Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 11, Issue 3, July 2020: 436 – 448

Qammer Distribution - Structural Properties and Applications in the Risk Management of Private Banks of India

Authors:-*Syed Roohul Islam Andrabi* [1]

Research Scholar Dept. of Management Studies DR. M.G.R Educational and Research Institute Chennai Tamil Nadu

Dr. G Brindha [2]

HOD/Professor Faculty of Management Studies DR. M.G.R Educational and Research Institute Chennai Tamil Nadu

Khalid ul Islam [3]

Research Scholar Division of Statistics and Computer Science SKUAST main Campus Chatha Jammu

ABSTRACT

The modified life time distribution called Qammer Distribution is derived in this article .This distribution is the mixture of Rayleigh-Ailamujia distribution ,which gives birth to this new three parameter distribution .The two parent distributions are mixed by adding the mixing parameter $'\lambda'$ through a proper method .The reliability analysis ,structural properties ,moment generating function, characteristic function and order statistics of the resulting distribution are derived .The PDF , CDF and reliability functions of the said distribution are represented by different graphs respectively. The estimates of parameters are obtained by the method of likelihood estimation .Apart from this the usefulness of the derived distribution has been explained through two bio science related data sets .On comparing to other related distributions , this new distribution (QD) gives the better results .

1. INTRODUCTION

Rayleigh distribution is one of the prominent probability distributions; it is named after Lord Rayleigh (1880). It has resplendent and preeminent applications for modeling data in engineering and medicine arena. Siddiqui (1962) studied the genesis and other features of this model. Given the importance it holds in diverse fields, myriad authors have extensively researched and worked on this model. To mention a few of them- Howlader and Hossain (1995), Voda (2005), Ahmad et al. (2014), Kandu and Raqab (2005), Merovci (2013), Ahmad et al (2017), Gazal and Hasaballah (2017), Ajami and Jahanshiahi (2017), Ateeq et al (2019) and Sofi et al (2019). Nonetheless, Ailamuja distribution is a newly suggested lifetime model improvised by H.Q.LV et al (2002). They have obtained its various characteristics subsuming mean, variance, median and maximum likelihood. Ailamuja distribution is a serviceable distribution used to design the repair time and affirm distribution procrastination time. Lately, several authors have worked on Ailamuja distribution. This distribution was also proposed by pan et al (2009) for interval estimation and hypothesis testing based on sample of small size. Yu et al. (2015) suggested a new strategy by applying Ailamuja distribution to iron out the conundrum in the production and distribution of battel field injury in campaign macrocosm. Uzma et al (2017) has presented the weighted analogue of Ailamuja distribution with exploration of its various characteristics. Moreover, they inferred that weighted analogue of Ailamuja distribution performs way better than Ailamuja distribution. B. Jaya Kumar et al (2019) suggested area biased distribution and have studied its various proprieties and applied the obtained distribution to bladder cancer data. Ahmad et al. (2020) proposed inverse analogue of Ailamuja distribution with statistical proprieties and application. Rather et al (7) introduced a size biased Ailamuja distribution and applied it for analysing data from engineering and medical science. Ahmad et al (2020) suggested Bayesian Estimation of inverse Ailamuja distribution using different loss functions.

Insofar as Rayleigh distribution is concerned, statistical distributions have received a broad attention in order to unfold flexible models for modeling diverse data sets. Since, the classical distributions lack superiority in modeling data sets with variable nature. This leads to the growth and expansion of generalized probability models. Designing a new probability model from the erstwhile constructed models by using disparate approaches has got enormous horizon in the recent past. Once such approach used by different researchers is power transformation technique by which the extra parameter is added to the parent distribution. Induction of an extra parameter in the parent model, more often than not, endows us with greater flexibility and ameliorates its goodness. There are researchers galore who worked on power generalization of probability models; some among them are Meniconi and Barry (1996), Ghitany et al (2013).

