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Research Article

Conceptual Vs. Procedural Knowledge In Mathematics: A Look At Tertiary Students

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Abstract

Strong conceptual knowledge supports the knowledge of different mathematical procedures. The research looked into the level of conceptual and procedural values and the relations to some profile variables. The study utilized the descriptive-correlational designs where 394 sophomore tertiary students of the University of Eastern Philippines served as respondents. Data from an adapted instrument were treated using frequency counts, percentages, weighted means, multiple regression analysis, and Pearson Product Moment Correlation. The respondents are mostly female, have inadequate learning resources at home and in school, have very high conceptual and procedural values, very good procedural knowledge, and poor conceptual knowledge. Conceptual knowledge is significantly related to learning resources at home, conceptual values, and procedural values, while procedural knowledge is significantly related to conceptual values. The conceptual and procedural knowledge was significantly related. There is a need to design lessons focused on the acquisition of conceptual knowledge to strengthen procedural knowledge.

Keywords: *conceptual values, conceptual knowledge, procedural values, procedural knowledge, tertiary students, mathematics education*

Introduction

In a typical mathematics class, procedural knowledge of mathematics dominates the learning of mathematics. Students recall and apply different formulas, they prefer to do computational methods and perform algorithms which are the common practices in solving math problems. Less emphasis is placed on an in-depth understanding of the underlying concepts, on how and why the procedures work, and on the connections between ideas. This situation is observable not only in elementary and secondary education but as well as in tertiary education.

It is a fact that conceptual and procedural knowledge cannot be separated. As such, it is necessary to distinguish between these two types of knowledge to better understand knowledge development. The New York State Education Department distinguishes between conceptual and procedural knowledge in its learning standard for mathematics. Conceptual knowledge involves an understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. It consists of relationships constructed internally and connected to already existing ideas. On the other hand, procedural knowledge is the skill in performing procedures flexibly, accurately, efficiently, appropriately and it includes, but is not limited to algorithms (NYSED, 2005). Generally speaking, procedural knowledge is focused on the understanding of mathematical rules and procedures, whereas, conceptual knowledge is more on the understanding of mathematical relationships.

The Principles and Standards for School Mathematics of the National Council of Teachers of Mathematics state that balance ought to exist between conceptual and procedural learning in mathematics classrooms (PSSM, 2000). Piaget postulated that conceptual knowledge and procedural knowledge are not separated and both are integral parts of a single cognitive schema. (Baker and Czarnocha, 2002). To solve a wide range of mathematical problems, there should be a balance and connection between conceptual and procedural understanding.

Significant challenges of mathematics education are thus related to developing and linking different competencies as well as to solving problems by relating the underlying conceptual and procedural knowledge (Kadijevich, 2002). Several studies were conducted on conceptual and procedural knowledge. It examined the relationship between concepts and procedures to better understand children's tendencies in learning algorithms by rote without developing any understanding of what they are doing (Hiebert, 1986). In this study, conceptual understanding could influence procedural knowledge was hypothesized. In the same manner, procedural knowledge can also lead to an increase in conceptual knowledge.

The study was carried out to ascertain the level of conceptual and procedural values and knowledge of the sophomore tertiary students in the University of Eastern Philippines. It also tried to look into the significant relationship between the profile of the respondents in terms of sex, learning resources at home, learning resources in school, and conceptual and procedural values to their level of conceptual and procedural knowledge, and to find out if there is a significant relationship between the students' conceptual and procedural knowledge in mathematics.

Methodology

The study utilized the descriptive-correlational design. The respondents consisted of 394 sophomore students from the different colleges of the University of Eastern Philippines identified through Slovin's formula and random sampling. This study used an adapted instrument (Kajander, 2007) to gather the needed data. The questionnaire consisted of two parts. Part I is the profile of the respondents, and Part II consists of 10-item mathematical exercises that will measure the respondents' level of conceptual and procedural knowledge. Using frequency counts, percentages, and weighted mean, profiles such as sex, learning resources at home, learning resources in the school, conceptual and procedural values, and conceptual and procedural knowledge were organized.

The learning resources used the following category: 0 - 2 (Inadequate); 3 - 4 (Adequate); and 5 (Very Adequate). The respondents' level of conceptual and procedural values was classified using the weighted mean as: 4.20 - 5.00 (Very High); 3.40 – 4.19 (High); 2.60 – 3.39 (Average); 1.80 – 2.59 (Low); and 1.00 – 1.79 (Very Low). Meanwhile, the scores for the conceptual and procedural knowledge used the following category: 5 (Excellent); 4 (Very Good); 3 (Good); 2 (Fair); and 0 - 1 (Poor). To determine the correlation of the profile of the respondents to their level of conceptual and procedural knowledge, multiple regression analysis was employed while Pearson product-moment of correlation was utilized to find out the relationship between procedural knowledge and conceptual knowledge.

