Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 8 July 2021: 5481 – 5496

Research Article

Number Theory

Deepak Tiwari[a*] and Dr Jaya Kushwah[b]
[a]Dept. of Mathematics, Himalayan University, Itanagar Arunachal Pradesh, India
Email. tdeepak703@gmail.com[a]
[b]Asst. Professor, Dept. of Mathematics Himalayan University Itanagar Arunachal Pradesh, India
Email. kushwahjaya@gmail.com[b]

Abstract
Maths is a universal language which allows us to think analytically. Mathematics is essentially the discipline that addresses the logic of form, volume, and organisation. The “number theory” is a field of basic arithmetic, which is mainly concerned with the research of binary, integer or arithmetic or arithmetic methods for older purposes. Helps to find and to prove that there are true fascinating partnerships between different numbers. The theory of numbers is often quantitative and sometimes analytical. The experimental aspect leads to questions and offers ways to react. The analytical component attempts to develop an argument that answers the questions decisively. The science of numbers has long intrigued amateurs and experts. While Solutions of the question to the theorem and proofs, many issues and numerical theorems by people can be understood in comparison to other branches of mathematics. The historical and unsolved issues of number theory, positive numerical numbers and natural numbers are presented in this paper.

Keywords: Mathematics; Number Theory; Natural Numbers; Positive Numbers.

Introduction

In Greek, the term "knowledge, study and learning" contains the research of such subjects as number, “algebra Structure, space, geometry, and modification (numerical assessment)” [11]. The science of form, quantity and arrangements is the science of mathematics. In all, maths is everywhere about us. It is the basis for everything in our day-to-day life, including mobile devices, (old and modern) architecture, art, money, engineering, and sports. Mathematical thinking may be used to offer information or conclusions about nature because mathematical systems are good representations of actual phenomena. Mathematics develops from the counting, estimation, measurement and rigorous study of the real objects' shapes and movements by using abstraction and logic. Practical maths has been a human endeavour since written documents exist. It can take years or even decades to investigate the analysis necessary to solve mathematical problems. “German mathematician Carl Friedrich Gauss (1777–1855) [2]” stated, The queen of science is Mathematics, and “number theory” is the ideal of mathematics [3] . The number theory is the mathematical branch dealing with the positive integer properties (1, 2, 3, ...). It is often referred to as "greater arithmetic," it is one of the oldest and natural studies in mathematics.

The science of numbers has always been fascinating for amateurs and professionals. While answers to the questions and evidence of statements also involve advanced numerical backgrounds, many of the issues and theorems of numbers theory can be grasped by the people.

The theory of number, which had little direct use for the real world, was the purest branch of mathematics until the mid-20th century. Digital machines and digital connectivity have shown that the principle of digital numbers can offer surprising solutions to challenges in the physical world [4]. Around the same time, information technological advancements have made it possible for countless theorists to take significant steps forward in vast quantities, determine premiums, evaluate guesses, and resolve numerical questions until those are not considered available to them.

A wide range of subjects, such as primary, “algebraic number theory”, “number theory” analytical number theory, geometry, and “probabilistic number theory is modern number theory”. These groups represent the approaches utilized to tackle integer questions [5].

This main text describes the brief introduction into mathematics and why it is essential to us both in our personal and in our daily life, and then offers background research and analysis of maths education through the systematic analysis of various interventions in various research papers that discuss empirical studies in enhanced mathematical education through different avenues and then, the new educational and technological developments on these mathematical education approaches and trends stop.

1. Background

- **Pre-history to classic Greece**

The desire to count is prehistoric. This is obvious from archaeological objects including a bone from Africa, Congo, which is ten thousand years old and has tally characteristics—signs of an ancient ancestor, which counts as something. The notion of "multiplicity" was grasped close to the dawn of civilization, paving the way for the earliest steps in numerical analysis.

There is no doubt that in ancient Mesopotamia, Egypt, China and India, there existed knowledge of numbers on tablets, papyrus, and the temples of these early civilizations. One case in point is that a Babylonian tablet called Plimpton 322 (1700 BCE). It shows three-fold x, y, and z in modern notation as well as the property $x^2 + y^2 = z^2$. One is 2.291, 2.700, and 3.541, with 2.2912 + 2.7002 = 3.5412. This surely demonstrates some theoretical numbers in old Babylon [6].

There was no overall theory of numbers while these findings were separate. For this, one must turn to the ancient Greeks, as with so many breakthrough achievements in theoretical mathematics that demonstrate the magical inclinations of the "Pythagoreans" and the harrow of "Euclid's Elements (c. 300 BCE)”.

Pythagoreans (c. 580-500 BC) traditionally served with dedicated adherents in southern Italy. The amount of his theory is the unifying principle needed to explain anything, from celestial movement to musical harmony. The Pythagoreans are not surprising give certain numbers of quasi-rational properties in this respect.

They placed a premium on ideal numbers, as equivalent to the total of the factors of their right. Examples include 6 (whose divisors are 1, 2, and 3) and 28 (1 + 2 +4 + 7 + 14). Examples include the Greek philosopher Nicholas of Grease (blooming about 100 CE), who claimed that the ideal numbers of "virtues, wealth, temperance, property, and elegance" served centuries after Pythagoreans but obviously in his philosophical debt. (Numerical theology is considered nonsensical by a few modern writers) [7].

