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Shear Stress and Heat Transfer Studies of Visco-Elastic Fluid Flowing Between Two Parallel Plates

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ABSTRACT:

In this paper we have carried out the "Shear Stress and Heat Transfer Studies of Visco-Elastic Fluid Flowing between Two Parallel Plates". In the proposed problem, upper plate moves with a constant velocity and lower one is at rest. Perturbation technique is used to find out the solution up to first order approximation. Effect of visco-elasticity and frequency of fluctuation on flow pattern is discussed. Velocity profiles and shear stresses at lower and upper plates are calculated. Variations of amplitude and phase angle with respect to frequency parameter are shown graphically. Heat transfer through lower and upper plates is discussed, also.

Keywords: Shear Stress; Perturbation Technique; Visco-elasticity.

Introduction:

The problem of heat transfer study of Newtonian and non-Newtonian liquids is of urgent interest owing to its many applications in various disciplines. Degan et al. [6] studied the problem of heat transfer of elastic-viscous liquid through parallel plates when both the plates are at uniform temperature. For a visco-elastic fluid in motion, a certain amount of energy is stored up in the material as strain energy and some energy is lost due to viscous dissipation. Singh [11] has studied the unsteady flow of visco-elastic fluid through parallel channel. Further, Acharya et al. [2], Beavers et al. [3], Bujurke et al. [4], Khan and Shaowei [9] and Nandeppanavar et al. [10] have investigated the magnetic field effect on the free convection, Rayleigh – Taylor instability, laminar flow in a uniform porous pipe, non-linear analysis and heat transfer in a Walters liquid-B fluid over an impermeable stretching with non-uniform heat and elastic deformation. Abd – el Naby et al. [1], Chen [5], Gokhale and Al Samman [7] and Israel – Cookey [8] have studied finite difference solution of radiation effect on free convection flow over a vertical plate, heat and mass transfer in MHD flow by natural convection, effect of mass transfer on the transient free convection flow and influence of viscous dissipation on unsteady MHD free convection flow past an infinite heated vertical plate.

Mathematical Analysis :

Velocity distribution –

Consider the motion of visco-elastic fluid characterized by the constitutive model

$$S^{ik} = -p\, g^{ik} + \tau^{ik} \tag{1}$$

where S^{ik} is the stress tensor, p the isotropic pressure, g^{ik} the metric tensor and τ^{ik} the deviatoric stress tensor given by

$$\tau^{ik} = 2\mu_0 \, e^{ik} - 2K_0 \frac{\delta e^{ik}}{\delta t} \qquad (2)$$

where $\mu_0 = \int_0^\infty N(\tau)\,dt$ is the limiting viscosity at small rate of shear, $K_0 = \int_0^\infty \tau\, N(\tau)\,d\tau$ is elastic parameter and $\frac{\delta}{\delta t}$ is convected derivative is given by

$$\frac{\delta e^{ik}}{\delta t} = \frac{\partial e^{ik}}{\partial t} + v^j \, e^{ik}_{\ ,j} - v^k_{\ ,j}\, e^{ij} - v^i_{\ ,j}\, e^{jk} + e^{ik}\, v^j_{\ ,j}$$

where v_i is the velocity vector and e^{ik} the rate of strain tensor given by

$$e^{ik} = \frac{1}{2}\left(v_{i,k} + v_{k,i}\right)$$

Pulsatile flow of non Newtonian liquid given by equation (2) is considered. Flow is caused due to the sudden motion of upper wall of the channel with velocity

$$U(t) = U_0\left(1 + \in e^{i\omega t}\right)$$

where U_0, $\in$ are constant and ω is frequency, t is time, flow is independent of walls direction.

The continuity and momentum equations for unsteady incompressible flow for liquid of density ρ are given by

$$\left(\rho\, v^i\right)_i = 0 \qquad (3)$$

$$\left(\rho \frac{\partial v^i}{\partial t} + v^j \, v^i_{\ ,j}\right) = -p_i + \tau^{ij}_{\ ,j} \qquad (4)$$

under Couette flow condition. Only x-component of velocity is considered and in a fully developed state at sufficiently large distance from the entrance region it can be taken to be a function of y and t only.

