Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 3, June 2021:269- 276

Research Article

Ranking of TraIFNs Based on Improved Accuracy Function and its Application

R. Sathya Bama[1], A. Farizan Begam[2] and A.Gowri Shankari[3]

Abstract

This research paper introduces an improved accuracy function (IAF) for the highest ranking order of Trapezoidal intuitionistic fuzzy numbers (TraIFNs) and to choose the better one in the multicriteria by using Trapezoidal intuitionistic fuzzy TOPSIS technique Problems in decision making. Finally, some illustrative examples are given in order to illustrate the utility of the proposed method.

Keywords: *Trapezoidal Intuitionistic Fuzzy numbers; Fuzzy set; Accuracy Function; Multi Criteria Decision Making and TOPSIS Method.*

[1,2]K. Ramakrishnan College of Engineering (Autonomous), Tiruchirappalli, Tamilnadu, India.
[3]Cauvery College for Women (Autonomous), Tiruchirappalli, Tamilnadu, India.
E-mail addresses:
sathyamatics@gmail.com, *farzakrce@gmail.com*, *gowrishankari.maths@cauverycollege.ac.in*
Received Accepted

Introduction

The definition of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov [1, 2], as the generalization of the Zadeh [17] proposed fuzzy set. Because, Characterizing IFSs by membership as well as non-membership Functions are considered more appropriate resources to manage than fuzzy sets. Most of the researchers chen [3, 4], Li [7, 8], Xu [12], geetha [10] and ye [13, 14] in multi criteria decision-making (MCDM), adopted the approach of IVIFSs.

In order to find the most suitable alternative in the course of a collection of acceptable alternatives, the technique for it is decision-making. A popular tool for multi-criteria decision-making was found by Hwang and Yoon [5]. The fuzzy TOPSIS method was presented based on arithmetic operations on fuzzy, Triantaphyllou and Lin [11], leading to a fuzzy relative closeness for each choice.Chen[3] extends the TOPSIS approach by using a compact Euclidean interval between any two fuzzy numbers to fuzzy group decision-making problems.[15, 16]Muthuperumal S & Venkatachalapathy M et. all. an algorithmic approach for using triangular fuzzy and trapezoidal fuzzy numbers of transportation problems. Prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets and their application to multi-criteria decision-making were introduced by Jun Ye[13].The majority of this paper incorporated as follows. In Section 2, some basic definitions are addressed as preliminaries. Section 3, An enhanced accuracy function of TrIFN has been proposed and correlates with current methods.. To test trapezoidal intuitionist fuzzy MCDM issues, a TOPSIS approach develops based on the improved accuracy value in section 4. Numerical examples are given in Section 5 to reflect the utility of the system proposed. There is a conclusion to the paper in Section 6.

Preliminaries

Definition 2.1. [1] Let D*[0; 1] be the set of all closed subintervals of the interval [0; 1]. Let $A(\neq \phi)$. An IVIFS is an expression in Z given by $A = \{z, M_A(z), N_A(z) : z \in Z\}$ where $M_A : Z \to D^*[0,1]$ $N_A : Z \to D^*[0,1]$ with the condition $0 < \sup_z M_A(z) + \sup_z N_A(z) \leq 1$. The degree of belongingness is denoted by $M_A(z)$ and the degree of non-belongingness is denoted by $N_A(z)$ of the element z to the set Z. We denote Z by $Z = \{z, [M_{AL}(z), M_{AU}(z)], [N_{AL}(z), N_{AU}(z)] : z \in Z\}$ where $0 < M_{AU}(z) + N_{AU}(z) \leq 1$, $M_{AL}(z) \geq 0$, $N_{AL}(z) \geq 0$.

Let TrIFS(Z) denotes the set of all TrIFSs in Z. We denote TrIFN by

$A=([\mu_p,\mu_q,\mu_r,\mu_s],[\vartheta_t,\vartheta_u,\vartheta_v,\vartheta_w])$ where $[\mu_p,\mu_q,\mu_r,\mu_s]$ denotes the degree of membership and $[\vartheta_t,\vartheta_u,\vartheta_v,\vartheta_w]$ denotes the degree of non membership.

Definition 2.2. [1] Let A^c be the complement of A and is defined by $A^C=\{z,\nu_A(z),\mu_A(z):z\in Z\}$.