Similarly, the Greeks regarded a pair of numbers, if either one was the addition of the appropriate divisions. The only one friendly pair they recognised was 220 and 284. The number of the correct 284 splitters can be conveniently checked to 1 + 2 + 4 + 71 + 142 = 220 and the amount of proper 220 splitters to 22 + 2 2 + 4 + 10 + 10 + 11 + 20 + 22 + 45 + 110 = 284. It must have seemed magical for a phenomenon.

- **Euclid**

Euclid started Buch VII of his Elements with a number described as "a multitude made up of units". The number "measurable by a single unit," that is, the sole valid divider was defined as the non-primary and ideal component equal to the number of parts) prime (i.e., its appropriate divisors).

Hence Euclid demonstrated a series of statements which mark the starting of the theory of numbers as a (rather than a numerological) mathematical undertaking. Special mention should be given to four Euclidean proposals.

The first book VII is a method for determining the greatest ordinal factor of two whole integers. This important finding is now known as the “Euclidean algorithm” in his honour.

Second, Euclidian established a basic theorem based on a version of the so-called special theorem for factoring or arithmetic. This means that in the product of primes, all numbers should be treated in the same way. While Euclid's debates on the unusual three-factoring cannot meet modern needs, Proposals 32 in Book VII and 14 in Book IX are at the core of this book [8].

Third, Euclid demonstrated that not everyone has a limited range of premiums. Proposition 20 of Book IX has long been one of the most beautiful pieces of evidence in all of mathematics, beginning with certain predictable primes a, b, c..., n. The number of primes forming Euclid suggested including one to the product: N = (b c to n) + 1, then it looks at the two options:

1. If N is primary, it is not recent since a, b, c... n is greater. For instance, suppose the new primes were 2, 3, and 7, then the prime was $N = (2 \times 3 \times 7) + 1 = 43$.
2. Instead, where N is composite, the primary element must be that which cannot be one of the originals, as shown by Euclid. The primes 2, 7 and 11 start to demonstrate, so that $N = (2 \times 7 \times 11) + 1 = 155$. This is complex but it does not appear in the originals in the primary factors 5 and 31. Anyway, there can still be increased a limited number of primes. It follows that the set of premiums is endless, with this lovely piece of logic.

Fourthly, the Book of Euclid ends with a blockbuster: when the digit N = 2k (1 + 2 + 4 + ...+ 2k) is 1 + 2 k (1 + 2 + 4 + ... + 2k), (1 + 2 + 4 = 7). For instance, the prime is perfect, so 4 (1 + 2 + 4) = 28. The "formula" of Euclid was a great success for perfect numbers.

- **Diophantus**

The author of Arithmetic is particularly notable among Greek mathematicians later to be noted. This paper contains several problems of which Diophantine equations have been the most important. For instance, Diophantus called for “two numbers”, “one square” and “one cube”, such that the squares themselves became a square [9] . Alexandria searched y, x and z in contemporary symbols such that $(x^2)^2 + (y^3)^2 = z^2$. Real numbers that meet this relationship is simple to find (for example, x = square root of 2, y = 1, and z = root of 2); however, the need to have integer solutions creates the problems more complex. (One answer shall be x = 6, y = 3, and z = 45.) The work of “Diophantus” inspired after mathematics [10].

- **Number theory in the East**

The “millennium” after Rome's fall saw no major European advancements, but Chinese and Indian academics contributed themselves to number theory. China's Sun Zi (sol Tzu), a Chinese mathematician about 250 CE), who dealt with some Diophantine calculations was motivated with questions of astronomy and the calendar. For example, China’s son asked a complete number for 2 to remain if divided by 3, if divided by 5, leaves 3 to remain, and 7 leaves 2 to remain (his answer: 23). Nearly 1000 years later the Chinese theorem Qin Jiu Shao (1202–61) provided a general method for the resolution of such problems [11].

Indian mathematicians have, in the meantime, been working difficult. “Brahmagupta” got up (wrongly) what is now known as the “Pell equation” in the 7th century. He raised the question of finding a perfect place that would produce another perfect place when multiplied by 92 and increased by 1. In other words, it looked for “whole numbers” x and y, $92x^2 + 1 = y^2$, a “Diophantine” equal to “quadratic terms". "Brahmagupta" indicated that someone who solved the problems in a year's time would be allowed to call him a mathematician. The x = 120 and y = 1 151 is the solution.

"Indian scholars" have created known as "Hindi-Arabic numerals", which are the foundation 10 of the mathematical and civil cultures around the world (see numerals and numeral systems) and these numbers won because of their simplicity and ease of use, even though they are numerical than theoretical. The Indians used this version, which included zero, around 800 CE.

The Islamic world became a mathematical giant at about this time. Situated on trading lines among "East and West", Islamic scholars took up and expanded the works of other cultures. For example, Qurrah was engaged in "Baghdad in the 9th century," reverted to the subject of friendly numbers in Greece and found a "second pair 17,296 and 18,416".

- **Modern Number Theory**

Mathematics was transmitted from the "Islamic culture to Renaissance Europe", thus the theory of numbers collected small interest. Significant advances in mathematics, algebra, and probability were made between 1400 and 1650 BC, not least in the detection of logs and analytical geometry. However, numbers theory, which was perceived as a difficulty.

- **Pierre de Fermat**

Pierre de Fermat, a French judge with spare time and a penchant for statistics, is credited for advancing this viewpoint. Despite the fact that he wrote nothing, Fermat posed and articulated questions that have influenced number theory ever since [12] .