From equation (3), we have

$$u = u(y, t) \qquad (5)$$

Equation (2.4) gives

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau^{xy}}{\partial y}$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu_0}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{K_0}{\rho}\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}\right). \qquad (6)$$

For upper plate motion u = U(t), we have

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial U}{\partial t}.$$

So equation (6) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{K_0}{\rho} \frac{\partial^3 u}{\partial y^2\, \partial t} \,. \tag{7}$$

To solve the equation (7), we put

$$u = U_h \left[u_0(\eta) + \in e^{i\omega t}\, u_1(\eta) \right] \tag{8}$$

where $\eta = \dfrac{y}{h}$ is a non-dimensional variable and $\nu = \dfrac{\mu_0}{\rho}$ is the kinematic viscosity.

$$U_h\; i\; \omega\; e^{i\omega t}\; u_1 = \in^{i\omega}\; U_h\, e^{i\omega t} + \frac{\nu}{h^2} U_h \left(u_0'' + \in e^{i\omega t}\; u_1'' \right)$$
$$- \frac{K_0\, U_h}{\rho\, h^2} \left\{ \in^{i\omega}\; e^{i\omega t}\; u_1'' \right\}$$

Comparing the coefficient of $\in$ and $\in$ on both sides of above equation, we get

$$u_0'' = 0 \tag{9}$$

$$(1 - i k \sigma)\; u_1'' - i \sigma u_1 = - i \sigma \tag{10}$$

where $\sigma = \dfrac{\omega h^2}{\nu}$, $\quad K = \dfrac{K_0}{\rho h^2}$, are frequency and visco-elastic parameters.

The boundary conditions are

$$\left.\begin{array}{lll} u_0 = u_1 = 0 & \text{where} & \eta = 0 \\ u_0 = u_1 = 1 & \text{at} & \eta = 1 \end{array}\right\} \tag{11}$$

In view of conditions (11), the solutions of (9) and (10) are

$$u_0 = C_1 \eta + C_2 \tag{12}$$

put $\quad \eta = 0, \quad u_0 = 0 \quad$ and $\quad C_2 = 0$

$\quad \eta = 1, \quad u_0 = 1 \quad$ and $\quad C_1 = 1.$

From equation (12), we have

$$u_0 = \eta \tag{13}$$

$$(1 - i k \sigma) u_1'' - i \sigma u_1 = - i \sigma$$

$$u_1'' - m^2 u_1 = - m^2 \qquad \text{where} \qquad m^2 = \frac{i \sigma}{1 - i k \sigma}$$

$$u_1 = C_1\; e^{m\eta} + C_2\; e^{-m\eta} + 1$$

Put $\quad \eta = 0, \quad u_1 = 0$

$$C_1 + C_2 + 1 = 0$$

$\quad \eta = 1, \quad u_1 = 1$

$$C_1\ e^{m} + C_2\ e^{-m} = 0.$$

Solving these two equations, we get

$$u_1 = 1 - \cosh m\ \eta + \coth m\ \sinh m\ \eta \tag{14}$$

where $m = \sqrt{\dfrac{i\ \sigma}{1 - i\,k\,\sigma}}$

Shear Stresses at Lower and Upper Plates –

$$\frac{\partial u_1}{\partial \eta} = m\left[-\sinh m\ \eta + \coth m\ \cosh m\,\eta\right] \tag{15}$$

put $\eta = 0$

$$\left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} = m\ \coth m \tag{16}$$

put $\eta = 1$

$$\left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=1} = m\ \operatorname{cosech} m \tag{17}$$

Temperature Distribution –

We consider the energy equation in the form

$$\rho\,\frac{De}{Dt} = -\frac{\partial q_i}{\partial x_i} + \sigma_{ij}\,\frac{\partial v_i}{\partial x_j} \tag{18}$$

where $e = C_v\,T$ is the internal energy

and $q_i = -K\,\dfrac{\partial T}{\partial x_i}$, is the heat flux for thermally isotropic fluid.

We treat the thermal conductivity K as constant for incompressible medium. Specific heat at constant volume and constant pressure to be equal $C_v = C_p = C$, only.

