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Research Article

A Petri Net Representation of Processing Model

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Abstract

In this paper, Petri net based model of production task, which can be used to analyse the issue we have addressed in production sequencing which consists of loading and processing. The weakness of classic assignment model is that only one job can be assigned to each machine. A task plan modelled by a bounded marked petri net can be extracted from the mathematical sequential model. The Petri net representing the closed model is suitable for being real time discussion. In order to capture the structure and dynamics of such a system, Petri nets offer a natural and effective modelling methodology. Finally it is demonstrated the investigation of structural and behavioral process analysis.

Subject Classification:
Keywords: *Petri net, system model, concurrent system and reachability tree.*

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Introduction:

Modelling is the base for all activities like the design, implementation, and deployment of systems. A model is a tool to understand the system going to be developing and the models enable to communicate the desired structure and behaviour of our system and provide a basis for designing high-performance systems [1]. Petri nets constitute a modelling tool applicable to many systems. They are graphical in nature and are backed up by a sound mathematical theory. Petri net model used to describe and study discrete event dynamical systems. A discrete event dynamical system is a system in which the evolution of the activities in time depends on the occurrence of discrete events [4]. In order to capture the structure and dynamics of such a system, Petri nets offer a natural and effective modelling methodology [2]. In this paper, we will first briefly trace the basic definitions and properties of Petri net modelling. By using an illustrative example that of a simple manufacturing mathematical model with two machine and three jobs are discussed. Following this, how Petri net models can present the behaviour or dynamics of a modelled system. And illustrate Petri net modelling through a representative example that of how important system properties are present by the dynamics of a Petri net model of the system. Following this, we investigate the structural and behavioral system analysis [5]. Coverability (reachability) tree method, Matrix-equation approach, and linear algebraic method, Reduction techniques are demonstrated.

Petri Net

Petri net is used to represent systems activity and dynamics graphically. It is bi-directed graph, whose nodes represent places and transitions, and the directed arcs represent flow between place and transition. The place & transition are denoted by circles and rectangle respectively. The events and the cause of events, of a system are representing in the petri net by transition and places respectively. A valuation function or weight of an arc is a value assigned for the arc.

Definition:

A Petri net is a five tuple, $PN = (P, T, F, W, M_0)$, where
$P = \{p_1, p_2, \dots, p_m\}$, set of places and non-empty,
$T = \{t_1, t_2, \dots, t_n\}$, set of transitions and non-empty,
$F \subset (P \times T) U (T \times P)$, set of arcs,
$W: f \rightarrow Z^+, weight\ function,$
$M: P \rightarrow N,$ marking function,
$P$and T are disjoint.

Definition:

A Petri net PN with associated marking is called marked Petri net. A Petri net with initial marling M_0 is denoted by (PN, M_0). Petri net with m places is marked as $m \times 1$row vector; number of tokens in each place is represented by black dots. The change in the states of system is represented by changing the marking black dot, which happens by the successive transition firings.

Firing Rule: "Firing an enabled transition removes one token, from each input place of the transition, and adds one token to each output place of the transition."

Definition:

Each input place **P** of an enabled transition **t**hat has least one token. One token is removed from each input and output of place **P** of enabled transition **t,** when it is fired. The enabling of transition always depends on the event takes place.

Definition:

A transition having no input place is called source transition and enabled unconditionally. A transition having no output place is called sink transition and get token by firing but produce nothing

- If in a marking, no transition enabled then it is called **deadlock** marking.
- If in a marking, one or more transitions have been permanently disabled then it is called **starvation** marking.
- If all transitions of a Petri net are enable to fire then Petri net is said to be in **live**.

Definition:

A self-loop is a place **P** that connects a transition **P** to itself. A *Pure Petri net* contains self-loop. The properties holds good in all marking is called invariants properties.

