of magnetic field dependent viscosity on thermal instability in a ferromagnetic fluid. Agarwal and Prakash [28] have discussed the impact of suspended particles on thermal instability of ferrofluids with rotation. Kumar et al [29, 30] have investigated the impact of magnetic field and rotation on a fluid layer of couple stress. The effects of inconstant gravity field, Rotation and magnetic field on thermal instability of couple stress ferromagnetic fluids have been studied by Pulkit et.al. [31, 32].

The influence of magnetic field on thermal instability of couple stress rivlin ericksen ferromagnetic fluid with inconstant gravity field through porous media is uninvestigated so far. The goal of this work is to investigate the impact of magnetic field on thermal instability of couple stress rivlin ericksen ferromagnetic fluid with inconstant gravity field in porous medium.

2.Mathematical Formualtion

Let us take a thin layer of infinite in compressible, electrically non conducting couple stress Rivlin ericksen ferromagnetic fluid. Which is bounded by two infinite horizontal planes at a distance d.

On the considered layer, a homogeneous magnetic field H (0, 0, H) and a variable gravity field g (0, 0,-g) are applied along the vertical direction (Z axis).. where $g = \lambda g_o$, g_o represents value of g at z=0, which is always positive. λ might be positive or negative As gravity climbs or declines upwards from go



The fluid layer under consideration is heated from below in order to maintain the uniform temperature gradient β = (|dT/dz|).

Because a ferromagnetic fluid reacts so quickly to a magnetic force, we can replicate the following criteria to sustain,

$$M \times H = 0 \qquad ....(1)$$

Where *M* represents magnetization and *H* represents the magnetic field intensity.

Ferromagnetic fluid also fulfils the Maxwell's equation. Because we are looking at an electrically nonconductive fluid, the displacement current is insignificant. So the equation of Maxwell is transformed into,

$$\nabla \cdot B = 0\,, \nabla \times H = 0 \qquad -------(2)$$

In the Chu formulation of electro dynamics, the strength H of the magnetic field, magnetization M, and magnetic induction B are all associated in such a way that,

$$B = \mu_0\,(H + M) \qquad ------(3)$$

Assume that magnetization is affected by the amplitude of the magnetic field and the temperature. Assume that magnetization is also aligned with the magnetic field, so that,

$$M = \frac{H}{H}\,M(H,T) \qquad -----(4)$$

Let us take pressure, density, temperature, coefficient of thermal expansion, kinematic viscosity, kinematic viscous elasticity, couple stress viscosity, medium porosity, medium permeability, magnetic permeability, thermal diffusivity, resistivity, and fluid velocity are denoted by p, ρ, T, α, ν, ν', μ',ε, K_1, μ_e , κ_T, η and q = (u1,u2,u3) respectively.

In presence of magnetic field, the equations of motion, continuity and heat conduction of couple stress Rivlin Ericksen ferromagnetic fluid in porous media are

$$\frac{1}{\varepsilon}\left[\frac{\partial q}{\partial t}+\frac{1}{\varepsilon}(q\cdot\nabla)q\right]=-\frac{1}{\rho_0}\nabla p+g\,\left(1+\frac{\delta\rho}{\rho_0}\right)+\frac{1}{\rho_0}\,M\,\nabla H+\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 q$$
$$-\frac{1}{K_1}\left(\nu+\nu'\,\frac{\partial q}{\partial t}\right)+\,\frac{\mu_e}{4\pi\rho_0}[(\nabla\times H)\times H]$$

------(5)

$$\nabla\cdot q=0$$ ------ (6)

$$\frac{\partial T}{\partial t}+(\mathrm{q}\cdot\nabla)\mathrm{T}=\kappa_{\mathrm{T}}\,\nabla^2\,\mathrm{T}$$ ------(7)

ectromagnetic Maxwell's equation is,

$$\varepsilon\frac{\partial H}{\partial t}=(H\cdot\nabla)q+\varepsilon\eta\nabla^2 H$$

----(8)

$$\nabla\cdot H=0$$ ------(9)

The equation of state is

$$\rho=\rho_0\,[\,1-\alpha(\,T-T_0)]$$ ------(10)

Where T_0 and ρ_0 represents the value of temperature and density at 0 distances on z axis respectively.

∇ H denotes the magnetic field gradient.