Fermat argued that if p is primary and a number is complete, in 1640. p is divided into ap – a similarly small theorem. As p = 7 and a = 12 thus, the far-reaching assumption is that 7 is 128 − 12 = 35,831,796. The results are not clear. This theorem is one of the major instruments of scientific theory of numbers [13].

Fermat examined the two categories of odd premiums: those which are more than four and those which are less than four. These are referred to as a 4k + 1 bonus and a 4k − 1 bonus. The first ones include 5 = 4 ×1+1 and 97=4 ×24+1; the latter include 3=4 ×1-1 and 79=4 ×20 −1. Fermat stated that every 4k + 1 type primary could be written in a single manner as a sum of two squares, whereas the 4k − 1 prime cannot be created in any way as the sum of the 2 squares. So, there are no alternate decomposition into sums of squares 5 = 22 + 12 and 97 = 92 + 42. 3 and 79 cannot, on the other hand, be so degraded. This dichotomy is one of the hallmarks of numerical theory among primes.

Fermat proclaimed that the total amount of four or less squares could be represented in 1638. Fermat had evidence but did not communicate it. Fermat claimed that the right triangle with integer length sides cannot be established with a perfect quadrant field. This is like telling us that x, y, z, it has no integer (such as $x^2 + y^2 = z2$) and $w^2 = 1/2$(base) x (height).

Fermat has presented uncharacteristic evidence of this latest outcome and used a method that was suitable to show the impossibility, called infinite descent [14]. The rational approach implies that the constraint in question is met in whole numbers and produces smaller whole numbers that would satisfy it. Fermat generated a continuous series of declining whole numbers, applying the statement over and over. However, this cannot be assumed since there would be less positive integer numbers. Fermat finished with this irony that in the first-place certain numbers can't occur.

There can be two more assertions from Fermat. First, any form number 22n + 1 must be primary. For formula 220 + 1 = 3, 221 + 1 = 5, 221 + 1 = 17, 223 + 1 = 257, and 224 + 1 = 65,537, the number has been right if it is = 0, 1, 1, 2, 3, and 4. This is also known as Fermat primes. The next number 225 + 1 = 231 + 1 = 4,294,967,297 is unfortunately not primal to his reputation (more about that later). It was also unbeatable to Fermat.

The second statement is one of the highly popular mathematical assertions in the past. Fermat scribbled near the edge of this book during Diophantus' reading of Arithmetic: To separate a 'cube into two cubes', a power or any other power that is not conceivable in two powers above the second one of the same kinds.

Fermat claimed in symbols that if $n > 2$, there will be no "whole numbers x, y, z, as $x^n + y^n = z^n$", Fermat's reasoning is known as the theorem. It defeated all opponents for three and a half centuries, earning it the title of the most popular unsolved mathematics problem. While Fermat was clever, the theory of numbers was mostly ignored. His reluctance to give proof was part of the problem, but the calculus in other eras of the "17th century" was much more detrimental. Calculus is the most flexible mathematical tool, and academics have solved a number of real-world issues by enthusiastically applying their ideas. In distinction, "number theory" appeared too "pure," too divided from the anxieties of "physicists", "astronomers', and "engineers".

- **Number Theory in 18th Century**

The recognition for introducing Fermat's dream number theory to the mainstream is owed to the dominant mathematics figure of the 18th century [15]. Euler was one of the profiles and most influential mathematicians ever, and the topic could no longer be disregarded as focused his attention on number theory.

Euler initially shared his colleagues' general ignorance, But in Christian Goldbach's correspondence (1690–1764) [16], A fan of numerical theory who knew the work of Fermat. Like a strong salesman, Goldbach was interested in numerical theory and finally paid off his insistence.

Goldbach wrote on 1 December 1729: "this will sure both number 22, n and number one are the prime numbers, this comment of Fermat is known. Euler was impressed. Indeed, he demonstrated that Fermat's claim was incorrect by dividing the 225 + 1 in 641 and 6700417 items.

Over the next five periods, Euler wrote over 1000 sheets on numerical analysis, many of which included evidence of the claims of Fermat. Euler proven the small theorem of Fermat in 1736 (cited above). By the middle of the century, Euler had discovered "Fermat's theorem" that primes of the way 4k + 1. It subsequently got up the question of "perfect numbers", showing that even a "perfect number" would take the form Euclid found 20 centuries ago (see above). Euler has extended the supply universe significantly by expanding 58 additional ones as turned its focus to nice numbers, only three couples of which have been named at the time.

Even Euler could not solve each dilemma, of course. The last theorem of Fermat for exponents n = 3 and n = 4 was supposed or almost shown, but desperate to find an overall solution. Goldbach's claim that even more than 2 numbers can be written as the total of two primes was totally stunning for him. Euler accepted the finding – today called the Goldbach theorem – but acknowledged it had failed to prove.

Euler brought scientific authority to number theory and fast advancement thereafter.

In 1770, for example, "Joseph-Louis Lagrange (1736–1813) [17]" Fermat has shown that any number as a sum of four or less squares can be printed. Soon afterwards Euler found a result which is identified as "Wilson's theorem": p will pre-eminent if and only if p split equally into "$(p - 1) \times [[p - 2] \times \ldots\ 3 \times 2 \times 1] + 1$".