To formulate the new distribution,let us define the parent distributions as under;

PDF of Rayleigh Distribution

If a random variable 'x' follows Rayleigh distribution, then its PDF is defined as under;

$f(x) = \frac{x}{a^2} e^{-x^2/2a^2}$, Where "a" is the parameter **(1.1)**

PDF of Ailamujia Distribution

If a random variable 'x' follows Ailamujia distribution, then the PDF is defined as follows;

$g(x) = 4xb^2e^{-2bx}$, Where 'b' is the parameter (1.2)

2. Qammer Distribution –The mixture of Rayleigh and Ailamujia Distribution

The combination of two density functions defined above as f(x) &g(x) gives birth to a new three parameter distribution with PDF as under;

$f(x; a, b, \lambda) = \lambda f(x) + (1 - \lambda)g(x)$, λ is the mixing parameter **(2.1)**

Substituting the values of $f(x)$ and $g(x)$ in the above equation we get;

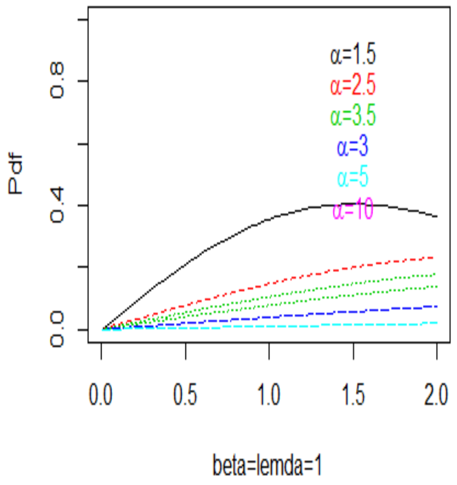
$f(x; a, b, \lambda) = \lambda \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} + (1 - \lambda)4xb^2e^{-2bx}$, **(2.2)**

And the corresponding CDF is given by;

$F(x; a, b, \lambda) = \lambda \left(1 - e^{-x^2/2a^2}\right) + (1 - \lambda)\left(1 - e^{-2bx}(2bx + 1)\right)$ **(2.3)**

The graphs of PDF and CDF of QD are represented below;

Fig ‘a’& Fig ‘b’ below shows the graph of PDF and CDF of QD respectively.



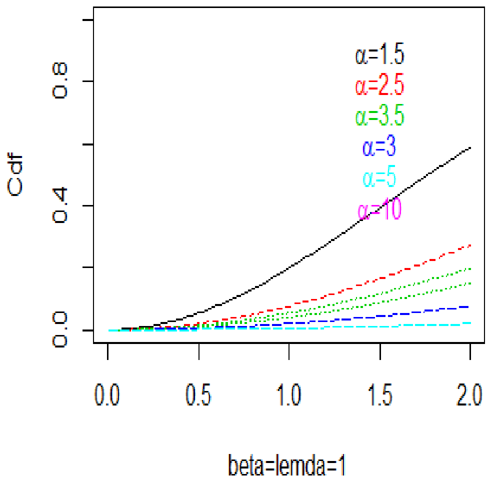


Fig ‘a’ **Fig ‘b’**

3. Reliability Analysis

In this section,we have derived the survival ratefunction ,hazard rate function , reverse hazard rate function ,cumulative hazard rate function and mills ratio.

3.1Survival Rate

The survival function of a random variable ‘x ‘is denoted by $S(x; a, b, \lambda)$ and is defined as ;

$$S(x; a, b, \lambda) = 1 - F(x; a, b, \lambda), x > 0; a, b, \lambda > 0 \qquad \textbf{(3.1.1)}$$

Substituting the value of $F(x; a, b, \lambda)$ from**(2.3)** we get;

$$S(x; a, b, \lambda) = 1 - [\lambda\left(1 - e^{-x^2/2a^2}\right) + (1 - \lambda)(1 - e^{-2bx}(2bx + 1)$$

$$S(x; a, b, \lambda) = (1 - \lambda)\{2 - (2bx + 1)e^{-2bx}\} + \lambda e^{-x^2/2a^2} \qquad \textbf{(3.1.2)}$$

3.2Hazard Rate Function

The hazard rate function of a random variable ‘x’ is denoted by $H(x; a, b, \lambda)$,and is defined as;$H(x; a, b, \lambda) = \frac{f(x;a,b,\lambda)}{S(x;a,b,\lambda)}$ **(3.2.1)**