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

$$H=|H|\,,B=|B|\,,M=|M|$$

In general, to complete the system a state equation is needed, which satisfy M in two thermodynamic variables H and T. In present paper we consider magnetization does not depend on magnetic field and depends only on temperature. Therefore,

$$M=M(T)$$

As the first approximation, we consider that

$$M=M_0[1-\gamma(T-T_0)]$$ ----(11)

Where M_0 is the magnetization at $T = T_0$ *and*

$$\gamma=\frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)_H$$

3.Basic State and Perturbation Equations

The basic state is supposed to be a quiescent state and is given by

$$q = (0,0,0), p = p(z), \rho = \rho(z) = \rho_0(1+\alpha\beta z), M = M(z) = M_0(1+\gamma\beta z)$$

$$T = T(z) = T_0 - \beta z \qquad \text{-----(12)}$$

To investigate the nature of equilibrium, consider that the system is slightly perturbed such that every physical quantity is assumed to be the sum of mean and fluctuating components, which are later designated as prime quantities and are thought to be very small in comparison to their equilibrium state values. Now consider that these minor disturbances are a function of time and space. The altered flow can be expressed as follows,

$$q = (0,0,0) + (u_1, u_2, u_3), T = T(z) + \theta, \rho = \rho(z) + \delta\rho,$$

$$p = p(z) + \delta p, \qquad \text{----(13)}$$

$$H = (0,0,H) + (h_x, h_y, h_z), \qquad M = M(z) + M$$

Where the perturbation in velocity of fluid q (0, 0, 0), density ρ, pressure p, magnetic field, temperature T and magnetization M are q (u_1, u2, u3*),* $\delta\rho$, δp, (h_x, h_y, h_z), θ, δM respectively.

Applying (12) in governing (5) to (11) and linearzing them, we obtain,

$$\frac{1}{\varepsilon}\frac{\partial q}{\partial t} = -\frac{\nabla\, \delta p}{\rho_0} + \frac{\delta\rho}{\rho_0} g + \frac{\delta M}{\rho_0}\nabla H + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 q - \frac{1}{K_1}\left(\nu + \nu'\,\frac{\partial}{\partial t}\right) q + \frac{\mu_e}{4\pi\rho_0}[(\nabla \times H)\times H] \qquad \text{....(14)}$$

$$\nabla \cdot q = 0 \qquad \text{....(15)}$$

$$\frac{\partial \theta}{\partial t} = \beta u_3 + \kappa_T\, \nabla^2\, \theta \qquad \text{....(16)}$$

$$\varepsilon \frac{\partial h}{\partial t} = (H \cdot \nabla) q + \varepsilon\eta\nabla^2 h \qquad \text{...(17)}$$

$$\nabla \cdot h = 0 \qquad \text{...(18)}$$

$$\delta\rho = -\alpha\, \rho_0\, \theta \qquad \text{...(19)}$$

$$\delta M = -\gamma\, M_0\, \theta \qquad \text{...(20)}$$

With the help of (19) and (20), Cartesian form of (14) - (15) can be expressed as following

$$\frac{1}{\varepsilon}\frac{\partial u_1}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial x}\delta p + \left(\nu - \frac{\mu'}{\rho_0}\,\nabla^2\right)\nabla^2\, u_1 - \frac{1}{K_1}\left(\nu + \nu'\,\frac{\partial}{\partial t}\right) u_1 + \frac{\mu_e}{4\pi\rho_0}\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right)$$

.....(21)

$$\frac{1}{\varepsilon}\frac{\partial u_2}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial y}\delta p + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 u_2 - \frac{1}{K_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)u_2 + \frac{\mu_e}{4\pi\rho_0}\left(\frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y}\right)$$

.....(22)

$$\frac{1}{\varepsilon}\frac{\partial u_3}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\delta p + \left(\alpha g - \frac{\gamma M_0 \nabla H}{\rho_0}\right)\theta + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 u_3 - \frac{1}{K_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)u_3$$

....(23)

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} = 0 \quad ...(24)$$

Taking z component from (16), we get,

$$\left(\frac{\partial}{\partial t} - \kappa_T \nabla^2\right)\theta = \beta u_3 \quad ....(25)$$

Transforming (17), (18) in Cartesian form, we obtain,

$$\varepsilon\frac{\partial h_x}{\partial t} = H \cdot \frac{\partial u_1}{\partial z} + \varepsilon\eta\nabla^2 h_x \quad ....(26)$$

$$\varepsilon\frac{\partial h_y}{\partial t} = H \cdot \frac{\partial u_2}{\partial z} + \varepsilon\eta\nabla^2 h_y \quad ....(27)$$

$$\varepsilon\frac{\partial h_z}{\partial t} = H \cdot \frac{\partial u_3}{\partial z} + \varepsilon\eta\nabla^2 h_z \quad ....(28)$$