- **18th Century**

Arithmetical disquisitions the 1801 publication of Carl Friedrich Gauss Arithmetical Disquisitions (1777–1855) was of considerable importance. It was in a way the divine writing of the philosophy of numbers. In it, Gauze organised the work of his predecessors and outlined them before they bravely moved to the scientific frontier. Gauss' first contemporary evidence of the special factorization theorem was that it noted the difficulty of solving "composite numbers" into primary aspects as "this one was the most important and useful in arithmetic." It presented the first evidence for the law of quadratic reciprocity, a profound finding that Euler had rarely seen. Gauss presented the concept of a congruence between numbers to speed up his work — that is, specified "a and b to be modulo m" congruent ("a =

b mod m"), when the variation a –b is equally divided. 39 § 4 mod 7, for example. Paired with outcomes like Fermat's small theorem, this innovation has become an essential element of numerical theory.

Another mathematician from the 19th century, inspired by Gauss, took up the task. Sophie Germain (1776–1831) and Peter Gustav Lejeune Dirichlet (1805–1833) [18] and Adrien-Marie Legendre (1752–1833), who once said "I have never stopped thinking in number theory," made significant contributions to Fermat's last theorem) [19]. The theorem verified for n = 5 — that is, the two fifth powers sum was shown as cannot be a fifth power. By 1847 Ernst Kumar (1810-1893) continued to show that the last theorem of Fermat was valid for a great group of exponents; Fermat could unfortunately not exclude the probability for a big club of exponents that it was incorrect, so the issue was unsolved.

The Dirichlet himself (who allegedly maintained a copy of Gauss's Dissuasions Arithmetic logical for evening reading by his bedside) made an important contribution by demonstrating how "arithmetical progression a, a + b, a + 2b and a + 3b". Infinitely many bonuses should be needed because a and b have the same element. This has shown that 4k + 1 primes are infinitely numerous and 4k − 1 first premium too is infinitely large. However, Dirichlet's method of proof was what made this theorem unique: Dirichlet used calculus methods to produce a result in the theory of numbers. This unexpected yet ingenious approach signalled the start of a new branch: the philosophy of analytical numbers.

- **Prime Number Theorem**

The main theorem of 19th-century mathematics, and it is worth a short digression, was one of the highest achievements. First, specify the number of primes below or equivalent to n by π (n). So, π (10) = 4 and the four primes not more than 10 are 2, 3, 5 and 7. Likewise (25) = 9 andπ (100) = 25. Next, consider the percentage of prime i.e., T(n) /n numbers lower or equal to n [20] . Of course, π (10)/10 = 0,40 is the highest of 40% of those who have not exceeded 10. Table 1 shows other proportions.

Table1: proportion of prime number

n	$\pi(n) = no. of\ primes \leq n$	$\frac{\pi(n)}{n}$ = proposition of primes among the first n numbers	$\frac{1}{log\ n}$ = predicted proposition of primes among the first numbers
10^2	25	0.2500	0.2172
10^4	1229	0.1229	0.1086
10^6	78498	0.0785	0.0724
10^8	5761455	0.0570	0.0543
10^{10}	455052511	0.0455	0.0434
10^{12}	37607912018	0.0377	0.0362
10^{14}	3204941750802	0.0320	0.0310

A trend is amazingly simple, but at least roughly the theorem of prime numbers defines one and requires a formula for dissemination of premiums between the "whole numbers". The theorem states that the ratio π(n)/n is approximately "1/log n" for the big n, where the "log n is the normal logarithm of n". It is nothing short of remarkable to bind primes to logs.

One of the first to see this was the young Gauss, who proposed it in his fertile mind to examine his log tables and prime numbers. Bernhard Riemann (1826–66) [21] after Dirichlet exploited analytical methods of theory of numbers and Panty Chebyshev (1821–94) [22] Significant advances made until Jacques Hadamard (1865–1963) [23] demonstrated the prime-number theorem in 1896 [24] and "Charles Jean de la Vallie-Poussin (1866–1962) [25]". The 19th century came to a successful conclusion.

- **20th Century**

Paul Erodes (1913-96) was a legendary name in the philosophy of numbers of the 1900s [26]. A "Hungarian genius" well-famous for his profound observations, his broad sphere, and individual peculiarities. Erodes released at the age of 18 are much simpler evidence of a Chebyshev theorem that, when n part 2 is present, the prime must be between n and 2n. This was the first in a series of theoretical findings which spanned most of the century. Dobie has written over 1500 papers and produced over 500 collaborators across the globe by Erodes – who have worked in combination theories, graph theories and theory of dimensionality. This astonished achievement Erode accomplished when it was working off a case and continuously going in search of new mathematics from one university to another. It was not unusual for him to come with a statement, "My brain is free," without being reported, and then to go through the last tasteful matter.

It is worth mentioning two later inventions. One was the invention of the electronic machine, which was extended to a variety of theoretical problems with benefit. Euler once hypothesised, for example, that at least 4th competencies had to be put together to make the number a 4th power. But in 1988, American Noam Ellies, in conjunction with the muscles of the machine, uncovered Euler's theory, 2682, 4404+15,365,6394 >18,796,7604 =20,615,6734. (There is a total of 30 digits on the right, so it is not surprising that Euler skipped).

Second, the principle of numbers gained a flavour that was extended when it was instrumental in the implementation of encryption systems commonly used in government and industry. This depends on the contribution of enormous numbers to premiums — an aspect which is familiar to the code user. This application contradicts the long-standing perception that the philosophy of numbers is nice but fundamentally pointless. (See encryption: encryption).

The number theory of the 20th century peaked greatly during the 1995 season when Fermat's last theorem, with the prompt support of his British equivalent Richard Taylor, was demonstrated by England man Andrew Wiles. Wiles flourished, when too many failed with an unbelievably intricate 130-page evidence that would surely not work anywhere else.