Definition 2.3. [9] Let $A=([\mu_{p1},\mu_{q1},\mu_{r1},\mu_{s1}],[\vartheta_{t1},\vartheta_{u1},\vartheta_{v1},\vartheta_{w1}])$ and $B=([\mu_{p2},\mu_{q2},\mu_{r2},\mu_{s2}],[\vartheta_{t2},\vartheta_{u2},\vartheta_{v2},\vartheta_{w2}])$ be two TrIFNs. A subset relation is defined by A $\subset$B if and only if $\mu_{p1}\le\mu_{p2},\mu_{q1}\le\mu_{q2},\mu_{r1}\le\mu_{r2},\mu_{s1}\le\mu_{s2}$ and $\vartheta_{t1}\ge\vartheta_{t2},\vartheta_{u1}\ge\vartheta_{u2},\vartheta_{v1}\ge\vartheta_{v2},\vartheta_{w1}\ge\vartheta_{w2}$ respectively.

Definition 2.4. [9] Let $A=([\mu_p,\mu_q,\mu_r,\mu_s],[\vartheta_t,\vartheta_u,\vartheta_v,\vartheta_w])$ be a TrIFN. Then the α - cut of the membership and the β - cut of the non membership function are defined by $[\alpha(\mu_q-\mu_p)+\mu_p,\alpha(\mu_r-\mu_s)+\mu_s]$ and $[\beta(\vartheta_t-\vartheta_u)+\vartheta_u,\beta(\vartheta_w-\vartheta_v)+\vartheta_v]$.

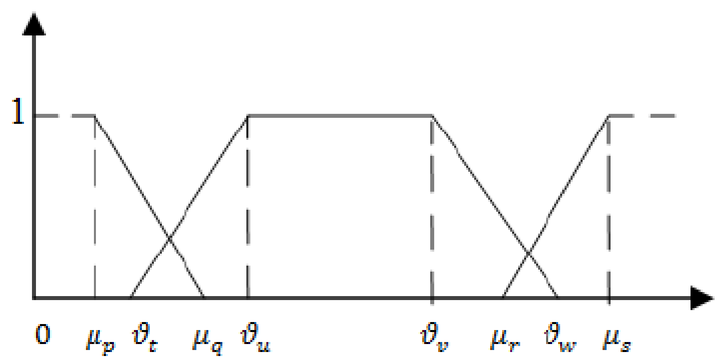

Figure 1: Graphical representation of membership and non membership functions of TrIFN

Definition 2.5. [12] Let $A=([\mu_p,\mu_q,\mu_r,\mu_s],[\vartheta_t,\vartheta_u,\vartheta_v,\vartheta_w])$ be a TrIFN. Then S_T, the score function is defined by $S_{Tr}(A)=\dfrac{\mu_p+\mu_q+\mu_r+\mu_s-[\vartheta_t+\vartheta_u+\vartheta_v+\vartheta_w]}{4}$.

Definition 2.6. [12] Let $A=([\mu_p,\mu_q,\mu_r,\mu_s],[\vartheta_t,\vartheta_u,\vartheta_v,\vartheta_w])$ be a TrIFN. Then H_T, the score function is defined by $H_{Tr}(A)=\dfrac{\mu_p+\mu_q+\mu_r+\mu_s+\vartheta_t+\vartheta_u+\vartheta_v+\vartheta_w}{4}$.

Definition 2.7. [14] Let $A=([\mu_p,\mu_q,\mu_r,\mu_s],[\vartheta_t,\vartheta_u,\vartheta_v,\vartheta_w])$ be a TrIFN. Then M_T, the score function is defined by $M_{Tr}(A)=\frac{\mu_p+\mu_q+\mu_r+\mu_s}{2}-1+\frac{\vartheta_t+\vartheta_u+\vartheta_v+\vartheta_w}{4}$.

Definition 2.8. [6] Let $A=([\mu_p,\mu_s],[\vartheta_t,\vartheta_w])$ be an IVIFN. Then T(A); an IAF is defined by $T(A)=\frac{\mu_p(1-\vartheta_t)+\mu_s(1-\vartheta_w)}{2}, T(A)\in[0,1].$

Ranking by IAF for TrIFN

Example 3.1. Let A_1 = ([.1, .2,.3,.5], [.2,.3,.3,.4]) and A_2 = ([.1,.3,.3,.4], [.2,.2,.3,.5]) be two TrIFNs for two alternatives.