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0 \quad ....(29)$$

Operate $\frac{\partial}{\partial x}$ on (21) and $\frac{\partial}{\partial y}$ on (22) and by adding them we obtain,

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t} - \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 + \frac{1}{K_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\right]\frac{\partial u_3}{\partial z} = \frac{1}{\rho_0}\delta p\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{\mu_e H}{4\pi\rho_0}\nabla^2 h_z$$

........(30)

Operate $\left[\nabla^2 - \left(\frac{\partial^2}{\partial z^2}\right)\right]$ on (23) and $\frac{\partial}{\partial z}$ on (30) to eliminate δp and adding them, we obtain,

$$\frac{1}{\varepsilon}\nabla^2\frac{\partial u_3}{\partial t} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\alpha g - \frac{\gamma M_0 \nabla H}{\rho_0}\right)\theta + \left(\nu - \frac{\mu'}{\rho_0}\ \nabla^2\right)\nabla^4 u_3$$
$$-\frac{1}{K_1}\left(\nu + \nu'\ \frac{\partial}{\partial t}\right)\nabla^2 u_3 + \frac{\mu_e H}{4\pi\rho_0}\nabla^2\frac{\partial h_z}{\partial z}\nabla^2 \qquad ....(31)$$

Operate $-\frac{\partial}{\partial y}$ on (21) and $\frac{\partial}{\partial x}$ on (22) and by adding them, we obtain,

$$\frac{1}{\varepsilon}\frac{\partial\zeta}{\partial t} = \left[\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 - \frac{1}{K_1}\left(\nu + \nu'\ \frac{\partial}{\partial t}\right)\right]\zeta + \frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z} \qquad ....(32)$$

Operate $-\frac{\partial}{\partial y}$ on (26) and $\frac{\partial}{\partial x}$ on (27) and by adding them, we obtain,

$$\varepsilon\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)\xi = H.\frac{\partial\zeta}{\partial z} \qquad ....(33)$$

Where $\zeta = \frac{\partial u2}{\partial x} - \frac{\partial u1}{\partial y}$

$\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$

4.Dispersion Relation

We employ the Normal Mode technique to breakdown the disturbances. We assume that the perturbation takes the following form:

$$(u_3, \theta, \zeta, \xi, h_z) = [W(z), \Theta(z), Z(z), X(z), K(z)] \cdot \exp(i\, k_x\ x + i\, k_y\ y + nt) \qquad .....(34)$$

Where k_x and k_y are wave number in X and Y direction and *k* is the resultant disturbances wave number such that,

$$k = \sqrt{\left[(k_x)^2 + \left(k_y\right)^2\right]}$$

n is the frequency of any arbitrary disturbances which is a complex constant.

Now by using relation (34) we obtain non dimensional form of (31) – (33), (28) and (25)

$$(D^2 - a^2)\begin{bmatrix}\frac{\sigma}{\varepsilon} + F(D^2 - a^2)^2 \\ -(D^2 - a^2) + \frac{(1 + \sigma F_1)}{P_t}\end{bmatrix} W + \frac{\alpha a^2 \lambda d^2}{\nu}\left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda}\right)\theta$$

$$-\frac{\mu_e H d}{4\pi\rho_0 \nu}(D^2 - a^2)DK = 0$$

....(35)

$$\begin{bmatrix} \frac{\sigma}{\varepsilon} + F(D^2 - a^2)^2 \\ -(D^2 - a^2) + \frac{(1 + \sigma F_1)}{P_t} \end{bmatrix} Z = \frac{\mu_e H d}{4\pi\rho_0 \nu} DX$$ ......(36)

$$\varepsilon[(D^2 - a^2) - \sigma P_2]\mathrm{X} = -\frac{Hd}{\eta} DZ$$ . ...(37)

$$\varepsilon[(D^2 - a^2) - \sigma P_2]\mathrm{K} = -\frac{Hd}{\eta} DW$$ ....(38)

$$[(D^2 - a^2) - \sigma P_1]\Theta = -\frac{\beta d^2}{\kappa_T} W$$ ....(39)

Where we have represent the coordinates x, y and z in new units of length d, time t and (d^2/κ_T). Let a = kd, $\sigma = nd^2 / \nu$, $F = \frac{\mu'}{\rho_0 d^2 \nu}$, $F_1 = \frac{\nu'}{d^2}$, $P_t = K_1/ d^2$, $P_1 = \nu/\kappa_T$, $P_2 = \nu/\eta$

x* = x/d, y* = y/d, z* = z/d, D* = dD and dropping * for convenience.

Where P1 and Pt denotes prandlt number and dimensionless medium permeability.