Definition:

Let PN be a Petri net, the input function ***I*** is defined by
$I: (P \times T) \rightarrow N$,

$$I(P_i, t_j) = \begin{cases} 1, & if\ a\ directed\ arc\ from\ place\ P_i\ to\ transition\ t_j \\ 0, & if\ no\ arc\ from\ place\ P_i\ to\ transition\ t_j \end{cases}$$

The *output function* $O: (P \times T) \rightarrow N$ is defined by

$$O(P_i, t_j) = \begin{cases} 1, & if\ a\ directed\ arc\ from\ transition\ t_j\ to\ place\ P_i \\ 0, & if\ no\ arc\ from\ transition\ t_j\ to\ place\ P_i \end{cases}$$

For the transition t_i the *input place and output* places are defined as follows
$IP(t_i) = \{P_j \in P\ /\ I(P_j, t_i) \neq 0\}$, $OP(t_i) = \{P_j \in P\ /\ O(P_j, t_i) \neq 0\}$
The *input and output* transition of places P_j is defined by
$IT(P_j) = \{t_i \in T\ /\ O(P_j, t_i) \neq 0\}$, $OP(P_j) = \{t_i \in T\ /\ I(P_j, t_i) \neq 0\}$

Statement of the Problem

"A manufacturing system consists of two machines $M_1$and $M_2$and three jobs, each job goes through one stage of operation, which can be done on either machine M_1or machine M_2. A fresh job is performing only after the completion of previous job and unloaded from system". ***Figure* 1**shows the Petri net model of this system and the places and transitions of this model are given in the ***Table 1***.

Places:	**Transitions:**
P_1: Job 1 P_2 : Machine M_1 available P_3: Job 2 P_4 : Machine M_2 available P_5: Job 3 P_6 : M_1 processing the Job 1 P_7 : M_1 processing the Job 2 P_8: M_1 processing the Job 3 P_9:M_2 processing the Job 1 P_{10} : M_2 processing the Job 2 P_{11} : M_2 processing the Job 3	t_1 : M_1 starts processing the Job 1 t_2 : M_1 starts processing the Job 2 t_3 : M_1 starts processing the Job 3 t_4 : M_2 starts processing the Job 1 t_5 : M_2 starts processing the Job 2 t_6 : M_2 starts processing the Job 3 t_7: M_1finishes processing the Job 1 t_8: M_1finishes processing the Job 2 t_9: M_1finishes processing the Job 3 t_{10}: M_2finishes processing the Job 1 t_{11}: M_2finishes processing the Job 2 t_{12}: M_2finishes processing the Job 3

Table 1

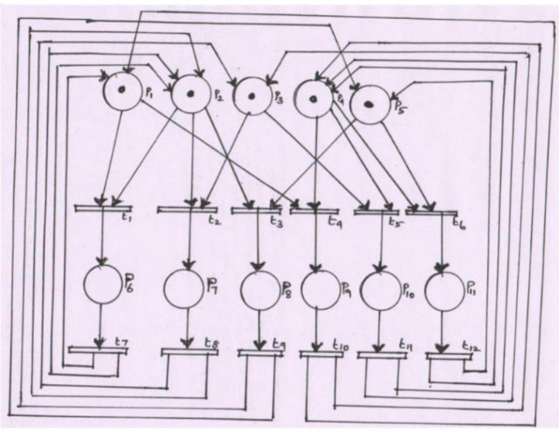


Figure1: Petri Nets with initial marking M_0

The Petri net of this system is, $PN = (P, T, F, M_0)$, where
"$P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}\}$,
$T = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}\}$,

$$F = \left\{ \begin{array}{c} (P_1,t_1),(P_1,t_4),(P_2,t_1),(P_2,t_2),(P_2,t_3),(P_3,t_2),(P_3,t_5),(P_4,t_4),(P_4,t_5),(P_4,t_6),\\ (P_5,t_3),(P_5,t_6),(P_6,t_7),(P_7,t_8),(P_8,t_9),(P_1,t_1),(P_9,t_{10}),(P_{10},t_{11}),(P_{11},t_{12}),(t_1,P_6),\\ (t_2,P_7),(t_3,P_8),(t_4,P_9),(t_5,P_{10}),(t_6,P_{11}),(t_7,P_1),(t_7,P_2),(t_8,P_2),(t_8,P_3),(t_9,P_2),\\ (t_9,P_5),(t_{10},P_1),(t_{10},P_4),(t_{11},P_3),(t_{11},P_4),(t_{12},P_4),(t_{12},P_5) \end{array} \right\}$$

and the initial marking $M_0 = (1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)$ which corresponds, the availability of machines and jobs waiting to be processed .”The input places of the all transitions of Petri net shown in Figure 1 are:

$IP(t_1) = \{P_1,P_2\}$, $IP(t_2) = \{P_2,P_3\}$, $IP(t_3) = \{P_2,P_5\}$, $IP(t_4) = \{P_1,P_4\}$, $IP(t_5) = \{P_3,P_4\}$, $IP(t_6) = \{P_4,P_5\}$, $IP(t_7) = \{P_6\}$, $IP(t_8) = \{P_7\}$,
$IP(t_9) = \{P_8\}$, $IP(t_{10}) = \{P_9\}$, $IP(t_{11}) = \{P_{10}\}$, $IP(t_{12}) = \{P_{11}\}$

Definition:

Let $T_1 \subset T$, T_1 is said to be ***conflicting*** if $\bigcap_{t\in T_1} IP(t) \neq \phi$, and T_1 is said to be ***concurrent*** if $IP(t_j) \bigcap IP(t_k) = \phi$ for all $t_j, t_k \in T_1$.