Substituting the value of $f(x; a, b, \lambda)$and $S(x; a, b, \lambda)$from equation **(2.2)&(3.1.2)** respectively, we get;

$$H(x;a,b,\lambda) = \frac{\lambda \frac{x}{a^2} e^{-x^2/2a^2} + (1-\lambda)4xb^2 e^{-2bx}}{(1-\lambda)\{2-(2bx+1)e^{-2bx}\} + \lambda e^{-x^2/2a^2}} \qquad \textbf{(3.2.2)}$$

The graphs of SRF and HRF of QD are represented below;

Fig 'c' & Fig 'd' below shows the graph of SRF and HRF of QD respectively.

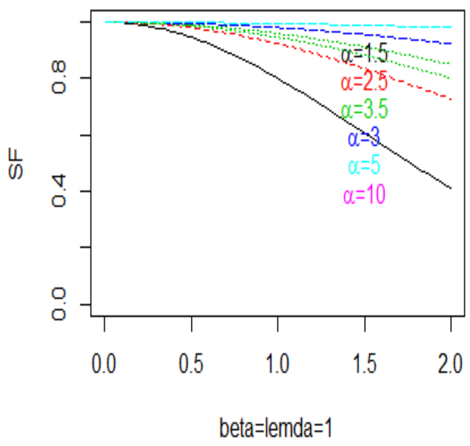


Fig 'c'

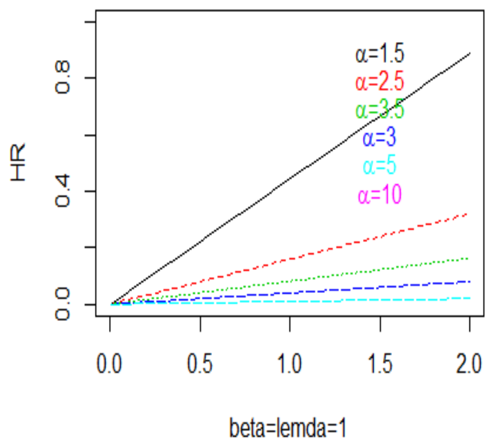


Fig 'd'

3.3 Reverse Hazard Rate

The reverse hazard rate function of a random variable 'x' is denoted by $RHR\ (x;a,b,\lambda)$, and is defined as;

$$\text{RHR}(x;a,b,\lambda) = \frac{f(x;a,b,\lambda)}{F(x;a,b,\lambda)} \qquad \textbf{(3.3.1)}$$

Substituting the value of $f(x;a,b,\lambda)$and $F(x;a,b,\lambda)$ from equation **(2.2)**&**(2.3)** respectively, we get;

$$\text{RHR}(x;a,b,\lambda) = \frac{\lambda \frac{x}{a^2} e^{-x^2/2a^2} + (1-\lambda)4xb^2 e^{-2bx}}{\lambda\left(1-e^{-x^2/2a}\right) + (1-\lambda)\left(1-e^{-2bx}(2bx+1)\right)}, \qquad \textbf{(3.3.2)}$$

The graph of RHR of QD is represented below;

Fig 'e' below shows the graph of RHR of QD respectively.

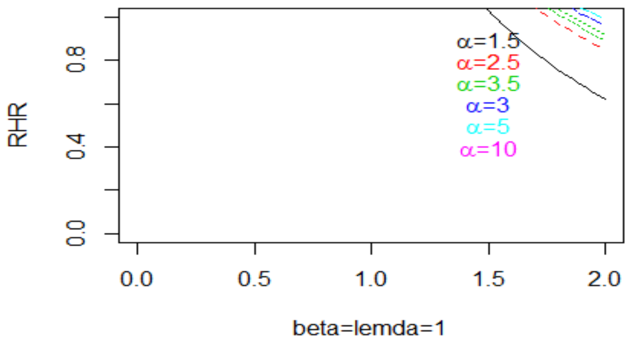


Fig 'e'