Now eliminate Θ, X, K and Z from (35) with the help of (36), (37), (38) and (39), then we get Stability governing equation

$$\lambda a^2 R_f \frac{1}{(D^2 - a^2) - \sigma P_1} = (D^2 - a^2)\left[\frac{\sigma}{\varepsilon} + F(D^2 - a^2)^2 + -(D^2 - a^2) + \frac{(1 + \sigma F_1)}{P_t}\right] W + \frac{Q}{\varepsilon(D^2 - a^2) - \sigma P_2}(D^2 - a^2)D^2 W = 0$$

......(40)

Where

$$R_f = \frac{\alpha\beta d^4}{\nu\kappa_T}\left[g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha\lambda}\right] = \frac{\alpha\beta d^4 g_0}{\nu\kappa_T}\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha\lambda g_0}\right] = R\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha\lambda g_0}\right]$$

R_f is the Rayleigh number for ferromagnetic fluid. R is the Rayleigh number for fluid.

. $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$ is the Chandrasekhar number

Because the boundaries are held at a constant temperature, the temperature perturbation on the boundaries is zero. As a result, the proper boundary condition is

W = 0, Z = 0, $\Theta = 0$ at z = 0 & z =1,

DZ = D^2W = D^4W = 0 at z = 0 & z = 1. ......(41)

All even order derivatives of W vanish on borders, according to (41). As a result, the correct solution to (40), which characterises the lowest mode, is as follows:

$W = W_0 \sin \pi z$, W_0 is a constant .....(42)

By using (41) with (40), we get,

$$R_1 = \frac{1}{\lambda x}(1+x)\,(1+x+i\sigma_1 P_1)\left[\frac{i\sigma_1}{\varepsilon} + F_2(1+x)^2 + (1+x) + \frac{(1+i\,\sigma_1 F_3)}{P}\right] + \frac{1}{\lambda x}\frac{Q_1(1+x)\,(1+x+i\sigma_1 P_1)}{\varepsilon(1+x+i\sigma_1 P_2)} \quad ..(43)$$

Where $x = \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^2}, F_2 = \pi^2 F, P = \pi^2 P_t,$

$$F_3 = \pi^2 F_1, R_1 = \frac{R_f}{\pi^4}, Q_1 = \frac{Q}{\pi^2}$$

5. Analytical Discussion

5.1 Stationary Convection: - at stationary convection, when stability sets, the marginal state will be characterized by $\sigma_1 = 0$. Put $\sigma_1 = 0$ in (43), we get,

$$R_1 = \frac{(1+x)^2}{\lambda x}\left[\left\{F_2(1+x)^2 + (1+x) + \frac{1}{P}\right\} + \frac{Q_1}{\varepsilon(1+x)}\right] \quad ...(44)$$

Clearly (44) shows the modified Rayleigh number R_1 as a function of F_2, P, $Q_1$and x parameters. Clearly viscoelastic parameter F_3 disappears with σ_1. It demonstrates that during stationary convection, the Rivlin Ericksen fluid behaves similarly to a Newtonian fluid.

To examine the effect of couple stress, permeability, magnetic field and magnetization we have to study the analytical behaviour of

$\frac{dR_1}{dF_2}, \frac{dR_1}{dP}, \frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dM_0}$.

$$\frac{dR_1}{dF_2} = \frac{(1+x)^4}{\lambda x} \qquad ......(45)$$

(45) demonstrates that couple stress has stabilizing effect on thermal instability of couple stress Rivlin Ericksen ferromagnetic fluid in presence of gravity field through porous medium if λ > 0, (when gravity grows from g_0) and destabilizing effect if λ < 0 (when gravity drops from g_0).

When magnetic field are not applied on considered fluid then (34) becomes

$$\frac{dR_1}{dF_2} = \frac{(1+x)^4}{\lambda x} \qquad ......(46)$$

(46) shows that couple stress has stabilizing effect on thermal instability of couple stress Rivlin Ericksen ferromagnetic fluid in presence of inconstant gravity field through porous medium if λ > 0, (when gravity grows from g_0) and destabilizing effect if λ < 0 (when gravity drops from g_0).

$$\frac{dR_1}{dP} = -\frac{(1+x)2}{P^2 \lambda x} \qquad .....(47)$$

(47) shows that permeability has stabilizing effect on the thermal instability of considered system if λ < 0, (when gravity drops from g_0) and destabilizing effect if λ > 0 (when gravity increases upwards from g_0).

