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Inventory Model with General Demand and Deterioration

Shelly[1] and **Dr. Rajender Kumar** (Supervisor)[2]

[1]Research Scholar, Department of Mathematics, Indira Gandhi University,
Meerpur (Rewari), 122502, (Haryana), India,
shelly.math.rs@igu.ac.in
[2]Assistant Professor, Department of Mathematics, Indira Gandhi University,
Meerpur (Rewari), 122502, (Haryana), India

ABSTRACT

In the present paper, an inventory model is generated for deteriorating items where Demand rate and Deterioration rate are taken as general and in the second part holding cost is also taken as general function of time.

Keywords :- Inventory, Demand, Deterioration, Cost, Shortage.

1 Introduction

Inventory management has become the most valuable, important, and undetachable part of business organizations, and deterioration of the stock goods has affected inventory management crucially. How much of the amount is to be ordered? and when to order the stock of goods? is the most important question to prevent the loss to the business organization. Thus, we need inventory models by also looking at the effect of deterioration. Many researchers have worked in this field, by creating models with different values of demand, deterioration, and holding cost. Power demand pattern for items that deteriorates with time is considered by Datta & Pal (1988) [3], Lee & Wu (2002) [7], Sharma, Sharma & Ramani (2012) [16] and Sharma & Preeti (2013) [15] using time varying deterioration in their respective models. Weibull distributed deterioration is considered by Wu (1999) [20], Wu (2002) [19], Lee & Wu (2002) [7], Skouri et. al. (2009) [18], Sharma et. al. (2012) [16] considered in their respective models. Time varying holding cost is used by Sharma et. al. (2012) [16], Karmakar et.al. (2014) [6], Ibe et. al. (2016) [5], Shah (2018) [14] in their respective models. Exponential distributed deterioration is used by Lee (2004) [8] in creating model and Wu (2002) [19] & Ghosh (2004) [4] created model with time varying quadratic demand. Wu (1999) [20] and Skouri (2009) [18] developed models with ramp type demand rate. Ouyang (2005) [12], Shah (2010) [13] and Aliyu (2020) [1] developed models with exponentially declining demand. Mukherjee (2010) [11] developed a model in which the time of duration of shortages varies directly with deterioration. Bhowmick (2011) [2] et. al., developed a model with continuous production model for deteriorating items with shortages. Maragatham (2017) [10] et. al., presented Model for Items in a single warehouse and assumed constant lead time. Sharma (2018) [17] developed a model for items that deteriorates with time, such as fruits, vegetables, and foodstuffs by considering demand as time-dependent. Long (2019) [9] demonstrated that structural deterioration affects the value of damage detection information. In the present paper, working is done based on the above papers specially on **Wu K.S.** (2002) [19] by considering general demand, general deterioration and general holding cost.

2 Assumptions and Notations

Notations

The Notations used here are given below :-

1. C_h = Cost per unit of inventory holding per unit time i.e., Holding cost

2. C_s = Shortage cost per unit per unit time.

3. C_d = Deterioration cost.

4. T = Each cycle length.

5. Q(t) = Inventory at any time t.

6. C(t) = Average total cost.

7. $D_R(t)$ = Demand Rate.

8. θ(t) = Deterioration Rate Function.

Assumptions

The Assumptions used here are given below :-

1. Demand Rate $D_R(t)$ is taken as general.

2. The deterioration rate function, θ(t) is taken as general.

3. There is constant Replenishment size and infinite replenishment rate.

4. There is zero Lead time.

5. Inventory system ends without Shortages.

6. Both the inventories, initial and final are taken as zero.

3 Analysis of Model

Let the level of Inventory at any time t be Q(t). Inventory level in the beginning is zero and shortages are permitted to fulfilled till t_0. Then refilling of stock is done at t_0. Stock of goods obtained at t_0 is first used to fulfil previous shortages and then for the demand and deteriorated stocks in [t_0, T]. Then at last inventory falls to zero at T. Differential equations which governs this inventory system during [0, T] are

$$\frac{dQ(t)}{dt} = -D_R(t); \quad [0, t_0] \tag{1}$$

&

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -D_R(t); \qquad [t_0,\ T] \tag{2}$$

Solutions of equations (1) and (2) are

$$Q(t) = -\int_0^t D_R(v)dv; \qquad [0, t_0] \tag{3}$$

&

$$Q(t) = e^{-k(t)} \int_t^T D_R(v)e^{k(v)}dv\,; \qquad [t_0, T] \tag{4}$$

Where $k(t) = \int \theta(t)dt$

and the boundary conditions are Q(0) = 0 and Q(T) = 0.

The entire amount of deteriorated units in [t_0, T] is

$$= Q(t_0) - \int_{t_0}^T D_R(t)dt$$

$$= e^{-k(t_0)} \int_{t_0}^T D_R(t)e^{k(t)}dt - \int_{t_0}^T D_R(t)dt \tag{5}$$

Inventory held over the period [t_0, T] is

$$= \int_{t_0}^T e^{-k(t)} \left[\int_t^T D_R(v)e^{k(v)}dv\right]dt \tag{6}$$

Shortage units Quantity,

$$= \int_0^{t_0}\int_0^t D_R(u)du\,dt$$

$$= \int_0^{t_0}(t_0 - u)D_R(u)du \tag{7}$$

Inventory Holding Cost = C_h * total inventory held

$$= C_h \int_{t_0}^T e^{-k(t)} \left[\int_t^T D_R(v)e^{k(v)}dv\right]dt \tag{8}$$

Deteriorating units cost = C_d * The entire amount of deteriorated units

$$= C_d[\,e^{-k(t_0)} \int_{t_0}^T D_R(t)e^{k(t)}dt - \int_{t_0}^T D_R(t)dt] \tag{9}$$