3.4 Cumulative Hazard Rate

The Cumulative Hazard Rate function denoted b CHR is defined as;

$$\text{CHR} = -log s(x; a, b, \lambda) \tag{3.4.1}$$

Substituting the value of $s(x; a, b, \lambda)$ from equation **(3.1.2), we get;**

$$\text{CHR} = -log\left((1-\lambda)\{2-(2bx+1)e^{-2bx}\} + \lambda e^{-x^2/2a^2}\right)$$

After doing a little algebra we have;

$$\text{CHR} = -log\{[2-(2bx+1)e^{-2bx}][\lambda(1-\lambda)]\} + \frac{x^2}{2a^2} \tag{3.4.2}$$

3.5 Mills Ratio

Mills Ratio is denoted by MR and the expression for it is given below;

$$\text{MR}= \frac{F(x;a,b,\lambda)}{S(x;a,b,\lambda)} \tag{3.5.1}$$

Substituting the value of $F(x; a, b, \lambda)$ and $S(x; a, b, \lambda)$ from equations (3.2) &(3.1.2), we get ;

$$\text{MR}= \frac{\lambda\left(1-e^{-x^2/2a^2}\right)-(1-\lambda)\left(1-e^{-2bx}(2bx+1)\right)}{(1-\lambda)[2-(2bx+1)e^{-2bx}]+\lambda e^{-x^2/2a^2}} \tag{3.5.2}$$

4.Structural Properties of Qammer Distribution

4.1 Moments of Qammer Distribution

Suppose a random variable 'x' follows Qammer Distribution, then the r^{th} moment denoted by μ_r' is defined as;

$\mu_r{}' = \int_0^\infty x^r f(x;a,b,\lambda)dx$ (4.1.1)

Substituting the value of $f(x;a,b,\lambda)$ from equation (2.2),we get ;

$$\mu_r{}' = \int_0^\infty x^r \ \lambda \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} + (1-\lambda)4xb^2 e^{-2bx} dx$$

$$= \lambda \frac{1}{a^2} \int_0^\infty x^{r+1} e^{\frac{-x^2}{2a^2}} dx + 4b^2(1-\lambda) \int_0^\infty x^{r+1} e^{-2bx} dx$$

Putting $\frac{x^2}{2a^2} = z$, and $2bx = t$in the above integral separately which reduces to;

$$\mu_r{}' = \lambda \int_0^\infty (2a^2 z)^{\frac{r}{2}} e^{-z} dz + \frac{1}{(2b)^r}(1-\lambda) \int_0^\infty t^{r+1} e^{-t} dt$$

$\mu_r{}' = \lambda a^r 2^{\frac{r}{2}} \Gamma\left(\frac{r}{2}+1\right) + \frac{1}{(2b)^r}(1-\lambda)\Gamma(r+2)$ (4.1.2)

Now putting $r = 1,2,3,4$ to get $\mu_1{}', \mu_2{}', \mu_3{}' \& \mu_4{}'$respectively

$\mu_1{}' = a\lambda\sqrt{\frac{\pi}{2}} + \frac{1}{b}(1-\lambda)$ (4.1.3)

$\mu_2{}' = 2a^2\lambda + \frac{3}{2b^2}(1-\lambda)$ (4.1.3)

$\mu_3{}' = 3a^3\lambda\sqrt{\frac{\pi}{2}} + \frac{3}{b^3}(1-\lambda)$ (4.1.4)

$\mu_4{}' = 8\lambda a^4 + \frac{15}{2b^4}(1-\lambda)$ (4.1.5)

4.2Moments about mean

The moments about mean are obtained by using the relationship between moments about mean and moments about origin

Variance$\mu_2 = \mu_2{}' - (\mu_1{}')^2$ (4.2.1)

$$\mu_2 = \left(2a^2\lambda + \frac{3}{2b^2}(1-\lambda)\right) - \left(a\lambda\sqrt{\frac{\pi}{2}} + \frac{1}{b}(1-\lambda)\right)^2$$