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{\varepsilon \lambda x} \qquad ...\ (48)$$

(48) shows that magnetic field has stabilizing effect on thermal instability of considered system if λ > 0, (when gravity grows from g_0) and destabilizing effect if λ < 0 (when gravity drops from g_0).

Replace R_1 by R_f / π^4 and R_f by $R\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0}\right]$

$$R = \frac{\frac{\pi^4 (1+x)^2}{\lambda x}\left[\left\{F_2(1+x)^2 + (1+x) + \frac{1}{P}\right\} + \frac{Q1}{\varepsilon(1+x)}\right]}{\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0}\right]} \qquad .....,(49)$$

From (49)

$$\frac{dR}{dM_0} = \frac{\frac{\pi^4 (1+x)^2}{\lambda x}\left[\begin{matrix}\left\{F_2(1+x)^2 + (1+x) + \frac{1}{P}\right\} \\ + \frac{Q1}{\varepsilon(1+x)}\end{matrix}\right]\left(\frac{\gamma \nabla H}{\rho_0 \alpha \lambda g_0}\right)}{\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0}\right]^2} \qquad .....(50)$$

Clearly (50) shows that magnetization has stabilizing effect on thermal instability of couple stress Rivlin Ericksen ferromagnetic fluid in presence of inconstant gravity field and magnetic field through porous medium for each value of λ either λ > 0, or λ < 0 (either gravity grows from g_0 or gravity drops from g_0).

When magnetic field is not applied on the system

$$\frac{dR}{dM_0} = \frac{\frac{\pi^4(1+x)^2}{\lambda x}\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}\right]\left(\frac{\gamma\nabla H}{\rho_0\alpha\lambda g_0}\right)}{\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]^2} \qquad ....(51)$$

Clearly (51) shows that magnetization has stabilizing effect on thermal instability of couple stress Rivlin Ericksen ferromagnetic fluid in presence of gravity field through porous medium for each value of λ either λ > 0, or λ < 0 (either gravity grows from g_0 or gravity drops from g_0).

5.2 Stability of the system & Oscillatory Modes:-

Multiplying (35) by the conjugate of W i.e. W* and integrate over the range of z and making use of (36) - (39) together with boundary condition (41), we obtain,

$$\left[\frac{\sigma}{\varepsilon}+\frac{(1+\sigma F_1)}{P_t}\right]I_1+I_2+FI_3$$

$$+\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}(I_4+P_2\sigma^*I_5)-\frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)[\,I_6+\sigma^*P_1I_7]=0 \qquad ....(52)$$

Where

$$I_1=\int(|DW|^2+a^2|W|^2)\,dz$$

$$I_2=\int(|D^2W|^2+a^4|W|^2+2a^2|DW|^2)\,dz$$

$$I_3=\int(|D^3W|^2+3a^2|D^2W|^2+3a^4|DW|^2+a6|W|^2)\,dz$$

$$I_4=\int(|D^2K|^2+a^4|K|^2+2a^2|DK|^2)\,dz$$

$$I_5=\int(|DK|^2+a^2|K|^2)\,dz$$

$$I_6=\int(|D\Theta|^2+a^2|\Theta|^2)\,dz$$

$$I_7=\int(|\Theta|^2)dz$$

All above specified integrals I_1- I_7 are +ve definite.

Put $\sigma = \sigma_r + i\,\sigma_i$ in (52) where σ_r and σ_i real. Equating real and imaginary part, we obtain,

$$\sigma_r\left[\begin{array}{c}\left(\frac{1}{\varepsilon}+\frac{F_1}{P_t}\right)I_1\\ +\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}P_2I_5-\frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)P_1I_7\end{array}\right]$$

$$= -\left[\begin{array}{c}\frac{1}{P_t}\,I_1+I_2+FI_3\\ +\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}I_4-\frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)I_6\end{array}\right] \qquad \text{........(53)}$$

$$\sigma_i\left[\begin{array}{c}\left(\frac{1}{\varepsilon}+\frac{F_1}{P_t}\right)I_1\\ -\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}P_2I_5+\frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)P_1I_7\end{array}\right]=0 \qquad \text{.........(54)}$$

In absence of magnetic field (54) becomes,

$$\sigma_i\left[\left(\frac{1}{\varepsilon}+\frac{F_1}{P_t}\right)I_1+\frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)P_1I_7\right]=0 \qquad \text{...........(55)}$$

From (55) the quantity is in bracket will be positive definite if $g_0 > \frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}$.

It means $\sigma_i = 0$, modes are non oscillatory or oscillatory modes are not permitted and principle of exchange of stability held if $g_0 > \frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}$.