Results And Dıscussıon

Profile of the Respondents

Table 1 shows the frequency distribution on the profile of the respondents regarding sex, learning resources at home, learning resources in the school, and level of procedural and conceptual values. Out of 394 respondents, 227 or 57.6 percent were female, and 167 or 42.4 percent were male. Both the respondents' learning resources at home and in school are inadequate. In this study, the respondents had a very high level of conceptual and procedural values in Mathematics. This finding indicates that the respondents strongly believe in the importance of both procedural well as conceptual learning in mathematics. This finding affirms the study of Kajander (2007). Tables 2 and 3 show the item analysis of the conceptual and procedural values.

Table 1. Profile of the Respondents

Profile	f	%
Sex		
Male	167	42.4
Female	227	57.6
TOTAL	394	100.0
Learning Resources at Home		
Inadequate	306	77.7
Adequate	84	21.3
Very Adequate	4	1.0
TOTAL	394	100.0
Learning Resources in School		
Inadequate	322	81.7
Adequate	62	15.7
Very Adequate	10	2.5
TOTAL	394	100.0
Level of Conceptual Values		
Low	1	0.3
Average	7	1.8
High	107	27.2
Very High	279	70.8
TOTAL	394	100.0
Level of Procedural Values		
Average	3	0.8
High	113	28.7
Very High	278	70.6
TOTAL	394	100.0

Table 2. Procedural values of the respondents by item

Procedural Values	Mean	Interpretation

It is important to get the correct answer to math questions.	4.58	Very High
It is the teacher's job to teach the steps in each new math method to the students before they have to use it.	4.51	Very High
It is important to recall math facts such as addition facts or times tables quickly and accurately.	4.50	Very High
There is usually one best way to write the steps in a solution to a math question.	4.50	Very High
It is important to accurately do basic math calculations such as addition or multiplication, without a calculator.	4.49	Very High
Most people learn math best if they are taught the methods step by step.	4.43	Very High
Learning to follow "the steps" to generate correct answers is very important.	4.37	Very High
Accurate and efficient calculation skills are highly important in mathematics.	4.29	Very High
Calculators shouldn't be used too much in school because they can lessen opportunities to practice computational skills.	4.05	High
It is important to practice on many familiar shorter math questions in school.	3.83	High
GRAND MEAN	**4.36**	**Very High**

Table 3. Conceptual values of the respondents by item

Conceptual Values	Mean	Interpretation
It is important to really understand how and why math procedures work.	4.50	Very High
There are often several correct ways to get a right answer.	4.49	Very High
When I'm learning math, I really want to know "how" and "why" the methods and ideas work.	4.41	Very High
It is important to develop connections between related ideas and models in mathematics.	4.39	Very High
To think about different ways to solve a problem enriches student understanding.	4.35	Very High
It is important to think through and understand a variety of different approaches to problems.	4.33	Very High
Children learn deeply by investigating new types of problems different from one's they've seen before.	4.32	Very High
Most people learn math best if they explore problems in small groups to discuss and compare different approaches.	4.30	Very High
It is important to deeply understand how and why math procedures work if I am going to make effective use of them.	4.30	Very High

It is important to develop connections between ideas by working on multi-step problems.	4.27	Very High
GRAND MEAN	**4.37**	**Very High**

Level of Conceptual and Procedural Knowledge

Table 4 presents the level of conceptual and procedural knowledge of the respondents. The majority of the respondents had very good procedural knowledge where 108 or 27.4 percent had very good performance; 105 or 26.6 percent had good performance; 83 0r 21.1 percent had fair performance; 50 or 12.7 percent had excellent performance, and only 48 or 12.2 percent had poor performance. The result indicates that the majority of the respondents are knowledgeable on the basic mathematical operations particularly on the items that entail multiplication of decimals, subtraction of integers, the addition of fractions, and computing for the perimeter and area of a figure. With regards to their level of conceptual skills, 352 or 89.3 percent had poor performance; 28 or 7.1 percent had fair performance; 9 or 2.3 percent had a good performance, and only 5 or 1.3 percent had a very good performance.