Shortage Cost = C_s * shortage units quantity

$$= C_s \int_0^{t_0}(t_0 - u)D_R(u)du \tag{10}$$

Average Total cost per unit time,

C(t) = $\frac{1}{T}$ $[Inventory\ holding\ Cost + Deteriorating\ units\ cost + Shortage\ cost]$

$$= \frac{C_h}{T} \int_{t_0}^T e^{-k(t)} \left[\int_t^T D_R(v)e^{k(v)}dv\right]dt + \frac{C_d}{T}\left[e^{-k(t_0)} \int_{t_0}^T D_R(t)e^{k(t)}dt - \int_{t_0}^T D_R\,(t)dt\right]$$

$$+ \frac{C_s}{T} \int_0^{t_0} (t_0 - u) D_R(u) du \tag{11}$$

For minimum cost the necessary and sufficient conditions are $\frac{dC(t_0)}{dt_0} = 0 \text{ and } \frac{d^2C(t_0)}{dt_0^2} > 0.$

Now

$$\frac{dC(t_0)}{dt_0} = \frac{1}{T} \left[C_s \int_0^{t_0} D_R(t)dt - \left(C_h + C_d\, \theta(t_0)\right) e^{-K(t_0)} \int_{t_0}^{T} D_R(t) e^{k(t)} dt \right] \tag{12}$$

and

$$\frac{d^2C(t_0)}{dt_0^2} = \frac{D_R(t_0)}{T} (C_h + C_s + C_d \theta(t_0))$$

$$+ \frac{1}{T} \left[\left(C_h \theta(t_0) + C_d \left(\theta^2(t_0) - \theta'(t_0) \right) \right) e^{-k(t_0)} \int_{t_0}^{T} D_R(t) e^{k(t)} dt \right] \tag{13}$$

Now $\frac{dC(t_0)}{dt_0} = 0 \; gives$

$$C_s \int_0^{t_0} D_R(t)dt - \left(C_h + C_d \theta(t_0)\right) e^{-k(t_0)} \int_{t_0}^{T} D_R(t) e^{k(t)} dt = 0 \tag{14}$$

Now Let

$$f(t_0) = C_s \int_0^{t_0} D_R(t)dt - \left(C_h + C_d \theta(t_0)\right) e^{-k(t_0)} \int_{t_0}^{T} D_R(t) e^{k(t)} dt$$

∃ a unique solution $t_0^* \in (0, T)$ satisfying equation (14), by the use of Intermediate Value Theorem, as here $f(0) < 0$ & $f(T) > 0$ and as $\frac{d^2C(t_0)}{dt_0^2} > 0$ for $0 \le \theta \le 1$, therefore, the optimized value of t_0 is t_0^*. Substituting $t_0 = t_0^*$, minimum value of $C(t_0)$ is

$$C(t_0^*) = \frac{C_h}{T} \int_{t_0^*}^{T} e^{-k(t)} \left[\int_t^T D_R(v) e^{k(v)} dv \right] dt + \frac{C_d}{T} \left[e^{-k(t_0^*)} \int_{t_0^*}^{T} D_R(t) e^{k(t)} dt - \int_{t_0^*}^{T} D_R(t) dt \right]$$

$$+ \frac{C_s}{T} \int_0^{t_0^*} (t_0^* - u) D_R(u) du$$

(15)

Now equation (15) gives the optimal value of cost. This equation can be further used for other values of Demand and Deterioration Rate.

Inventory Model with Holding Cost as time Dependent

Holding Cost is taken constant in above part. Now, taking time dependent holding cost i.e., h(t). Thus, equation for average total cost becomes

$$C(t_0) = \frac{1}{T}\int_{t_0}^{T} h(t)e^{-k(t)}\left[\int_{t}^{T} D_R(v)e^{k(v)}dv\right]dt + \frac{C_d}{T}\left[e^{-k(t_0)}\int_{t_0}^{T} D_R(t)e^{k(t)}dt - \int_{t_0}^{T} D_R\,(t)dt\right]$$

$$+\frac{C_s}{T}\int_{0}^{t_0}(t_0-u)D_R(u)du \qquad (16)$$

Now, for minimum cost C(t_0) the necessary condition and sufficient conditions are $\frac{dC(t_0)}{dt_0} = 0\ and\ \frac{d^2C(t_0)}{dt_0^2} > 0$.

Now $\frac{dC(t_0)}{dt_0} = 0\ gives$

$$C_s\int_{0}^{t_0} D_R(t)dt - \big(h(t_0) + C_d\theta(t_0)\big)e^{-k(t_0)}\int_{t_0}^{T} D_R(t)e^{k(t)}dt = 0 \qquad (17)$$

and $\frac{d^2C(t_0)}{dt_0^2} > 0\ gives$

$$\frac{d^2C(t_0)}{dt_0^2} = \frac{D_R(t_0)}{T}\big(h(t_0) + C_s + C_d\theta(t_0)\big)$$

$$+\frac{1}{T}\left[\Big(h'(t_0) + h(t_0)\theta(t_0) + C_d\big(\theta^2(t_0) - \theta'(t_0)\big)\Big)e^{-k(t_0)}\int_{t_0}^{T} D_R(t)e^{k(t)}dt\right] > 0$$

For $0 < \theta < 1$

Similarly, like done previously ∃ a unique solution t_0^* ∈ [0, T] satisfying equation (17), therefore the optimized value of t_0 is t_0^*. Minimum value of cost C (t_0^*) can be find out by putting t_0^* in place of t_0 in (16).

4 Conclusion

In this paper, an inventory model is generated for items that deteriorates with time by considering demand and deterioration as general function of time whereas in second part holding cost is also taken as general function of time. This paper is useful for different values of Demand, Deterioration and Holding Cost.

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