In a Petri net, the concurrency and nondeterministic activities are identified through *concurrent* and *conflicting* transitions.

In Figure 1 (Petri net),

conflicting **transitions**	***concurrent*** **transitions**
$(t_1,t_4),(t_1,t_2,t_3),(t_2,t_5),$ $(t_4,t_5,t_6),(t_3,t_6).$	$(t_1,t_5),(t_1,t_6),(t_2,t_4).(t_2,t_6)(t_3,t_4),$ $(t_3,t_5),(t_7,t_{11}),(t_7,t_{12}),(t_1,t_{11}).$

The initial marking $M_0 = [1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ ]$

Dynamic propertiesof Petrinet shown in Figure 1:

Graphical representation of input-output activities by places and transition is the structural modelling and the execution of the firing rule is the behavioural modelling [6].

Behavioral Modelling:

Dynamic behavior of proposed model is decribed by the following properties, the logical correctness of the proposed model can be verified by using these properties.

Definition:

If $M(P_i) \geq I(P_i, t_j)$, $for\ all\ P_i \in IP(t_j)$then transition t_j enabled in marking M. Transition $t_j$may be *fire* at any time. The next marking M'obtained by firingt_j, $M'(P_i) = M(P_i) + O(P_i, t_j) - I(P_i, t_j)$, $for\ all\ P_i \in P$.

The above marking reachability is denoted by $M \overset{t_j}{\rightarrow} M'$. And it is clear that in marked set the relation reachability is reflexive and transitive.

Definition:

The set of all markings reachable by the successive firing of the transitions from initial marking M_0 is called **reachability *set*** and denoted by $\boldsymbol{R}[\boldsymbol{M_0}]$.

Definition:

Let (PN, M_0) be marked Petri, whose ***reachability graph*** is defined by $\boldsymbol{RG} = (\boldsymbol{V}, \boldsymbol{E})$where the vertices set $V = R[M_0]$and the edge set $E$is given by: $(M_i, M_j) \in E$ such that

(i) $M_i, M_j \in R[M_0]$

(ii) Either$\exists\ t \in T$ such that $M_i \overset{t}{\rightarrow} M_j$ or there exists a set $T_i \subseteq T$, where $T_i = \left\{t_i \ /\ M_i \overset{t}{\rightarrow} M_j\right\}$, the *concurrent transitions.*

In$\boldsymbol{RG} = (\boldsymbol{V}, \boldsymbol{E})$, the nodes are labeled by the markings, and edges are labelled by the transition or concurrent transitions.

I. Reachability:

Structural and behavioural characteristics of a complex system (Decision making ,Synchronization , Priorities &Concurrent) can modeled by Petri nets[6]. The relation between the current marking and the initial marking can be cleared by reachability property.

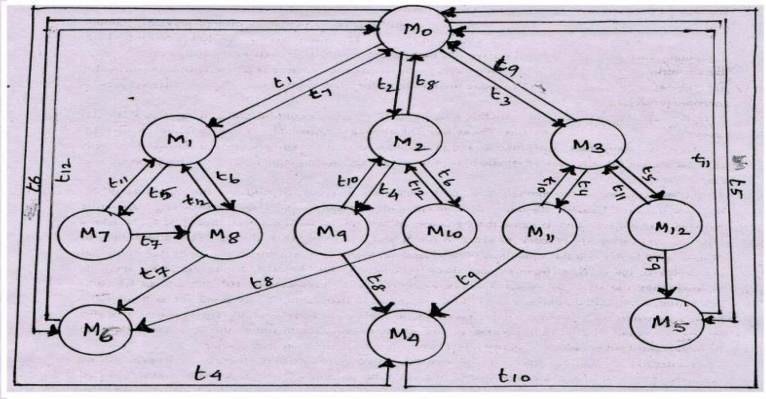


Figure 2:Reachability graph of Figure 1 (Petri net)