Using equation (2), the equation (18) reduces to

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C}\,\frac{\partial^2 T}{\partial y^2} + \frac{1}{C}\left[\nu\left(\frac{\partial u}{\partial y}\right)^2 - \frac{K_0}{\rho}\,\frac{\partial^2 u}{\partial t\ \partial y}\left(\frac{\partial u}{\partial y}\right)\right]$$

or

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C}\,\frac{\partial^2 T}{\partial y^2} + \frac{1}{C}\left[\nu\left(\frac{\partial u}{\partial y}\right)^2 - K_0^{*}\,\frac{\partial^2 u}{\partial t\ \partial y}\,\frac{\partial u}{\partial y}\right], \tag{19}$$

where $K_0^{*} = \dfrac{K_0}{\rho}$ and $\nu = \dfrac{\mu}{\rho}$.

We look for the solution in the form

$$T = T_0(\eta) + \in e^{i\omega t} T_1(\eta) \qquad (20)$$

and equating steady and unsteady part, we get

$$T_0'' = -\frac{\mu}{K} U_h^2 u_0'^2 \qquad (21)$$

$$T_1'' - i\sigma P_r T_1'' = U_h^2 P_r (iK\sigma - 2) u_0' u_1' \qquad (22)$$

where $P_r = \frac{\mu_0 C}{K}$ is Prandtl number.

Solution of equation (21) with boundary conditions

$$\eta = 0 \quad T_0 = T_w , \qquad \eta = 1 \quad T_0 = T_h$$

$$T_0 = -\frac{\mu}{2K} U_h^2 \eta^2 + C_1 \eta + C_2$$

Put $\eta = 0$, $T_0 = T_w$,

$$C_2 = T_w$$

Put $\eta = 1$, $T_0 = T_h$,

$$C_1 = \frac{\mu}{2K} U_h^2 - (T_w - T_h).$$

Then $$T_0 = -\frac{\mu}{2K} U_h^2 (\eta^2 - \eta) - (T_w - T_h)\eta + T_w$$

$$T_0 - T_w = -\frac{\mu}{2K} U_h^2 (\eta^2 - \eta) - \eta (T_w - T_h)$$

$$\frac{T_0 - T_w}{T_w - T_h} = -\frac{\mu}{2K} \frac{U_h^2}{(T_w - T_h)} (\eta^2 - \eta) - \eta$$

$$= -\frac{1}{2} P_r E (\eta^2 - \eta) - \eta$$

$$\frac{T_0 - T_w}{T_w - T_h} = -\frac{1}{2} P_r E (\eta^2 - \eta) - \eta \qquad (23)$$

where $E = \frac{\mu}{2K} \frac{U_h^2}{T_w - T_h}$ is Eckert number.

Solution of equation (22) with boundary condition

$$\eta = 0, \quad T_1 = T_w, \quad \eta = 1, \quad T_1 = T_h.$$

$$T_1'' - i\sigma\rho T_1 = U_h^2 \rho (ik\sigma - 2) u_0' u_1'$$

$$T_1 = C_1 e^{\ell\eta} + C_2 e^{-\ell\eta} + \frac{U_h^2 \; P_r \left(i k \sigma - 2\right) m}{\left(m^2 - \ell^2\right)}$$

$$\left[-\sinh m\eta + \coth m \; \cosh m\eta\right]$$

Put $\eta = 0$, $T_1 = T_w$, $\eta = 1$, $T_1 = T_h$

$$C_1 = \frac{T_h - T_w \; e^{-\ell}}{2 \; \sinh \; \ell} - \frac{U_h^2 \, P_r \left(i k \sigma - 2\right) m}{\left(m^2 - \ell^2\right) \sinh \ell \; \sinh m} \left(e^{-\ell} \; \cosh m - 1\right)$$

$$C_2 = \frac{\left(T_w \; e^{\ell} - T_h\right)}{2 \sinh \ell} + \frac{U_h^2 \, P_r \left(i k \sigma - 2\right) m}{\left(m^2 - \ell^2\right) \sinh \ell \; \sinh m} \left(e^{\ell} \cosh m - 1\right).$$

Then

$$\frac{T_1}{T_w - T_h} = -\frac{\sinh \ell\eta}{\sinh \ell} - \frac{m \; P_r \; E \left(i k \sigma - 2\right)}{\left(m^2 - \ell^2\right)}$$

$$\left\{\frac{\sinh \ell\eta - \cosh m \; \sinh (\eta - 1)\ell}{\sinh \ell} + \coth m \; \cosh m\eta - \sinh m\eta\right\} \qquad (24)$$

where primes denote differentiation w. r. t. η

and $\ell^2 = i \sigma \rho$, $\quad P_r = \dfrac{\mu_0 \, C}{K}$ Prandtl number

$$E = \frac{U_h^2}{C\left(T_w - T_h\right)}$$ Eckert number.