Definition 2.5, $\Rightarrow$ $S_T(A_1) = -0.025$ and $S_T(A_2) = -0.025$

Definition 2.6, $\Rightarrow$ $H_T(A_1) = 0.575$ and $H_T(A_2) = 0.575$

Definition 2.7, $\Rightarrow$ $M_T(A_1) = -0.15$ and $M_T(A_2) = -0.15$

Thus, we can not tell which alternative is the right one. The current tasks have also struggled to illustrate their usefulness in selecting the right alternative. The suggested approach is very useful in trying to solve this sort of situation.

In certain cases the values in the details for membership and non-membership might be intervals instead of absolute numbers. Moreover, the interval can be extended to form TrIFN. So the AF T specified in IVIFNs [6] is applied to the TrIFNs in this section in order to solve the decision problem when the given information is TrIFN. Using the definition of T_{Tr} for $\alpha\in[0,1]$, we get

$$T_{Tr}=\frac{1}{2}\int_0^1[p+\alpha(q-p)-(p+\alpha(q-p))(u+\alpha(t-u)) \\ +(s+\alpha(r-s)-(s+\alpha(r-s))(v+\alpha(w-v))]d\alpha$$

$$=\frac{1}{2}\left[\begin{array}{l} p+(q-p)\left[\frac{\alpha^2}{2}\right]_0^1-\left[p+(q-p)\left[\frac{\alpha^2}{2}\right]_0^1\right]\left[u+(t-u)\left[\frac{\alpha^2}{2}\right]_0^1\right] \\ \qquad +\left[s+(r-s)\left[\frac{\alpha^2}{2}\right]_0^1\right]-\left[s+\alpha(r-s)\left[\frac{\alpha^2}{2}\right]_0^1\right]\left[v+(w-v)\left[\frac{\alpha^2}{2}\right]_0^1\right]\end{array}\right]$$

$$T_{Tr}(A) = \frac{p+q+r+s}{4} - \frac{qt+pu+rw+sv}{6} - \frac{pt+qu+sw+rv}{12}$$

Definition 3.2. Let A = ([p, q, r, s], [t, u, v,w]) be a TrIFN. The improved accuracy function T_{Tr} is defined $T_{Tr}(A) = \frac{p+q+r+s}{4} - \frac{qt+pu+rw+sv}{6} - \frac{pt+qu+sw+rv}{12}$ …………..(1)

If we apply **Definition** 3.2 to Example 3.1, we get $T_{Tr}(A_1) = 0.1875$ and $T_{Tr}(A_2) = 0.1858$: Clearly, we can say that alternative A_1 is the best one compared with A_2,It shows the significance of the proposed method.

Theorem 3.3. For any TrIFN A = ([p, q, r, s], [t,u, v,w]), the IAF $T_{Tr}(A)$ 2 [0; 1]:

Theorem 3.4. For a fuzzy subset A = t = ([t, t, t, t], [1-t, 1-t, 1-t, 1-t]), the IAF is $T_{Tr}(A) = t$. Moreover,

If A = ([1, 1, 1, 1], [0, 0, 0 0]), $T_{Tr}(A) = 1$

If A = ([0, 0, 0,0], [1, 1, 1, 1]), $T_{Tr}(A) = 0$.

Theorem 3.5. Let A = ([p, q, r, s], [t, u, v,w]) be a TrIFN. Then $T_{Tr}(A) + T_{Tr}(A^c) \leq 1$

TOPSIS Method Based on the IAF for TrIFN

We try to assess Fuzzy MCDM problems where the data generated by decision makers are represented as TrIF decision matrix by using TOPSIS technology, that each entry is a TrIFN, even the known standard weights is taken. By using the proposed accuracy function, we will de_ne the separation measures of each alternative from the optimal negative and positive solutions to evaluate the relationship. In a MCDM problem, there are set of alternatives $Z = \{Z_1, Z_2, \ldots Z_m\}$ Each one is valued on n criteria, which are represented by $Y = \{Y_1, Y_2, \ldots Y_n\}$ Each entry in the DM can be indicated by a TrIFN $K_{ij} = ([p_{ij}, q_{ij}, r_{ij}, s_{ij}], [t_{ij}, u_{ij}, v_{ij}, w_{ij}])(i = 1, 2, \ldots m; j = 1, 2, \ldots n)$. The TrIF DM $D_{mxn}(K_{ij})$ is defined as below

$$\begin{pmatrix} ([p_{11}, q_{11}, r_{11}, s_{11}], [t_{11}, u_{11}, v_{11}, w_{11}]) & \ldots & ([p_{1n}, q_{1n}, r_{1n}, s_{1n}], [t_{1n}, u_{1n}, v_{1n}, w_{1n}]) \\ \vdots & \vdots & \vdots \\ ([p_{m1}, q_{m1}, r_{m1}, s_{m1}], [t_{m1}, u_{m1}, v_{m1}, w_{m1}]) & \ldots & ([p_{mn}, q_{mn}, r_{mn}, s_{mn}], [t_{mn}, u_{mn}, v_{mn}, w_{mn}]) \end{pmatrix}.$$