2. Theorem and Unsolved Problems

Table 2: Unsolved problems of theorem

Category	Name	Function /Related Concept
Algebric Number Theorems	Albert–Brauer–Hasse–Noether theorem	Central simple algebra Algebraic number field
	Ankeny–Artin–Chowla congruence	Class number Quadratic field
	Bauerian extension	Filed Extention Prime ideals
	Brauer–Siegel theorem	Class Number Imaginary Quadratic filed
	Chebotarev theorem on roots of unity	Lacunary Series Uncertainty Principle
	Chebotarev's density theorem	Splitiing of Primes Ideal primes
	Dirichlet's unit theorem	Unit if groups' rank Primitive elements
	Ferrero–Washington theorem	Iwasawa's μ invariant Class field theory
	Gras conjecture	Ideal Class group Cyclotomic units

	Gross–Koblitz formula	Gauss Sum P-addic gamma function
	Grunwald–Wang theorem	Local global principle p-adic number
	Hasse norm theorem	Cyclic extension Local global principle
	Hasse–Arf theorem	Local class field theory Galois Extension
	Hasse's theorem on elliptic curves	Elliptic curve Finite field
	Herbrand–Ribet theorem	Class group, number Bernoulli number
	Hermite–Minkowski theorem	Striling’s formula Field extension
	Hilbert–Speiser theorem	Abelian extension Cyclotomic filed
	Hilbert's Theorem 90	Cyclic extension Kummer theory
	Kronecker–Weber theorem	Abelian extension Galois group
	Lafforgue's theorem	General linear groups Automorphic forms
	Landau prime ideal theorem	Prime number theorem Prime ideals
	Local Tate duality	Absolute Galois group Non- Archimedean local field
	Main conjecture of Iwasawa theory	p-adic L- functions Ideal class group
	Mazur's control theorem	Selmer group Abelian variety Number filed
	Minkowski's bound	Complex embeddings Real embeddings
	Mordell–Weil theorem	Finitely generated abelian group Diophantine geometry
	Neukirch–Uchida theorem	Absolute Galois group Anabelian geometry
	Octic reciprocity	Reciprocity Law Modulo Primes
	Ostrowski's theorem	p-adic absolute value rational number
	Principal ideal theorem	Class field theory Hilbert class field
	Reflection theorem	Isotypic components Cyclotomic field

	Scholz's reciprocity law	Reciprocity law Quadratic number fields
	Shafarevich–Weil theorem	Galois extension Galois group
	Shintani's unit theorem	Minkowski space Number field
	Stark–Heegner theorem	Unique factorization Ring of integers
	Stickelberger's theorem	Galois module Cyclomatic filed
	Takagi existence theorem	Abelian extension Algebraic closure
	Thaine's theorem	Real abelian fields Mazur wiles theorem
	Yamamoto's reciprocity law	Reciprocity law Class number
Analytical Number Theorems	Barban–Davenport–Halberstam theorem	Prime number Arithmetic progression
	Bombieri–Vinogradov theorem	Arithmetic progression Large sieve method
	Brun–Titchmarsh theorem	Prime number in arithmetic progression
	Chebyshev's bias	Prime number theorem Arithmetic progression extension
	Chen's theorem	Semiprime Sieve method
	Friedlander–Iwaniec theorem	Prime numbers Landau's problem
	Fundamental lemma of sieve theory	Sieve method Weighted average
	Hardy Ramanujan theorem	Normal order Prime factors
	Jurkat–Richert theorem	Sieve theory Goldbach's conjecture
	Kronecker limit formula	Real analytical Eisenstein series Dedekind eta function
	Kuznetsov trace formula	Kloosterman sums Automorphic forms
	Landau prime ideal theorem	Prime number theorem Arithmetic progression
	Landsberg–Schaar relation	Jacobi Poisson summation formula
	Linnik's theorem	Prime in the arithmetic progression

		Coprime
	Maier's theorem	Probabilistic model of primes Prime counting function
	Perron's formula	Arithmetic function Mellin transform
	Petersson trace formula	Modular form Orthonormal basis
	Prime number theorem	Prime counting function Natural logarithm
	Ramanujan's master theorem	Infinite series Quantum physics
	Riemann–Siegel formula	Z function Dirichlet series
	Riemann–von Mangoldt formula	Riemann zeta function Prime number
	Vaughan's identity	Primes in arithmetic progression Euler’s totient function
	Vinogradov's mean-value theorem	Sum of powers Diophantine equations
Fermat's Last Theorem	Fermat's Last Theorem	Fermat’s Conjecture Arithmetic, positive integer Fermat's Last Theorem in fiction Fermat's right triangle theorem Fermat's right triangle theorem Proof of Fermat's Last Theorem for specific exponents
Lemmas in Number Theroy		Euclid's lemma Fundamental lemma (Langlands program) Gauss's lemma (number theory) Heegner's lemma Ihara's lemma Krasner's lemma Lifting-the-exponent lemma Thue's lemma Zolotarev's lemma

3. Methodology

Methodology is a "centred scientific model". A coherent and objective approach based on the views, beliefs and concepts of researchers and other consumers. It includes theoretical analysis of the group of methods and principles related to an expert branch, which varies in various disciplines due to its historical development. It offers a range of methodologies extending over the opposite interpretation of reality and fact. This puts approaches into broad philosophies and methods.