Marking	$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$	$\mathbf{P_4}$	$\mathbf{P_5}$	$\mathbf{P_6}$	$\mathbf{P_7}$	$\mathbf{P_8}$	$\mathbf{P_9}$	$\mathbf{P_{10}}$	$\mathbf{P_{11}}$
M_0	1	1	1	1	1	0	0	0	0	0	0
M_1	0	0	1	1	1	1	0	0	0	0	0
M_2	1	0	0	1	1	0	1	0	0	0	0
M_3	1	0	1	1	0	0	0	1	0	0	0
M_4	0	1	1	0	1	0	0	0	1	0	0
M_5	1	1	0	0	1	0	0	0	0	1	0
M_6	1	1	1	0	0	0	0	0	0	0	1
M_7	0	0	0	0	1	1	0	0	0	1	0
M_8	0	0	1	0	0	1	0	0	0	0	1
M_9	0	0	0	0	1	0	1	0	1	0	0
M_{10}	1	0	0	0	0	0	1	0	0	0	1
M_{11}	0	0	1	0	0	0	0	1	1	0	0
M_{12}	1	0	0	0	0	0	0	1	0	1	0

Table 2: Reachable markings of Figure 1(Petri net)

II. ***Boundedness:***

A Petri net (PN) is said to be bounded if “number of tokens in any place does not exceed a finite number ***m*** for any marking reachable from the initial marking M_0”.The Petri net shown in Figure1 is bounded since at each stage from P_1 to P_{11}, $m = 1$.

III. ***Liveness:***

If Petri net(PN) having no deadlocks then it is in Live.The Petri nets shown in Figure 1 have no dead lock state, so this net is live.

IV. *Reversibility:*

If the initial marking M_0can be reachable from all possible markings then the Petri net is called reversible Petri net. The Petri nets shown in Figure1 have satisfied this property.

V. *Persistence:*

A Petri net (PN) is said to be persistent if every pair of enabled transitions having independent firing and the enabled transitions will stay until it fires [3]. Petri net shown in Figure 1is not Persistent.

Analysis Methods

There are three methods of analysis, namely

1) Reachability or Coverability method,
2) Matrix-equation approach,
3) Reduction techniques.

By reachability tree, we can enumerate all reachable markings. And it will applicable to all classes of Petri nets. The matrix equations approach and reduction method are not suitable for all classes of Petri nets, these methods applicable to specific of Petri nets subclasses.

(i)Reachability tree method:

Algorithm for the construction of coverability tree for a Petri net (PN).

Step 1.The tag of the tree be the initial marking M_0
Step 2.While (Marking = "new")
Step 2.1.Select a new marking M.
Step2.2. If M is equal to a marking on the sequence marking from M_0, then tag M "old" and go to another new marking.
Step2.3. If no transitions are enabled at M, tag M "dead- end."
Step2.4. While there exist enabled transitions at M, do the following for each enabled transition that M:
Step 2.4.1. Obtain the marking M' that results from firing tat M.
Step 2.4.2. On the path from M_0 to M if there exists a marking M" such that $M'(p) \geq M''(p)$for each place**P** and $M' \neq M''$, that is M'' is coverable, then replace $M'(p)$ by ω for each place **P** such that $M'(p) > M''(p)$.
Step 2.4.3.Introduce M' as a node , draw an arc with label **t** from M to M', and tag M' "new"

A Petri net (PN) is bounded and thus $R(M_0)$ is finite and only if $\omega$is not exists in T and the coverability tree having all possible reachable markings. Petri net (PN) shown in *Figure 1* is bounded and whose coverability tree is *Figure 2.*

(ii)*Incidence matrix & State equation:*

The dynamic behaviour of systems modelled by Petri nets (PN) can be governed by a matrix equation [7]. The matrix equations method applicable only for pure Petri nets (PN).

Definition:

The incidence matrix of a Petri net with **n** transitions and **m** places is a $n \times m$ matrix defined by $A = [a_{ij}]_{n\times m} = O - I,$ where

$O = [a_{ij}{}^{+}] =$ "*number* of arc from transition i to its output place j",

$I = [a_{ij}{}^{-}] =$ "*number* of arc from transition i to its input place j" **.**

Definition:

The state equation $M_n = M_{n-1} + C_n A^T$, where

A- is the incidence matrix and

M_n - is a$m \times 1$ column vector whose i^{th} entry represent the number of tokens In place *i* immediately after the nth firing in some firing sequence.