This result shows that majority of the respondents did not perform well in the items that require conceptual knowledge. An interesting finding was the respondents were able to obtain the correct answers on items that require procedural understanding but failed to illustrate the procedure using diagrams or models or justify and support the calculation they were able to do and resorted only to restating rules by way of explanation. The result suggests limited conceptual understanding and confirms the study of Mewborn (2001) that many elementary teachers lack the conceptual understanding of mathematics. This is also in conformity with the findings of the study of Kajander (2007) where the preservice teachers scored higher on math problems that require procedural knowledge than with conceptual knowledge and Kajander & Holm‘s (2013) study about Pre-service teachers' mathematical understanding which revealed that procedural computational skills of the respondents were adequate, but their conceptual understanding was extremely low.

Table 4. Level of Conceptual and Procedural Knowledge

Level	f	%
Procedural Knowledge		
Poor	48	12.2
Fair	83	21.1
Good	105	26.6
Very Good	108	27.4
Excellent	50	12.7
TOTAL	394	100.0
Conceptual Knowledge		
Poor	352	89.3
Fair	28	7.1
Good	9	2.3
Very Good	5	1.3
TOTAL	394	100.0

Test of Relationship between Respondents' Profile and Level of Conceptual Knowledge

This study used multiple regression analysis to test the relationship between respondent's profiles and level of conceptual knowledge. Generally, the analysis result showed a significant R-value of 0.281 with a coefficient of determination equal to 0.079 (Table 5) which means that only 7.9 percent of the variance in conceptual knowledge percentage could be attributed to the independent variables. An F-value of 6.662 and a significance value of 0.000 (Table 6) suggested a significant relationship between the respondents' profile and level of conceptual knowledge because the p-value is lesser than the 0.05 alpha level. Thus, the effect of the independent variables of this study is significant.

The Beta coefficient in Table 7 indicated that the independent variables, learning resources at home (β = 0.180, sig. = 0.000), procedural values (β = -0.308, sig. = 0.014) and conceptual values (β = 0.233, sig. = 0.027) significantly predicted conceptual knowledge. This data entails that the conceptual knowledge is affected by the respondents' availability of learning resources at home and their procedural and conceptual values.

Table 5. Model Summary

R	**R Square**	**Adjusted R Square**	**Std. Error of the Estimate**
.281[a]	.079	.067	.79595

a. Predictors: (Constant), Conceptual Values, Sex, Learning Resources in School, Learning Resources at Home , Procedural Value

b. Dependent Variable: Conceptual Knowledge

Table 6. Analysis of Variance

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Regression	21.102	5	4.220	6.662	.000[a]
Residual	245.812	388	.634		
Total	266.914	393			

Table 7. Beta Coefficients

	Unstandardized Coefficients		
	B	**Std. Error**	Sig.
Sex	-.066	.082	.419
Learning Resources at Home	.180	.041	.000
Learning Resources in School	-.009	.039	.812
Procedural Values	-.308	.125	.014
Conceptual Values	.233	.105	.027

Test of Relationship between Respondents' Profile and Level of Procedural Knowledge

Multiple Regression Analysis was employed to test the relationship of the respondent's profile and level of procedural knowledge. Generally, the analysis result showed a significant R-value of 0.220 with a coefficient of determination equal to 0.049 (Table 8) which means that the independent variables attributed to the 4.9 percent of the variance in procedural knowledge percentage. An F-value of 3.965 and a significance value of 0.002 (Table 9) suggested a significant relationship between the respondents' profile and level of procedural knowledge because the p-value is lesser than the 0.05 alpha level. Hence, the effect of the independent variables is generally significant.

The Beta coefficient in Table 10 indicated that only the independent variable conceptual values (β = 0.412, sig. = 0.014) showed a significant relationship with the procedural knowledge while the rest of the independent variables showed a not significant relationship with the procedural knowledge. This finding entails that the respondent's procedural knowledge is affected by their conceptual values. The result further indicates that the respondents with very high beliefs on the importance of conceptual understanding in dealing with mathematical problems will likely have a higher procedural knowledge as far as this study is concerned.

Table 8. Model Summary

R	R Square	Adjusted R Square	Std. Error of the Estimate
.220[a]	.049	.036	1.26483

a. Predictors: (Constant), Conceptual Values, Sex, Learning Resources in School, Learning Resources at Home , Procedural Value

b. Dependent Variable: Procedural Knowledge

Table 9. Analysis of Variance

	Sum of Squares	df	Mean Square	F	Sig.
Regression	31.712	5	6.342	3.965	.002[a]
Residual	620.717	388	1.600		
Total	652.429	393			

Table 10. Beta Coefficients

	Unstandardized Coefficients		
	B	**Std. Error**	Sig.
Sex	.148	.130	.256
Learning Resources at Home	.088	.065	.174
Learning Resources in School	.087	.062	.159
Procedural Values	-.114	.199	.565
Conceptual Values	.412	.167	.014

Test of Relationship between Respondents' Level of Procedural and Conceptual Knowledge

The Pearson Product Moment Correlation was used in testing the significant relationship between the respondents' level of procedural and conceptual knowledge in mathematics. Table 11 shows the analysis of the computed r-value of 0.396 and significance value of 0.000 which is lesser than the 0.05 alpha level which means that there is a statistically significant correlation between the respondents' level of procedural and conceptual knowledge. It can be inferred that procedural knowledge increase leads to an increase in conceptual knowledge. Similarly, an increase in conceptual knowledge can lead to improved procedural knowledge.