The technique of study describes the pattern of research results. The analysis on this paper is focused on the chosen methods. There is a certain methodology available that can be selected to adequately assess questions and conclusions for the standards and requirements set by an individual.

- **Research Philosophy**

A scientific theory is an interpretation of how evidence about a phenomenon should be collected, interpreted, and used. The term epistemology encompasses the various philosophies for the analysis, unlike doxology (what is considered true). The aim of science is therefore to transform creeds into known things: doxa to episteme. There are two major research theories developed in the Western tradition of research, namely positivists and interpreters (also known as positivist). There are two major research philosophies within the scope of market studies:

Positivism. Positivism is a philosophical principle which states that "genuine" knowledge (familiarity of rather that is not acceptable by description) is derived solely from experience with and the properties and associations of “natural phenomena”. Therefore, sensory information, as understood by reason and logic, is the primary source of all such intelligence.

Interpretivism. Interpretations argue that this fact can be totally comprehended only by circumstantial sensitivity and interference. The study of phenomena in their natural context and the understanding that science cannot support but affect the phenomena they study are critical for important philosophy.

- **Research Approach**

This thesis proposes a top-to-bottom deducting process. Theory and factual arguments should be combined by that. Deductive and inductive methods have a significant distinctive role in research hypotheses. Deductive approach looks for hypothesis (or hypothesis), while inductive approach contributes to the creation of novel theories and generalisations. The "surprising facts" or "puzzles" begins with the test of abduction.

- **Time Horizon**

The study takes around 65 days. After data has been compiled for each branch of mathematics, theorems of the mixture are settled. Each part describes the hypothesis and thresholds over time. Examples are taken and analysed at an early stage. The supplies are labelled according to the milestones, and results of optimum efficiency are obtained.

- **Research method**
- **Theoretical Method**

To limit the scope of similar results, a theoretical framework is used. However, a few inventions could demonstrate that scientific processes happen one thing. In fact, any declaration specifying what general statements involving cause or area theory based largely on what is being evaluated or represented is, indirectly, minimum.

- **Rule-based mathematical model**

The guidelines ensure that the entry of the data is not altered and 100% of the results are obtained. Two or three input data sources may be used to simulate the system. Data can be evaluated using real-world tables and one or two outputs can be used for the parameters and gates used to construct real-world tables.

- **Practical Method**

Practical refers to an individual, concept, mission, etc., as more concerned, or applicable to practise than to theory. Practical analysis refers to awareness building and is helpful in doing so. It adheres to science study fundamental principles — specific studies identified. Issues, legitimate behaviour, compilation and review of systemic data and associated conclusions (i.e., not overstated).

- **Sampling**

Random forecasts and results according to the selected methodology increase improbable sampling. It is assumed that the results achieved for the whole region under survey are correct. The overall result will be to calculate a combination of the 3 statistical divisions and to use them for other purposes. The improbable sample space allows flexibility when selecting conical equations, calculus, arithmetic, trigonometric equations, etc.

Probability sampling: Probability samples are characterised by the probability theory approach of selecting samples as a broad population sampling technique. A delegate must be selected by random choice to be a probability sample. Each society is included in this method, and everybody has the same selection opportunities.

Nonprobability sampling: The improbable sample solution requires the researcher's judgement to select a sample. This kind of survey was mostly taken from the researchers' or statisticians' ability to receive this sample. The main objective of this sample approach is to conclude the analysis.

- **Analysis and interpretation of data**

The data obtained by the theoretical method depended on the default polynomials of the current system. It can be linear, quadratic and so on. Because of the constantly shifting inputs and inconsistent results, the data obtained from the mathematical modules based on the law automatically changed its value. Due to variables and a formal approach to analysis.

- **Tools used for research work**

For scholarly studies on a few portals, journals and webinars are used. The theorems and predefined postulates of all three hypotheses are used for the research work. The Google Scholar, the Science Gate, books and other web pages used newspapers written by other writers. Many web portals are available to carry out the research technology and translate and test the data obtained.

- **Research objective**

The objectives of the study are,

- To obtain interconnection between the algebraic number theory and algebraic geometry.
- To understand the relationship between commutative rings and number theory.
- To understand the interaction of commutative rings with algebraic geometry.
- To deploy the current findings of combined mathematical proof with real-time applications and bring significant change in the practical aspects of a theoretical basis.

4. Result Analysis

Objective 1. To obtain interconnection between the algebraic number theory and algebraic geometry.

Analysis. “Algebraic number theory”, geometry and depiction theory. A finite extension space of rational numbers is an “algebraic number field”. There is an arithmetic ring in an algebraic number space integer, playing a role as the standard integer. Mathematicians such as Kronecker, Kumar, Dedekind and Dirichlet began their studies on the concept of “algebraic numbers" in the 19th century. Gauss said that: the Algebraic Number Theory is the "Queen of Mathematics' '. One explanation for this thesis was an attempt to improve the Last Theorem of Fermat (proved a few years ago by Andrew Wiles). The researchers in the theory of algebraic numbers also investigate square forms (quadric function field, general division, Milnor K theory) and arithmetical forms (integrative properties, class number, zeta functions). Theory also examines quadratic forms. The techniques used in this research are techniques from different fields of mathematics.

Through this study, it concluded that an optional means of strengthening the connectivity and consistently increasing the profound comprehension of the topic is the relation between theory and algebra geometry.

Objective 2. To understand the relationship between commutative rings and number theory.