$= a^2\lambda\left[2 - \frac{\lambda}{2}\pi\right] + \frac{(1-\lambda)}{b}\left[\frac{3}{2b} - \frac{(1-\lambda)}{b} - 2a\lambda\sqrt{\frac{\pi}{2}}\right]$ (4.2.2)

$$\mu_3 = \mu_3{}' - 3\mu_2{}' \mu_1{}' + 2\mu_1{}'^2$$

$$= 3a^3\lambda\sqrt{\frac{\pi}{2}} + \frac{3}{b^3}(1-\lambda) - 3\left(2a^2\lambda + \frac{3}{2b^2}(1-\lambda)\right)\left(a\lambda\sqrt{\frac{\pi}{2}} + \frac{1}{b}(1-\lambda)\right) + 2\left(a\lambda\sqrt{\frac{\pi}{2}} + \frac{1}{b}(1-\lambda)\right)^2$$

$= \frac{1}{2b^2}\left[\sqrt{\frac{\pi}{2}}\{6a^3b^2\lambda(1-2\lambda^2)+a\lambda(1-\lambda)(8b-9)\}+\frac{(1-\lambda)}{b}\{3(3\lambda-1)+4b(1-\lambda)-12a^2b^2\lambda\}+\right.$
a2λπ (4.2.3)

$$\mu_4 = \mu_4{}' - 4\,\mu_3{}'\,\mu_1{}' + 6\,\mu_2{}'\,\mu_1{}'^2 - 3\,\mu_1{}'^4$$

$= \left(8\lambda a^4 + \frac{15}{2b^4}(1-\lambda)\right) - 4\left(3a^3\lambda\sqrt{\frac{\pi}{2}} + \frac{3}{b^3}(1-\lambda)\right)\left(a\lambda\sqrt{\frac{\pi}{2}} + \frac{1}{b}(1-\lambda)\right) + 6\left(2a^2\lambda + \right.$
32b21−λaλπ2+1b1−λ2−3aλπ2+1b1−λ4

$= \frac{4a\lambda}{b^3}\left[2a^3b^3 - 3a^2b^2(1-\lambda)\sqrt{\frac{\pi}{2}}(1-2\lambda) + 3(1-\lambda)\left(ab\lambda - \sqrt{\frac{\pi}{2}}\right) + \frac{(1-\lambda)}{b^2}\left(\frac{15}{2b^2} - 12(1-\lambda)\right) + \right.$
6a4λ2πλ−1−3aλπ2+1b(1−λ)4 (4.2.4)

4.3 Standard Deviation

Standard deviation of Qammer Distribution is denoted by σ and is defined as ;

$\sigma = \sqrt{Variance}$

$$\sigma = \sqrt{a^2\lambda\left[2 - \frac{\lambda}{2}\pi\right] + \frac{(1-\lambda)}{b}\left[\frac{3}{2b} - \frac{(1-\lambda)}{b} - 2a\lambda\sqrt{\frac{\pi}{2}}\right]} \qquad (4.3.1)$$

4.4 Coefficient of Variation

Coefficient of variation denoted by C.V is defined as ;

C.V= $\frac{\sigma}{\mu_1{}'}$

$$= \frac{\sqrt{a^2\lambda\left[2 - \frac{\lambda}{2}\pi\right] + \frac{(1-\lambda)}{b}\left[\frac{3}{2b} - \frac{(1-\lambda)}{b} - 2a\lambda\sqrt{\frac{\pi}{2}}\right]}}{a\lambda\sqrt{\frac{\pi}{2}} + \frac{1}{b}(1-\lambda)}, \qquad (4.4.1)$$

4.5Index of Dispersion

The index of dispersion denoted by ID is defined as under;

$$\text{ID} = \frac{\sigma^2}{\mu_1{}'} = \frac{a^2\lambda\left[2 - \frac{\lambda}{2}\pi\right] + \frac{(1-\lambda)}{b}\left[\frac{3}{2b} - \frac{(1-\lambda)}{b} - 2a\lambda\sqrt{\frac{\pi}{2}}\right]}{a\lambda\sqrt{\frac{\pi}{2}} + \frac{1}{b}(1-\lambda)}, \qquad (4.5.1)$$