Heat transfer rates for lower and upper walls are given by

$$\frac{T_1'(0)}{T_w - T_h} = -\frac{1}{\sinh \ell} + \frac{m \ell P_r \; E \left(2 - i k \sigma\right)}{\left(m^2 - \ell^2\right)} \left\{\frac{1 - \cosh m \cosh \ell}{\sinh \ell}\right\} \qquad (25)$$

$$\frac{T_1'(1)}{T_w - T_h} = -\ell \; \cot \ell + \frac{m \ell P_r \; E \left(2 - i k \sigma\right)}{\left(m^2 - \ell^2\right)} \left\{\cot \ell - \frac{\cosh m}{\sinh \ell}\right\} \qquad (26)$$

Table – 1
Variation of Steady and Unsteady Velocity for K = 1, σ = 3

η	u_0	u_1
0.0	0.0	0.0
0.1	0.1	0.1253
0.2	0.2	0.2312
0.3	0.3	0.3729
0.4	0.4	0.4315

0.5	0.5	0.5541
0.6	0.6	0.6139
0.7	0.7	0.7833
0.8	0.8	0.8346
0.9	0.9	0.9015
1.0	1.0	1.0000

Table – 2

Amplitude and Phase Lag at Lower Wall

σ	$\left\|\frac{\partial u_1}{\partial \eta}\right\|_{\eta = 0}$	ϕ_1
1	1.3129	0.3927
2	1.3174	14.3522
3	1.3198	10.2377
4	2.0694	56.5699
5	2.3891	40.3192

Table – 3

Amplitude and Phase Lag of Shear Stress at Upper Plate for Fixed Value of K = 1

σ	$\left\|\frac{\partial u_1}{\partial \eta}\right\|_{\eta = 1}$	ϕ_2
1	0.8509	1.3890
2	1.8878	14.7699
3	2.3493	10.2340
4	2.9433	45.7934
5	3.0159	34.3214

Table – 4

Heat Transfer Variation at Lower Walls with respect to Frequency Parameter σ and Fixed Values of K = 0, ρ = 10 and E = 2

σ	$\frac{T_1'(0)}{T_w - T_h}$	$\|\beta\|$	tan α
5	– 5.0826 – 14.0132	6.57001	8.1010
10	– 0.0478 – 14.0484	4.0486	84.6060

Table – 5
Heat Transfer Variation at Upper Walls with respect to Frequency Parameter σ and Fixed Values of K = 0, ρ = 10 and E = 2

σ	$\dfrac{T_1'(1)}{T_w - T_h}$	$\lvert \beta' \rvert$	tan α′
5	1.1398 – i 10.8598	1.4277	– 0075
10	4.9845 – 15.8225	7.6722	– 0.0863

Table – 6
Heat Transfer Variation at Lower Walls with respect to Frequency Parameter σ and Fixed Values of K = 0.1, ρ = 10 and E = 2

σ	$\dfrac{T_1'(0)}{T_w - T_h}$	$\lvert \beta \rvert$	tan α
5	– 0.0810 – i 0.0678	0.0105	0.0835
10	– 0.0029 – i 0.0132	0.0013	4.5514

Table – 7
Heat Transfer Variation at Upper Plate Wall with respect to Frequency Parameter σ and Fixed Values of K = 0.1, ρ = 10 and E = 2

σ	$\dfrac{T_1'(1)}{T_w - T_h}$	$\lvert \beta' \rvert$	tan α′
5	– 0.0001 + i 0.0096	0.0009	– 1.5532
10	– 4.9829 + i 6.2145	7.9655	– 0.0283