From (56),

$$\sigma_r\left[2\left(\frac{1}{\varepsilon}+\frac{F_1}{P_t}\right)I_1+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}P_2I_5\right]$$

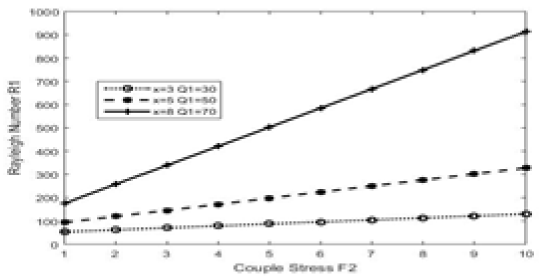
$$= -\left[\begin{array}{c}\frac{1}{P_t}\,I_1+I_2+FI_3+d^2\left(\frac{1}{P_t}\,I_4+I_5+FI_6+\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}I_7\right)\\ +\frac{\mu_e\eta\epsilon}{4\pi\rho_0\nu}I_9-\frac{\alpha a^2\lambda\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)I_{11}\end{array}\right] \qquad \text{.........(56)}$$

From (56) clearly σ_r is negative if $\lambda < 0$.

As a result, The system is stable while the viscoelastic parameter F1 and the magnetic field are present. When the viscoelastic parameter F1 is combined with a magnetic field, oscillatory modes emerge, the system loses stability, and the principle of exchange of stability does not hold in the system if $\lambda < 0$.

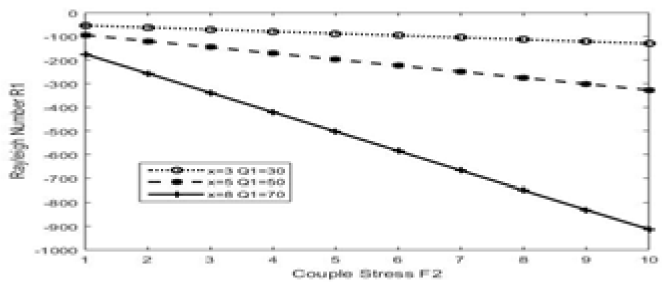
6.Numerical Computations

Dispersion equation (44) is analysed also. The critical Rayleigh number R_1 & Rayleigh number for ferromagnetic fluid is calculated for various numeric values of F_2, permeability P, magnetic field Q_1 and Magnetization M_0.



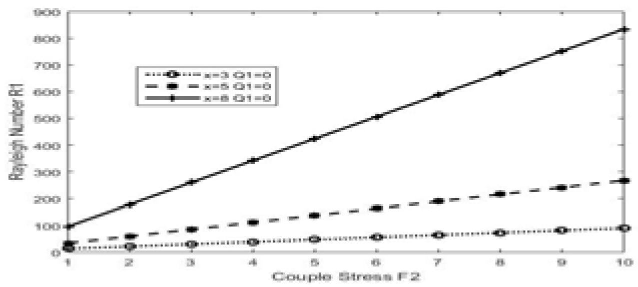
λ=10, x=[3,5,8] P=0.5, ε=0.5, Q1=[30, 50, 70]

Figure 2



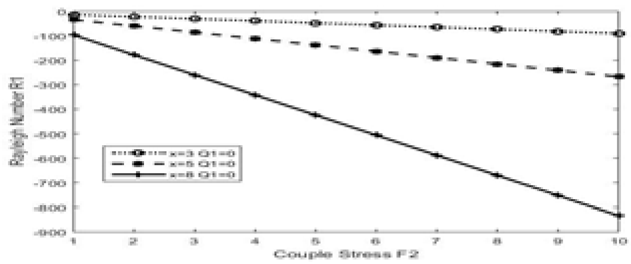
λ=-10, x=[3,5,8] P=0.5, ε=0.5, Q1=[30, 50, 70]

Figure 3



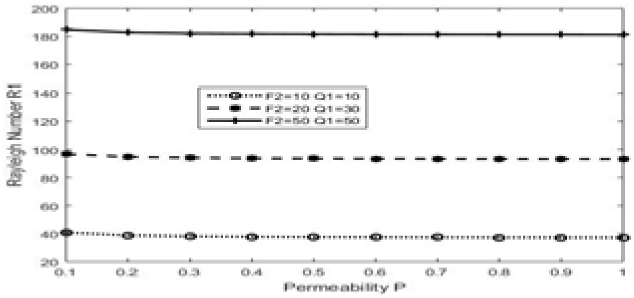
λ =10, x=[3,5,8] P=0.5,ε=0.5, Q1=[0, 0, 0] Q1=[10, 30, 50]