4.6. Moment Generating Function and characteristic function

Let the random variable 'x' follows Qammer Distribution, then the moment generating function of 'x' denoted by $M_x(t)$ is defined as;

$M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x;a,b,\lambda)dx$ (4.6.1)

Substituting the value of $f(x;a,b,\lambda)$ from equation (2.2) in the abo ve equation ,we have;

$$M_x(t) = \int_0^\infty \sum_{r=0}^{\infty} \frac{t^r}{r!} x^r \left(\lambda \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} + (1-\lambda)4xb^2 e^{-2bx}\right) dx$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \int_0^\infty x^r \left(\lambda \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} + (1-\lambda)4xb^2 e^{-2bx}\right) dx$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu_r{}'$$

Inserting the value of $\mu_r{}'$ we get ;

$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \left(\lambda a^r 2^{\frac{r}{2}} \Gamma\left(\frac{r}{2}+1\right) + \frac{1}{(2b)^r}(1-\lambda)\Gamma(r+2)\right)$ (4.6.2)

Similarly,

$$\phi_x(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \mu_r{}'$$

$\phi_x(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \left(\lambda a^r 2^{\frac{r}{2}} \Gamma\left(\frac{r}{2}+1\right) + \frac{1}{(2b)^r}(1-\lambda)\Gamma(r+2)\right)$ (4.6.3)

Which gives the characteristic function of Qammer Distribution.

4.7 Harmonic Mean

The Harmonic Mean denoted by $\frac{1}{H}$ is defined as;

$\frac{1}{H} = \int_0^\infty \frac{1}{x} f(x;a,b,\lambda)dx$ (4.7.1)

$$= \int_0^\infty \frac{1}{x}\left(\lambda \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} + (1-\lambda)4xb^2 e^{-2bx}\right) dx$$

$$= \int_0^\infty \left(\lambda \frac{1}{a^2} e^{\frac{-x^2}{2a^2}} + (1-\lambda)4b^2 e^{-2bx}\right) dx$$

$= 2b(1-\lambda)$ (4.7.2)

5. Method of Likelihood Estimation

The parameters of Qammer Distribution are estimated by method of likelihood estimation .Let $x_1, x_2, x_3, \dots \dots ., x_n$ be random samples of size n from Qammer Distribution then the likelihood function of Qammer Distribution is given by ;

$$L = \prod_{i=1}^{n} f(x; a, b, \lambda) \tag{5.1}$$

Substitute the value of $f(x; a, b, \lambda)$ in the above equation ,we get;

$$=\prod_{i=1}^{n} \left(\lambda \frac{x}{a^2} e^{\frac{-x^2}{2a^2}} + (1-\lambda)4xb^2 e^{-2bx}\right)$$

$$= \left(\frac{\lambda}{a^2}\right)^n \prod_{i=1}^{n} x_i e^{-\frac{\sum_{i=1}^{n} x_i^2}{2a^2}} + [4b(1-\lambda)]^n \prod_{i=1}^{n} x_i e^{-2b\sum_{i=1}^{n} x_i}$$

Applying log on both sides, we get;

$$\text{Log L} = nlo\ g\lambda - 2nloga + \sum_{i=1}^{n} logx_i - \frac{1}{2a^2}\sum_{i=1}^{n} x_i^2 + nlog[4b(1-\lambda)] + \sum_{i=1}^{n} logx_i - 2bi{=}1nxi \tag{5.2}$$

Differentiating the above equation partially w.r.t. λ, a and b respectively,then equating to zero we get ;

$$\frac{\partial \boldsymbol{logL}}{\partial \boldsymbol{\lambda}} = \mathbf{0}$$

$$\frac{\boldsymbol{n}}{\boldsymbol{\lambda}} - \frac{\boldsymbol{n}}{(\mathbf{1}-\boldsymbol{\lambda})} = \mathbf{0}$$

$$\boldsymbol{n}\left[\frac{\mathbf{1}-\mathbf{2}\boldsymbol{\lambda}}{\boldsymbol{\lambda}(\mathbf{1}-\boldsymbol{\lambda})}\right] = \mathbf{0} \tag{5.3}$$