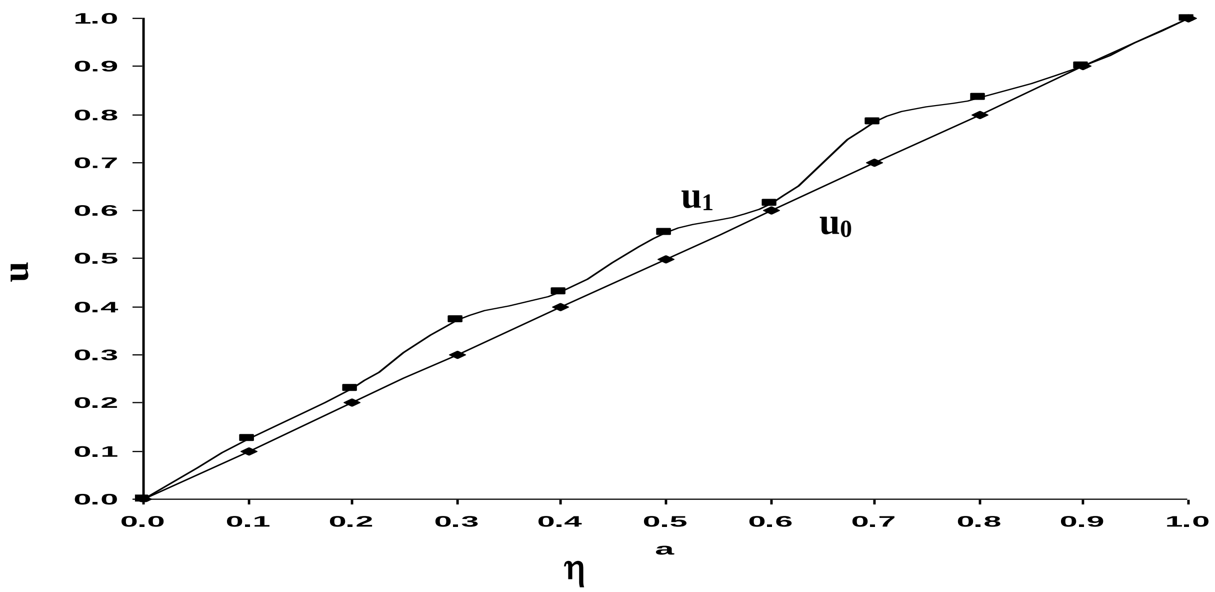

Figure – 1 : Variations of Steady and Unsteady Velocity

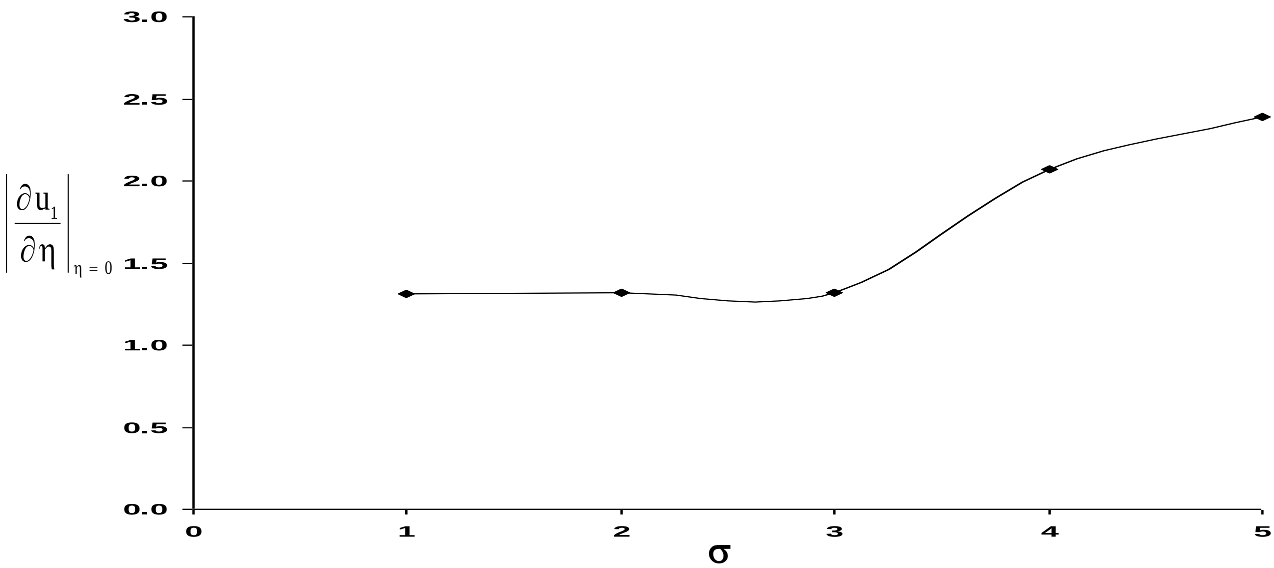

Figure – 2 : Variation of