C_k - is a $n \times 1$ column vector (control vector) having $n - 1$ 0's and 1 nonzero entry, 1 in the jth position indicating that transition j fires at the nth firing.

The control vector $C_n$cannot be taken arbitrarily since only enabled transitions may be fired. For each n, $C_n$is constrained by $M_{n-1} + C_n A^T \geq 0$.

If the destination marking M_d is reachable from M_0 through a firing sequence$\{C_1, C_2, \ldots, C_d\}$ then state equation is $\nabla M = A^T X$ where $\nabla M = M_d - M_0$, and$X = \sum_{i=1}^{k} C_i$. X is a$n \times \mathbf{1}$column vector of positive integers and is called firing count vector. The number of times that transition *i* must fire to transformM_0 *to* $M_d$is the i^{th} entry of X .The incident matrix A take an important role in characterizing both the dynamic behaviour and structure of a Petri net [PN].

Linear Algebraic Technique:

For the Petri net shown in Figure 1, the state equation is $M_k = M_{k-1} + C_k A^T$, and $A = O - I$, where

$$I = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad O = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The Incident matrix A is

$$A = O - I = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$\boldsymbol{t_1}$ $\boldsymbol{Fired}$:

$$M_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \times \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad M_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boldsymbol{t_2}$ $\boldsymbol{Fired}$:

$$M^1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \times \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad M_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boldsymbol{t_3}$ $\boldsymbol{Fired}$:

$$M^1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \times \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad M_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$\boldsymbol{t_4}$ $\boldsymbol{Fired}$:

$$M^1 = \begin{bmatrix}1\\1\\1\\1\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}^T + \begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}^T \times \begin{bmatrix}-1&-1&0&0&0&1&0&0&0&0&0\\0&-1&-1&0&0&0&1&0&0&0&0\\0&-1&0&0&-1&0&0&1&0&0&0\\-1&0&0&-1&0&0&0&0&1&0&0\\0&0&-1&-1&0&0&0&0&0&1&0\\0&0&0&-1&-1&0&0&0&0&0&1\\1&1&0&0&0&-1&0&0&0&0&0\\0&1&1&0&0&0&-1&0&0&0&0\\0&1&0&0&1&0&0&-1&0&0&0\\1&0&0&1&0&0&0&0&-1&0&0\\0&0&1&1&0&0&0&0&0&-1&0\\0&0&0&1&1&0&0&0&0&0&-1\end{bmatrix} M_4 = \begin{bmatrix}1\\1\\1\\1\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}^T + \begin{bmatrix}-1\\0\\0\\-1\\0\\0\\0\\0\\1\\0\\0\end{bmatrix}^T = \begin{bmatrix}0&1&1&0&1&0&0&0&1&0&0\end{bmatrix}$$

t_5 ***Fired***:

$$M^1 = \begin{bmatrix}1\\1\\1\\1\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}^T + \begin{bmatrix}0\\0\\0\\0\\1\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}^T \times \begin{bmatrix}-1&-1&0&0&0&1&0&0&0&0&0\\0&-1&-1&0&0&0&1&0&0&0&0\\0&-1&0&0&-1&0&0&1&0&0&0\\-1&0&0&-1&0&0&0&0&1&0&0\\0&0&-1&-1&0&0&0&0&0&1&0\\0&0&0&-1&-1&0&0&0&0&0&1\\1&1&0&0&0&-1&0&0&0&0&0\\0&1&1&0&0&0&-1&0&0&0&0\\0&1&0&0&1&0&0&-1&0&0&0\\1&0&0&1&0&0&0&0&-1&0&0\\0&0&1&1&0&0&0&0&0&-1&0\\0&0&0&1&1&0&0&0&0&0&-1\end{bmatrix} M_5 = \begin{bmatrix}1\\1\\1\\1\\1\\0\\0\\0\\0\\0\\0\end{bmatrix}^T + \begin{bmatrix}0\\0\\-1\\-1\\0\\0\\0\\0\\0\\1\\0\end{bmatrix}^T = \begin{bmatrix}1&1&0&0&1&0&0&0&0&1&0\end{bmatrix}$$

Similarly we can find all other firing and next markings of the Petrinet.