$$\frac{\partial \boldsymbol{logL}}{\partial \boldsymbol{a}} = \mathbf{0}$$

$$\frac{\sum_{i=1}^{n} \boldsymbol{x_i}^{\mathbf{2}}}{\boldsymbol{a}^{\mathbf{3}}} - \frac{\mathbf{2}\boldsymbol{n}}{\boldsymbol{a}} = \mathbf{0}$$

$$\text{Estimate of } \hat{\boldsymbol{a}} = \sqrt{\frac{\mathbf{1}}{\mathbf{2}}\left(\frac{\sum_{i=1}^{n} \boldsymbol{x_i}^{\mathbf{2}}}{\boldsymbol{n}}\right)} \tag{5.4}$$

$$\frac{\partial \boldsymbol{logL}}{\partial \boldsymbol{b}} = \mathbf{0}$$

$$\frac{\boldsymbol{n}}{\boldsymbol{b}} - \mathbf{2}\sum_{i=1}^{n} \boldsymbol{x_i} = \mathbf{0}$$

$$\text{Estimate of } \hat{\boldsymbol{b}} = \frac{\mathbf{1}}{\mathbf{2}}\left(\frac{\boldsymbol{n}}{\sum_{i=1}^{n} \boldsymbol{x_i}}\right) \tag{5.5}$$

6. Order Statistics

Let us suppose $x_{1,}x_2, x_3, \dots \dots x_n$ be the random samples of size n from Qammer Distribution with PDF$f(x; a, b, \lambda)$and CDF $F(x; a, b, \lambda)$.Then the probability density function of K^{th}order statistics is given as.

$$f_{x(k)}(x) = \frac{n!}{(k-1)!(n-k)!}[F(x)]^{k-1}[1\ - F(x)]^{n-k}f(x) \qquad (6.1)$$

Substituting the values of pdf and cdf ,we get ;

$$f_{x(k)}(x) = \frac{n!}{(k-1)!(n-k)!}[\lambda\left(1 - e^{-x^2/2a^2}\right) + (1-\lambda)\left(1 - e^{-2bx}(2bx+1)\right)]^{k-1}[1\ - [\lambda\left(1 - \right.$$
$e{-}x22a2{+}1{-}\lambda1{-}e{-}2bx2bx{+}1\ ]n{-}k\lambda xa2e{-}x22a2{+}1{-}\lambda4xb2e{-}2bx$ (6.2)

Putting k=1 in the above equation we get the ist order statistic of Qammer Distribution as under ;

$$f_{x(1)}(x) = n[1\ - [\lambda\left(1 - e^{-x^2/2a^2}\right) + (1-\lambda)\left(1 - e^{-2bx}(2bx+1)\right)]^{n-1}\left[\lambda\frac{x}{a^2}e^{\frac{-x^2}{2a^2}} + (1 - \right.$$
$\lambda 4xb2e{-}2bx$ (6.3)

Putting k = n in equation (6.2) we get the n^{th} order statistic of Qammer Distribution as under ;

$$f_{x(n)}(x) = n[\lambda\left(1 - e^{-x^2/2a^2}\right) + (1-\lambda)\left(1 - e^{-2bx}(2bx+1)\right)]^{n-1}\left[\lambda\frac{x}{a^2}e^{\frac{-x^2}{2a^2}} + (1 - \right.$$
$\lambda 4xb2e{-}2bx$ (6.4)

Data Analysis

The usefulness of the formulated distribution has been explained through two Banking related data sets. The data sets are as follows

Data set 1:- The data set represents the remission times (in months) of a random sample of 128 Defaulted Accounts. The observations are follows

0.08, 2.09, 2.73, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.22, 3.52, 4.98, 6.99, 9.02, 13.29, 0.40, 2.26,3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 15.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.93, 8.65, 12.63, 22.69

The summary of the data is given in Table 1.