Amplitude at Lower Wall with Frequency Parameter

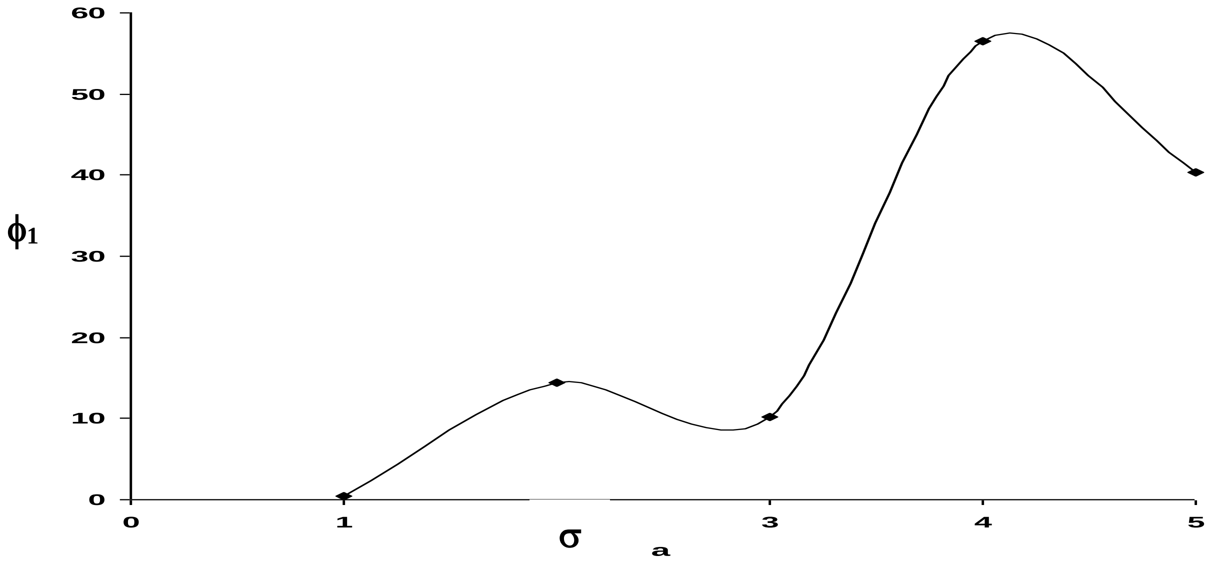

Figure – 3 : Variation of Phase Lag at Lower Wall with Frequency Parameter.