Figure 4



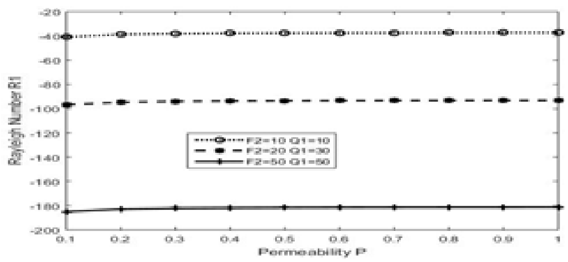
λ =10, x=[3,5,8] P=0.5, ε=0.5, Q1=[0, 0, 0]

Figure5



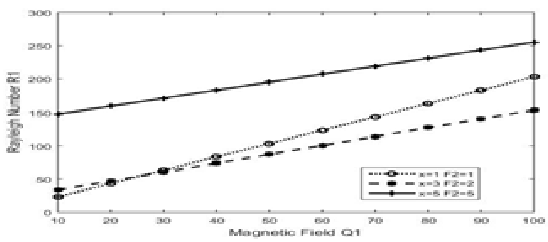
λ=10, x=1, ε=0.5, F2=[10,20,50], Q1=[10, 30, 50]

Figure 6



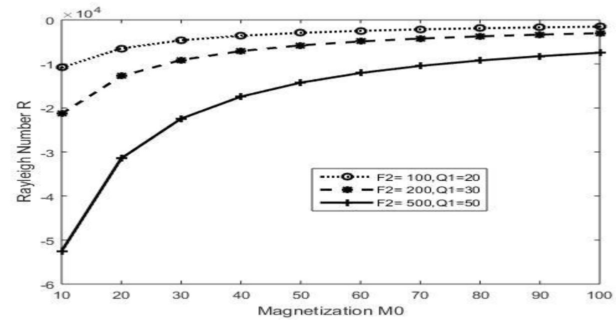
λ=10, x=1, ε=0.5, F2=[10,20,50], Q1=[10, 30, 50]

Figure 7



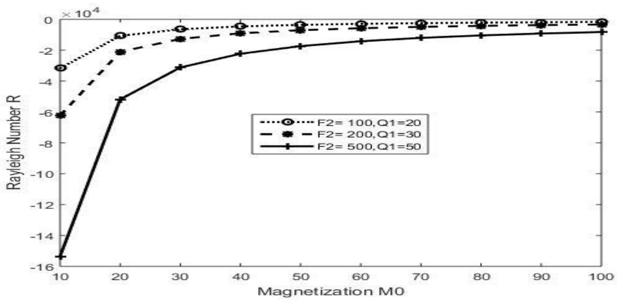
λ=10, x=[1,3,5], P=0.5, ε=0.5, F_2=[1,2,5]

Figure 8



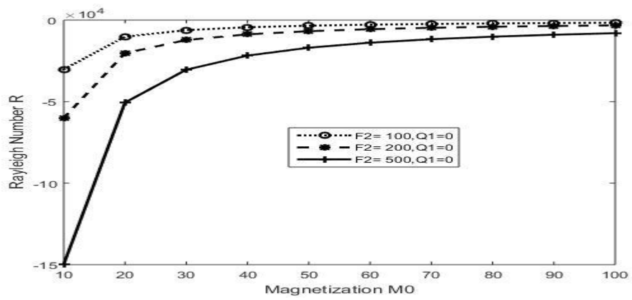
λ=5, x=1,P=0.5,ε=0.5, γ=1, ρ0=1, g=9.8, α=1, Q1=[20,30,50], F_2=[100,200,500]

Figure 10



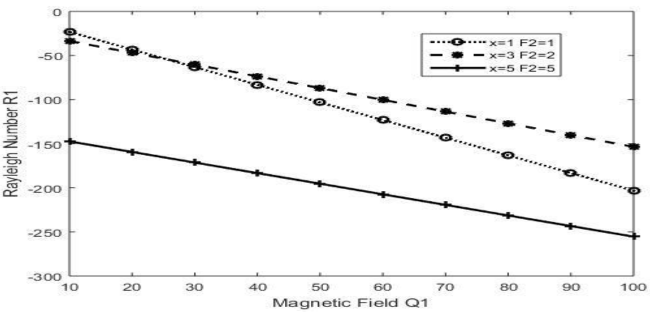
λ=-5, x=1,P=0.5,ε=0.5, γ=1, ρ0=1, g=9.8, α=1, Q1=[20,30,50], F_2=[100,200,500]

Figure 11



λ=5, x=1,P=0.5,ε=0.5, γ=1, ρ0=1, g=9.8, α=1, Q1=[0,0,0], F_2=[100,200,500]

Figure 12



λ=-10, x=[1,3,5],P=0.5,ε=0.5, F_2=[1,2,5]

Figure 9

In figure 2, the critical Rayleigh number R1 and the couple stress parameter F2 have been shown on a graph for x = [3, 5, 8], Q1 = [30, 50, 70] and $\lambda = 10 > 0$, which shows R_1 grows with increases in F_2. So couple stress stabilizes the system for $\lambda > 0$.