DATA 1	Min	Mean	Median	Variance	1st Qu.	3rd Qu.	max
	0.080	9.311	6.050	112.178	3.295	11.678	79.050

Table 2: Performance of the distributions:

Distribution	AIC	BIC	AICC	HQIC
AD	828.94	831.76	829.14	836.38
WAD	980.71	983.53	980.91	988.15
IAD	1007.63	1010.45	1007.83	1015.07
PRO	803.41	811.88	803.61	806.84

Table 3: The estimation of parameters for the first data set:

Distribution	Estimated Parameters
AD	0.107404257
WAD	0.214803804
IAD	2.4395558
PRO	24.42251460 0.11388888 0.14964964

Data set II: This data represents the Risk Rating Migration times, in weeks of 33 accounts identified from defaulters as Insolvents. The data is as follows: 65,156,100,134,16,108,121,4,39,143,56,26,22,1,1,5,65,56,65,17,7,16,22,3,4,2,3,8,4,3,30,4,43

The summary of the data is given in Table 4.

DATA 2	Min	Mean	Median	Variance	1st Qu.	3rd Qu.	max
	1.00	40.88	22.00	2181.17	4.00	65.00	156.00

Table 5: Performance of the distributions:

Distribution	AIC	BIC	AICC	HQIC
AD	349.554	354.044	350.382	351.065
WAD	437.589	439.086	438.417	443.100
IAD	366.123	367.619	366.950	371.633
PRO	345.543	347.039	346.370	351.054

Table 6: The estimation of parameters for the second data set:

Distribution	Estimated Parameters
AD	0.024468886
WAD	0.048926949
IAD	6.0070564
PRO	22.355941413 0.001000000 0.24451324

Conclusion

In this paper , the new distribution is introduced by combining the two frequently used statistical distributions namely Rayleigh and Ailamujia distributions by the mixing the third parameter λ.The resulting distribution is titled as ‘Qammer Distribution’.Some structural properties ,reliability analysis,moment generating function ,characteristic function, order statistics ,maximum likelihood estimation of Qammer Distribution are derived. To check the application of our derived distribution two data sets related to Indian Private Banks are inserted which gives the better fits as compared to other distributions .

References

1. Aijaz, A. Ahmad and Tripathi R, Inverse Analogue of Ailamujia Distribution with Statistical Properties and Applications. Asian Research Journal of Mathematics.16(9): 36-46, 202.
2. A. Aijaz, S. Qurat ul Ain, A.Ahmad, Tripathi.R,Bayesian Estimation of Inverse Ailamujia Distribution using different Loss Functions, Journal of Xi'an University of Architecture &Technology, Pages:226-235
3. Jayakumar B, Elangovan R. A new generalization of Ailamujia distribution with applications in bladder cancer data. IJSRMSS. 2019;6(1), 61-68.
4. Balakrishnan, N. and Basu, A. P. (1996). The Exponential distribution, Theory, Methods and Applications. New York: Gordon and Breach.
5. Belisle, C. J. P. (1992). Convergence theorems for a class of simulated annealing algorithms on Rd. Journal of Applied Probability. 29; 885-895.
6. Bera, W. T. (2015). The Kumaraswamy Inverse Weibull Poisson Distribution with Applications. Theses and Dissertations, 1287. Indiana University of Pennsylvania. http://knowledge.library.iup.edu/etd/1287
7. Bhati, D., Malik, M. A. And Vaman, H. J. (2015). Lindley-exponential distribution: Properties and applications. Metron, 73(3), pp. 335-357.
8. Ekhosueho, N. and Opone, F. (2018). A three Parameter generalized Lindley Distribution; Properties and Applications. STATISTICA, anno LXXVIII, 3; 233-249.

9. Ghitany, M., Al-Mutairi, D., Balakrishnan, N. and Al-Enezi, I. (2013). Power Lindley distribution and associated inference. Computational Statistics and Data Analysis, 64, pp. 20-33.

10. Gulshan, A. M. and Ahamd S. A. (2019). Lindley Stretched Exponential distribution with Applications. Global Scientific Journals. 7(1). ISSN 2320 – 9186.

11. Henningsen, A, and Toomet, O. (2011). MaxLik: A package for maximum likelihood estimation in R. Comput Stat, 26; 443-458. DOI: 10.1007/s00180-010-0217-1.