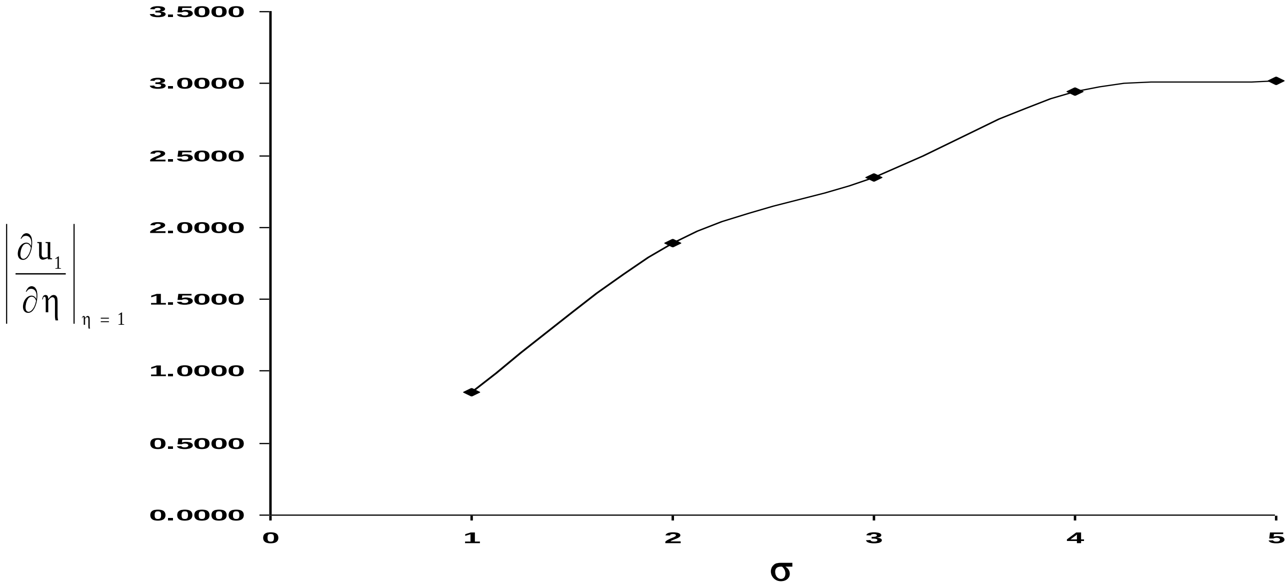

Figure – 4 : Variation of Amplitude at Upper Wall with σ

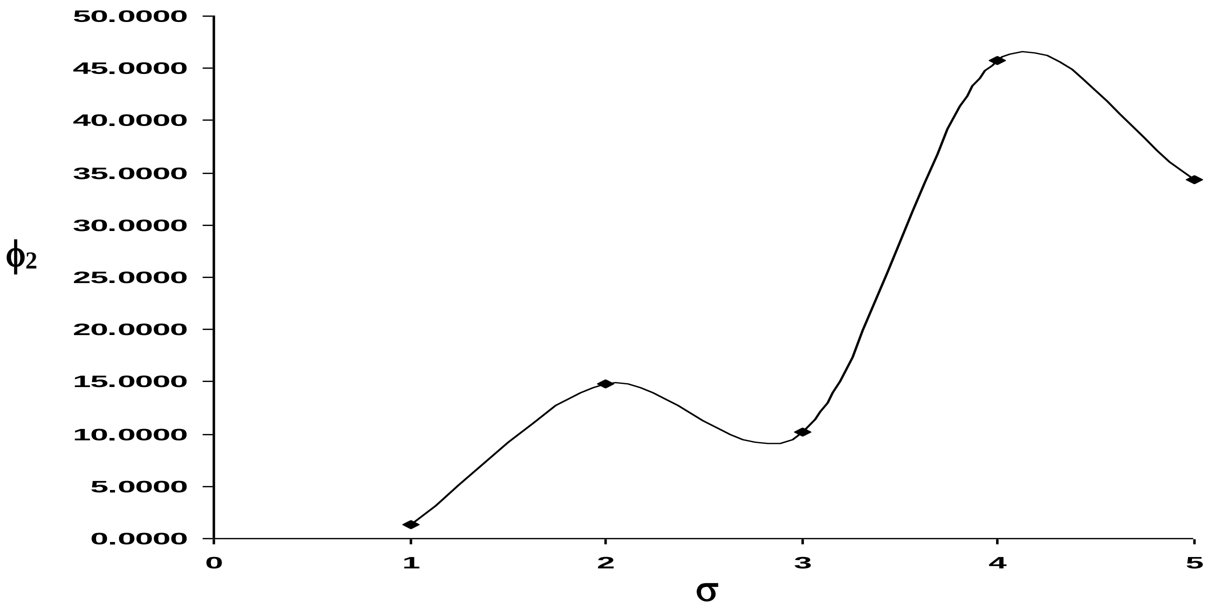

Figure – 5 : Variation of Phase Lag at Upper Wall with σ

Results and Discussion :

We have discussed the effect of visco-elastic parameter and frequency parameter on velocity field, shear stress and heat transfer variation at lower and upper plates. Results are illustrated with the help of tables – I. We see that steady velocity increases when η increases. And for fixed value of the K and σ, the unsteady velocity increases rapidly. From tables – 2 and 3, it is observed that at lower plate amplitude of the shear stress increases with frequency parameter, at upper plate the same effect is observed also.

From equations (25), (26) we have discussed the effect of oscillation on elastico-viscous energy dissipation. From table – 4, 5, 6 and 7, for the liquid of small elasticity (K = 0.1) heat transfer amplitude at lower wall decreases and phase increases with frequency parameter but at upper wall the same is not true. At lower wall the amplitude of non-Newtonian visco-elastic fluid is always less than that of Newtonian fluid (K = 0), where as at upper wall it is larger at high frequency and smaller at lower frequency.

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