Analysis. ‘Number theory” is a mathematical area that is concerned mostly with the analysis of integer and numerical characteristics. The first mathematical growth of the human being is the positive integer. In the 17th century, Fermat was the first person to discover the deep features of integers. In the theory of numbers, Nathanson developed several arithmetical ideas, in particular the idea of congruous numbers, and inspired the creation of a class of nudity, flat, arithmetic graphs. A ring is one that fills the axiom “a × b = b × a for all a, b = r”. In fundamental ring theory, study analyse the principal effects of ring principles (here commutative, although many of the key aspects carry over to general associative rings).

Through analysing the relationship between the ring and the number principle.

Objective 3. To understand the relationship between commutative rings and algebraic geometry.

Analysis. Study of algebraic and algebraic geometry rings is the most general algebraic algebra. Allow R to become a ring. R is an integral domain; this recalls no non-trivial zero dividers exist. For multiple positive integers n, the unit x = R is nilpotent when x(n) = 0. Therefore, R is a null frication of a component not strong (if R is not an intermediate ring, that is, R = 0). Unit x is the y in R of which x y= 1, 1 is the multiplicative R identity. The device may be inverted. The y factor is x-unique, and x^{-1} marked. The portion,

$$U(R): = \{x \in R \mid \exists\ y \in R \text{ s. t. } x\, y = y\, x = 1\}$$

The R is a multiplicative group, and its components are referred to as R units (in respect of multiplication in R). A ring R shall be a camp if all the elements that are 1 non-Null are a unit, e.g., U(R) = R return = R \ {0}. One well-known example is an integer ring denoting Z, which is visually the integral domain but not the domain, in fact U(Z) = {1, −1} is morphically to a cyclic group of order two, i.e., $C_2 =< x \mid x^2 = 1 >$. The ring of the whole group is the most common example.

If the K ring has zero dividers, i.e., x y is equal to 0 in all nil x and y is equal to K, so the ring group with all the nil elements is a groupoid for multiplication. The ring is a skew if all the uncertain elements are composed of a multiplication group. When K ring complies with the associative rule (x × y), z = X (and z) is regarded as an associative ring (“for all x, y, z in K”). If the ring is switchable, x × y = y × x for all x and y in K is the name of a ring. Element 1 is known as an identity.

$$x.1 = 1.x\ = x$$

A ring in general does not need an identity for all k. Each field is a connected ring with no zero dividers, connected to an identity. An integral domain without null splitters and identities is known to be a commutative associative ring.

Research Objective 4. To deploy the current findings of combined mathematical proof with real-time applications and bring significant change in the practical aspects of a theoretical basis.

Analysis. A statistical proof suggests that the above assumptions guarantee a mathematical hypothesis. In philosophy, any evidence can be constructed by only applying such basic or initial rules such as axioms and the accepted laws on inferences. Other grounds, for instance theorems, can be used for the argument.

The theory of mathematical proof is not very divided into deductive thinking, which is a kind of reasoning which draws concrete conclusions from general knowledge. The opposite feature of deductive thought is an inductive reasoning which allows abstract inferences from concrete knowledge. Euclid's Elements, a thesis that remains the blueprint for a proposed construction of a mathematical industry, is the early example of the power of a mathematical proof (and deductive argument).

It starts with vague words and axioms, unclear proposals which are clearly true (from Greek "axios", something worthy). This can be shown by theorems which use deductive inferences. Someone in the west who was considered learned before the mid-20th century read Euclid's book The Elements. Besides geometric theorems such as the “Pythagorean”, the Portions also cover the theory of number and proof of an infinitely large quantity, namely that the square root is irrational.

Approach

1. Direct proof.

A logical mixture of axioms, definitions and previous theorems is the inference of clear proof. Direct proof can, for instance, be used to show that two even integer numbers sometimes are identical. Take two x and y integer even. Since they are identical, "x = 2a, and y = 2b" can be entered respectively for entries a and b. The complete x + y = 2a + 2b = 2 (a + b). Therefore x +y is a factor of 2 and of itself. Therefore, there are also two integer numbers.

This proof uses both the principle of integral properties as well as the multiplication and distribution of the closure used.

2. Proof by mathematical induction.

The mathematical inference shows one "base case" and an "induction rule" shows that any random issue means the next one. The theory of the application of the induction law is that all the cases (usually endless) are shown to be so (starting from the shown case). Each scenario does not need to be justified. A method of mathematical inference is the endless descent, which can be used to show the irrationality of two squares.

A common application of maths is to prove that a property with one number contains "all-natural numbers": "Let N= {1,2,3,4...}" be a collection of naturals numbers. Let N= {1,2,3, 4} be a set of "natural numbers".

a. P (1) is true, i.e., for n=1 it is true that P(n) is true.

b. Where P(n+1) is true, i.e., P(n) is true means the truth of P(n+1).

3. Probabilistic proof.

A situation is a probable case, in which the use of probability theory techniques obviously proves an example. Probabilistic proof is one way of explaining theorems of nature as proof by architecture. In probabilistic terms, an entity with a particular property, starting with several applicants, is found. One gives the candidate a certain likelihood and then reveals that the candidate has a non-zero chance of possessing the property that he requires. This does not presuppose which candidates are present, but the likelihood cannot be certain unless at least one is present.

4. Contrapositive proof.

Contrapositive evidence is essentially a 'sub-method' of evidence of contradiction. The reasoning in both cases starts with the presumption that the statement is the contrary. So, if it is seen,

statement A ⇒ statement B,

To properly understand this relationship, it wants to know more about Wason, a logic puzzle formulated by Peter Wason in 1966. The above is based on his thesis and the logic the counter positive uses are very well shown.