………(2)

By using the IAF $T_{Tr,}$ we change the TrIF DM $D_{mxn}(Kij)$ the score matrix $V_{mxn}(T_{Trij}(Kij))$

$$\begin{pmatrix} (T_{Tr11}(K_{11})) & (T_{Tr12}(K_{12})) & ..... & (T_{Tr1n}(K_{1n})) \\ . & . & . & . \\ . & . & . & . \\ (T_{Trm1}(K_{m1})) & (T_{Trm2}(K_{m2})) & ... & (T_{Trmn}(K_{mn})) \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Fix the weights $W_j(j = 1; 2; : : : ; n) \in [0; 1]$ and $\sum_j W_j = 1$

$Z^- = \{(Y_j, [0, 0, 0, 0], [1, 1, 1, 1]) / Y_j \in C\}$ and $Z^+ = \{(Y_j, [1, 1, 1, 1], [0, 0, 0, 0])/Y_j \in Y\}$ respectively represent the negative and positive ideal solutions of the alternatives. Then the IAF based separation measures (S_i) are obtained by

$$S_i(Z^-, Z_i) = \sqrt{\sum_{j=1}^{m}\left[w_j T_{Trij}(K_{ij})\right]^2} \text{ and } S_i(Z^+, Z_i) = \sqrt{\sum_{j=1}^{m}\left[w_j(1 - T_{Trij}(K_{ij}))\right]^2}$$

$T_{Tr}(Z^-) = 0$ and $T_{Tr}(Z^+) = 1$:

The relative closeness of an alternative Z_i is then defined by the following general formula, based on the beneficial ideal solution Z^+

$$Y_i(Z_i) = \frac{S_i(Z^+, Z_i)}{S_i(Z^-, Z_i) + S_i(Z^+, Z_i)}$$; Where $Y_i(Z_i)(i = 1; 2; :::;m)$ and $0 \le Y_i(Z_i) \le 1$.

Then as per the values of $Y_i(Z_i)$, the alternatives are listed in a descending order. Finally, the alternative, which is put first in that order, is chosen as the better one among the alternatives.

Numerical Examples

Based on the following three requirements, a private organization needs to pick one applicant out of four for a position in their organization: (1). Y_1 is the Level of intelligence. (2). Y_2 is the Scholarly Papers. (3). Y_3 is Experience in the sector. The weight of the criterion is taken as W = (0:4; 0:3; 0:3).The Z_i alternatives (i = 1; 2; 3) are to be determined by the decision-maker to use the TrIFN on the basis of the above three. $D_{3x3}(K_{ij})$ =

Table 1: ATrIFNs

–	Y1	Y2	Y3
Z1	(0.12,0.16,0.2,0.26),(0.3,0.32,0.4,0.43)	(0.13,0.2,0.26,0.3),(0.31,0.42,0.45,0.5)	(0.22,0.24,0.31,0.35),(0.41,0.45,0.5,0.6)
Z2	(0.01,0.12,0.15,0.22),(0.25,0.32,0.35,0.43)	(0.11,0.16,0.22,0.26),(0.31,0.36,0.42,0.46)	(0.21,0.32,0.35,0.42),(0.45,0.52,0.55,0.6)
Z3	(0.16,0.22,0.28,0.3),(0.36,0.44,0.45,0.5)	(0.14,0.24,0.25,0.33),(0.35,0.45,0.5,0.6)	(0.22,0.34,0.35,0.4),(0.44,0.52,0.55,0.6)

Then we can achieve the most appropriate one(s) by using the suggested method. Then the accuracy matrix $R_{3x3}(T_{Trij}(K_{ij}))$ is attained by using (1).

$$R_{3x3}(T_{T\,rij}(K_{ij})) = \begin{pmatrix} 0.1157 & 0.1263 & 0.1400 \\ 0.0802 & 0.1123 & 0.1500 \\ 0.1334 & 0.1230 & 0.1539 \end{pmatrix} \quad \text{………(4)}$$

Obtain the attributes of positive ideal solutions and negative ideal solutions using (2) as follows.