In figure 3 a graph has been plotted between critical Rayleigh number R_1 and couple stress parameter F_2 for x = [3, 5, 8], Q1 = [30, 50, 70] and $\lambda = -10 > 0$,which shows R_1 decreases with increases in F_2. So couple stress destabilizes the system for $\lambda > 0$.

In figure 4, a graph has been plotted between critical Rayleigh number R_1 and couple stress parameter F_2 for x = [3, 5, 8] and $\lambda = 10 > 0$, which shows R_1 increases with increases in F_2. While magnetic field is not applied couple stress stabilizes the system for $\lambda > 0$.

In figure 5, a graph has been plotted between critical Rayleigh number R_1 and couple stress parameter F_2 for x = [3, 5, 8] and $\lambda = -10 < 0$, which shows R_1 decreases with increases in F_2. As a result while magnetic field is not applied couple stress destabilizes the system for $\lambda < 0$.

In figure 6, a graph has been plotted between critical Rayleigh number R_1 and permeability of the medium P for F_2= [10, 20, 50], Q_1= [10, 30, 50] and $\lambda = 10 > 0$, which shows R_1 decreases with increases P. So permeability destabilizes the system for $\lambda > 0$.

In figure 7, a graph has been plotted between critical Rayleigh number R_1 and permeability of the medium P for F_2= [10, 20, 50], Q_1= [10, 30, 50] and $\lambda = -10 < 0$, which shows R_1 increases with increases P. So permeability has stabilizes on the system for $\lambda < 0$.

In figure 8, a graph has been plotted between critical Rayleigh number R_1 and Magnetic Field for F_2 = [1, 2, 5], x = [1, 3, 5] and $\lambda = 10 > 0$, which shows R_1 increases with increases in Q_1. So Magnetic Field stabilizes the system for $\lambda = 10 > 0$.

In figure 9, a graph has been plotted between critical Rayleigh number R_1 and Magnetic Field for F_2 = [1, 2, 5], x = [1, 3, 5] and $\lambda = -10 < 0$, which shows R_1 decreases with increases in Q_1. So Magnetic Field destabilizes the system for $\lambda = -10 < 0$.

In figure 10, a graph has been plotted between Rayleigh number R and magnetization of ferromagnetic fluid M_0 for F_2= [100, 200, 500], Q1 = [20, 30, 50] and $\lambda = -5 < 0$, which shows R increases with increases in M_0. So magnetization stabilizes the system for $\lambda < 0$.

In figure 11, a graph has been plotted between Rayleigh number R for ferromagnetic fluid and magnetization of ferromagnetic fluid M_0 for F_2= [100, 200, 500], Q1 = [20, 30, 50] and $\lambda = 5 > 0$, which shows R_1 increases with increases in M_0. So magnetization stabilizes the system for $\lambda > 0$.

In figure 12, a graph has been plotted between Rayleigh number R and magnetization of ferromagnetic fluid M_0 for F_2= [100, 200, 500], Q1 = [0, 0, 0] and $\lambda = 5 > 0$, which shows R increases with increases in M_0. When magnetic field is not applied, magnetization stabilizes the system for $\lambda > 0$.

7.**Conclusion**

In the present paper, the effect of different parameters such as couple stress, viscoelastic, permeability, magnetic field and magnetization has been examined on thermal instability of couple stress rivlin ericksen ferromagnetic fluid in presence of inconstant gravity field in porous medium.

Results of investigation are as follows:-

(1) Couple stress stabilizes the system if $\lambda > 0$ and destabilizes the system if $\lambda < 0$.
(2) permeability of porous medium stabilizes the system if $\lambda > 0$ and destabilizes the system if $\lambda < 0$.
(3) Magnetic field stabilizes considered fluid layer if $\lambda > 0$ and destabilizies if $\lambda < 0$.
(4) Magnetization always stabilizes the system.
(5) for stationary convection Rivlin Ericksen fluid acts like a simple Newtonian fluid .
(6) When a viscoelastic parameter is present, the magnetic field creates oscillatory modes, and the exchange principle is violated.

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