Only imagine four cards, first and secondly.

A D 9 5

"If five as on a spreadsheet," it has to say the card (s) to turn to check if the rule is in effect.

While the majority send "A" and "5" automatically, the right answer is "A" and "9." Notice that if the card displays "A" on one side, it should be "5" on the other side, according to our guideline. However, the law states nothing about the "5" card.

Therefore, it is only possible to verify the law by inspecting "A" and "9."

- The maxim is broken if A does not have 5 on the other side.

- If on the other hand D has (or does not) 5, it does not mean anything to us.
- The maxim is not violated whether 5 is on (or not) A on the other hand – again.
- The law is violated if 9 has A on the other line.

In this study, it also acquired various mathematical facts and improved them based on assumptions and real time execution.

5. Conclusion

The theory of numbers is one of the basic elements of impartial and interment arithmetic management. There are also ethics of good scientific questions regarding predictions only. The analysis of different formulas that have been found in whole areas really conflicts with our theory and must face heuristic assumptions. It used several theorems and resolved unanswered problems based on the principle of the number, algebraic geometry, commuting rings and so on. The philosophy and understanding of numerical theory in some fundamental ways. The report template is a step-by-step guide for research to emphasise the methodology of the project, including data collection and interpretation. It also obtained various statistical facts and made many hypothesis- and real-time developments. In selecting the research design and analytic methodology, the reasons, intent and circumstances of the project apply to analytical methods used for conducting the study.

Reference

[1]. The science of space, number, quantity, and arrangement, whose methods involve logical reasoning and usually the use of symbolic notation, and which includes geometry, arithmetic, algebra, and analysis. Oxford, November 16, 2019.

[2]. Gauss, Carl Friedrich". Encyclopedia.com. Retrieved 17 Feb 2020.

[3]. German original: "Die Mathematic its die Kaingin der Disenchanted, und die Arithmetic its die Kaingin der Mathematic.

[4]. Neugebauer (Neugebauer 1969, pp. 36–40) discusses the table in detail and mentions in passing Euclid's method in modern notation.

[5]. Van der Waerden, Bartel L.; Dresden, Arnold (trans) (1961). Science Awakening. Vol. 1 or Vol 2. New York: Oxford University Press.

[6]. The term tilikum is problematic. Robson prefers the rendering "The holding-square of the diagonal from which 1 is torn out, so that the short side comes up...".Robson 2001.

[7]. Life of Pythagoras, in Guthrie 1987Van der Warden (van der Warden 1961, pp. 87–90) sustains the view that Thales knew Babylonian mathematics.

[8]. Number Theory: An Approach Through History from Hammurabi to Legendre. Boston: Birkhäuser. ISBN 978- 0-8176-3141-3. Retrieved 2021-02-28.

[9]. Katz, Mikhail G.; Schaps, David; Shnider, Steve (2013), "Almost Equal: The Method of Adequality from Diophantus to Fermat and Beyond".

[10]. Mathematics in India. Princeton University Press. ISBN 978-0-691-12067-6.

[11]. Bachet, 1621, following a first attempt by Xylander, 1575.

[12]. Cole brooke 1817, p. lxv, cited in Hopkins 1990, p. 302. See also the preface in Sachau 1888 cited in Smith 1958, pp. 168.

[13]. This was more so in number theory than in other areas (remark in Mahoney 1994, p. 284).

[14]. Faulkner, Nicholas; Hosch, William L. (2017-12-15). Numbers and Measurements. Encyclopaedia Britannica.

[15]. Neugebauer, Otto E.; Sachs, Abraham Joseph; Götze, Albrecht (1945). Mathematical Cuneiform Texts. American Oriental Series. 29. American Oriental Society etc.

[16]. Christian Goldbach". Encyclopaedia Britannica. Encyclopaedia Britannica, Inc. 16 August 2013. Retrieved 26 October 2014.

[17].joseph Louis Lagrange (1736–1813)," A Short Account of the History of Mathematics, 4th ed. pp. 401–412. Complete article online, p.338 and 333.
[18].An elementary proof of Dirichlet's theorem about primes in an arithmetic progression", Annals of Mathematics, 50 (2): 297–304.
[19].Gowers, Timothy; June Barrow-Green; Mire Leader (2008). The Princeton companion to mathematics Princeton University Press. pp. 764–765.
[20].Prime Curios. University of Tennessee at Martin.
[21].Hardy, G. H.; Littlewood, J. E. (1916). Contributions to the Theory of the Riemann Zeta-Function and the Theory of the Distribution of Primes.
[22].Costa Pereira, N. (August–September 1985). "A Short Proof of Chebyshev's Theorem". American Mathematical Monthly.
[23].Hadamard, Jacques (1954). An essay on the psychology of invention in the mathematical field / by Jacques Hadamard.
[24].Charles-Joseph de la Vallie Poussin Obituary: Journal of the London Mathematical Society 39 (1964) pp.
[25].Alladi, Krishnaswami; Elliott, P. D. T. A.; Granville, A. (30 September 1998). Analytic and Elementary Number Theory: A Tribute to Mathematical Legend Paul Erdos. Springer Science & Business.
[26].Correspondence mathmatique et physique de queues celebres geometers St.-Pétersbourg 1843.

About Author:
Deepak Tiwari
Research Scholar
Himalayan University Itanagar, Arunachal Pradesh, India
tdeepak703@gmail.com