S1(Z+;Z1) = 0:5185; S2(Z+;Z2) = 0:5121; S3(Z+;Z3) = 0:4978

S1(Z^-, Z1) = 0:0657; S2(Z^-, Z2) = 0:0710; S3(Z^-, Z3) = 0:0853

Then we can determine the values of Y i(Zi) as

Y 1(Z1) = 0:1125; Y 2(Z2) = 0:1218 and Y 3(Z3) =0:1463 by using (3).

The order of ranking for the three alternatives is $Z_1 < Z_2 < Z_3$. So, obviously, Z3 should be selected.

Conclusion and Scope

In this research paper, we have expanded the IAF for IVIFNs to the IAF for TrIFNs. To rank the alternatives by using the proposed accuracy function, a TrIF TOPSIS methodology is applied and the most appropriate one(s) will therefore be selected. Finally, numerical examples are presented in order to explain the usefulness of the proposed technique. In future, we could start working on the implementations of our developed method in completely different domains.

References

K. T. Atanassov, Intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87{96, 1986.

K. Atanassov and G. Gargov, Interval-valued intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 31, pp. 1{17, 1989.

C. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment," Fuzzy Sets and Systems, vol. 114, no.1, pp. 1{9, 2000.

T. Chen, Multi-criteria decision-makingmethod with leniency reduction based on interval-valued fuzzy sets," Journal of the Chinese Institute of Industrial Engineers, vol. 28, no. 1, pp. 1{19, 2011.

C. L. Hwang and K. Yoon, \Multiple Attribute Decision Making:Methods and Applications", Springer, Berlin, Germany, 1981.

D. Joshi and S. Kumar,\Improved Accuracy Function for Interval-Valued Intuitionistic Fuzzy Sets and Its Application to Multi{Attributes Group Decision Making," Cybernetics and Systems, vol. 49, no. 1, pp. 64{76, 2018.

D. Li, Multi attribute decision making models and methods using intuitionistic fuzzy sets," Journal of Computer and System Sciences, vol. 70, no. 1, pp. 73{85, 2005.

D. Li, Multi attribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets," Expert Systems with Applications, vol. 37, no. 12, pp. 8673{8678, 2012.

V. L. G. Nayagam, S. Jeevaraj and G. Sivaraman, Ranking of Incomplete Trapezoidal Information," Soft Computing, Volume 21 No.23, pp 7125{7140, 2016.

G. Sivaraman, P. Vishnukumar and M. Edwin Antony Raj,MCDM based on new membership and non-membership accuracy functions on trapezoidal-valued intuitionistic fuzzy numbers," Soft Computing, Volume 24, pp 4283{4293, 2020.

E. Triantaphyllou and C. T. Lin, Development and evaluation of ve fuzzy multi attribute decision-Making methods" , International Journal of Approximate Reasoning, vol. 14, no. 4, pp. 281{310,1996.

Z. Xu , J. Chen, Approach to group decision making based on interval valued intuitionistic judgement matrices, System Eng Theory Pract,vol. 27, no. , pp. 126{133, 2007.

Jun Ye, Prioritized aggregation operators of trapezoidal intuitionistic fuzzy sets and their application to multicriteria decision-making", Neural Comput & Application,vol. 25, no. 6, pp.1447-1454, 2014.

J. Ye, Multicriteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment," Expert Systems with Applications, vol. 36, no.3, pp. 6899{6902, 2009.

Muthuperumal S, Titus P & Venkatachalapathy M,(2020), An algorithmic approach to solve unbalanced triangular fuzzy transportation problems, Soft Computing, Springer-Verlag GmbH Germany, part of Springer Nature, https://doi.org/10.1007/s00500-020-05103-3.(0

Venkatachalapathy M, Pandiarajan R & Ganeshkumar S,(2020), A Special Type Of Solving Transportation Problems Using Generalized Quadratic Fuzzy Number, International Journal Of Scientific & Technology Research Volume 9, Issue 02,6344-6348.

L. A. Zadeh, Fuzzy sets," Information and Control, vol. 8, no. 3,pp. 338{353, 1965.