This finding was in agreement with Hiebert & Lefevre (1986) and Rittle-Johnson & Siegler (1998) who revealed that there is a positive correlation between knowledge of concepts and knowledge of procedures. Both are learned in tandem rather than independently.

Table 11. Relationship between Level of Procedural and Conceptual Knowledge

Correlations

		Procedural Knowledge	Conceptual Knowledge
Procedural Knowledge	Pearson Correlation	1	.396**
	Sig. (2-tailed)		.000
	N	394	394
Conceptual Knowledge	Pearson Correlation	.396**	1
	Sig. (2-tailed)	.000	
	N	394	394

Conclusions

The respondents manifested very high conceptual and procedural values in mathematics which implies a substantial belief in the importance of both procedural and conceptual learning in mathematics. The procedural knowledge was very good. However, the respondents have poor conceptual skills. This result shows that majority of the respondents were able to get the correct answers on the given math problems but failed to explain the connections between the concepts. Getting the correct answers and fluency with procedures, however, are not sufficient for teaching and learning (Ball, et al. 2008). The poor conceptual knowledge can be associated with the fact that the respondents have inadequate learning resources at home and in school. Learning resources at home, procedural and conceptual values are significantly related to conceptual knowledge while only conceptual values are significantly related to procedural knowledge. A positive correlation exists significantly between conceptual knowledge and procedural knowledge. There is a need to create well-structured lessons and courses that focus on the acquisition of conceptual knowledge from primary education for problem-solving procedures. The Mathematics teachers are encouraged to craft procedural lessons to encourage the discovery of underlying concepts that can improve a stronger connection between procedural knowledge and conceptual knowledge.

Recommendations

Based on the findings and conclusions of this study, the following are hereby recommended: (1) There is a need to create well-structured lessons and courses focus on the acquisition of conceptual knowledge from basic education for problem-solving procedures. (2) Mathematics teachers are encouraged to craft procedural lessons to encourage the discovery of underlying concepts that can improve a stronger connection between procedural knowledge and conceptual knowledge. (3) Teachers are encouraged to use the instructional approach that will give importance in developing both conceptual and procedural knowledge. (4) Further researches are recommended considering other factors that will give an impact on the relations of conceptual and procedural knowledge. Future studies should be conducted that will validate tasks and measures of conceptual and procedural knowledge.

References

[1] Baker, W. & Czarnocha, B. (2002). Written meta-cognition and procedural knowledge. Proceedings of the 2nd International Conference on the Teaching of Mathematics (at the undergraduate level) University of Crete, July 2002, Hersonissos, Crete.

[2] Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

[3] Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Lawrence Erlbaum.

[4] Hiebert, J., & Lefevrre, P. (1986). Conceptual and procedural knowledge in mathematics: an introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

[5] Kadijevich, Dj. (2002). Towards a CAS promoting links between procedural and conceptual mathematical knowledge. *The International Journal of Computer Algebra in Mathematics Education*, 9, 1, 69-74.

[6] Kajander, A. (2007). Unpacking mathematics for teaching: A study of pre-service elementary teachers' evolving mathematical understandings and beliefs. *Journal of Teaching and Learning*, 5(1), 33-54.

[7] Kajander, A., & Holm, J. (2013). Pre-service teachers' mathematical understanding; Searching for differences based on curriculum background. *Fields Mathematics Education Journal*, 1, 3-20.

[8] Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Teacher Education and Development* 3, 28-36.

[9] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

[10] New York State Education Department. (2005). *Learning standards for mathematics.* Retrieved on 17 April 2017 from http://www.p12.nysed.gov/ciai/mst/math/documents/mathcore.pdf

[11] Rittle-Johnson, B., & Schneider, M. (2014). Developing Conceptual and Procedural Knowledge of Mathematics. Oxford Handbook of Numerical Cognition. Oxford University Press.

[12] Rittle-Johnson, B., & Siegler, R. S. (1998). The relation between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 75-110). East Sussex, UK: Psychology Press.