T-invariant (Transition invariant):

T- Invariant $YA = 0$, where Y is the number of firing transitions

$$\begin{bmatrix}y_1\\y_2\\y_3\\y_4\\y_5\\y_6\\y_7\\y_8\\y_9\\y_{10}\\y_{11}\\y_{12}\end{bmatrix}^T \begin{bmatrix}-1&-1&0&0&0&1&0&0&0&0&0\\0&-1&-1&0&0&0&1&0&0&0&0\\0&-1&0&0&-1&0&0&1&0&0&0\\-1&0&0&-1&0&0&0&0&1&0&0\\0&0&-1&-1&0&0&0&0&0&1&0\\0&0&0&-1&-1&0&0&0&0&0&1\\1&1&0&0&0&-1&0&0&0&0&0\\0&1&1&0&0&0&-1&0&0&0&0\\0&1&0&0&1&0&0&-1&0&0&0\\1&0&0&1&0&0&0&0&-1&0&0\\0&0&1&1&0&0&0&0&0&-1&0\\0&0&0&1&1&0&0&0&0&0&-1\end{bmatrix} = 0$$

$$-y_1 - y_4 + y_7 + y_{10} = 0; \qquad -y_1 - y_2 - y_3 + y_7 + y_8 + y_9 = 0;$$
$$-y_2 - y_5 + y_8 + y_{11} = 0; -y_4 - y_5 - y_6 + y_{10} + y_{11} + y_{12} = 0;$$
$$-y_3 - y_6 + y_9 + y_{12} = 0; \; y_1 - y_7 = 0;$$
$$y_2 - y_8 = 0; y_3 - y_9 = 0;$$
$$y_4 - y_{10} = 0; y_5 - y_{11} = 0;$$
$$y_6 - y_{12} = 0$$

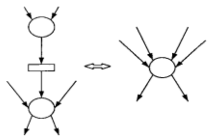
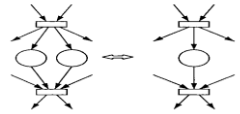
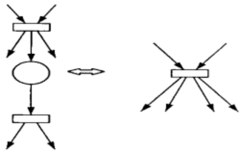
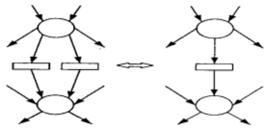
By solving above equations,

$$\begin{bmatrix}y_1\\y_2\\y_3\\y_4\\y_5\\y_6\\y_7\\y_8\\y_9\\y_{10}\\y_{11}\\y_{12}\end{bmatrix}^T = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}^T$$

Thus the T-invariant (of Figure 1) is: $(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$ Similarly ,we can find the place invariant P-invariant(of Figure 1) from the equation $AX^T = 0$, where X is the weights of each place and n vector as (0 0 0 0 0 0 0 0 0 0 0).

(iii)*Reduction Technique*

This technique will facilitate the analysis of a large system, by reducing existing system into simpler one, while preserving the system properties (like livens, safeness, and boundedness) to be analysed.

Transformation	**Pictorial representation**	**Transformation**	**Pictorial representation**
"Fusion of Series Places (FSP)"		*"Fusion of Parallel Places (FPP)"*	
"Fusion of Series Transitions (FST)"		*"Fusion of Parallel Transitions (FPT)"*	

By using the above transformation rules the Petri net [PN] shown in Figure 1 is reduced as follows.

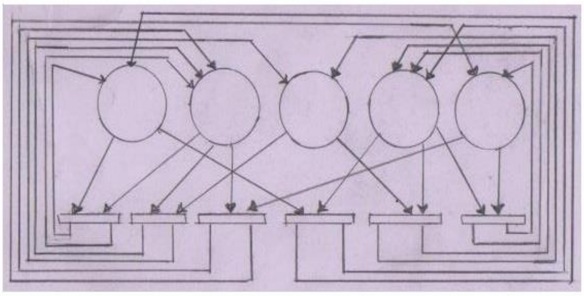


Figure 3: Reduced Petri net of Figure 1(Petri net)

Conclusion

In this paper, we have been able to model, analyse, the allocation of three jobs in two machines dynamically. This work was aided with the use of Petri nets model. Petri net is a(mathematical) graphical tool suitable to modelling many systems. PN is used to setup the state equation and mathematical model governing the behaviour of a defined system. Finally, the structural properties such as reachability, reduction and linear Algebraic technique and behavioural properties such as boundedness, reversibility, and persistence are explained for the Petri net of given system model.

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