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Generalised Recurrent Spaces

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Abstract:

The generalised recurrent spaces have been discussed in this paper. As holomorphic sectional curvature of a Kaehlerian space plays an important role in its geometry, the notion of generalised recurrent spaces has not been considered identically the same as in Riemannian geometry but has been modified in a sense to involve the above mention holomorphic sectional curvature. In different articles of this paper we have studied Ricci-recurrent, generalised Ricci-recurrent and generalised Ricci-2-recurrent Kaehlerian spaces.

Keywords: Kaehler space, Generalised Ricci-recurrent and generalised Ricci-2-recurrent Kaehlerian space.

1. Introduction:

The idea of recurrent Kaehler space has been introduced by Mathai[7] in an analogy with the concept of recurrent Riemannian space([10],[16]) in 1969. With the help of well-known holomorphically projective curvature tensor[15], Bochner curvature tensor[14], and H-conharmonic curvature tensor[12] the idea of recurrency has been defined and studied by many investigators like Singh and Lal[13], Prasad[9] etc.

Recently in 1991, U. C. De and others introduced the idea of a Generalised recurrent Riemannian space[1]. In this paper we have extended this concept to a Kahlerian space. The nature and existence of K-torse-forming vector fields[20] in such spaces have also been discussed.

Let $K_n(n=2m>2)$ be an n-dimensional Kaehlerian space with $F_i^h=0$ and g_{ji} as the components of the structure tensor and Hermitian metric tensor respectively, then the Riemannian curvature tensor, Ricci tensor and its transformation by the structure tensor satisfies

(1.1) $F_i^i=0$

(1.2) $\nabla_l R_{kji}{}^l = \nabla_k R_{ji} - \nabla_j R_{ki}$

(1.3) $\nabla_l R_k{}^l = \frac{1}{2}\nabla_k R$

(1.4) $R_{kji}{}^h F_h{}^i = 0$

Further holomorphically projective curvature tensor or briefly HP-curvature tensor $P_{kji}{}^h$ which is an invariant under any holomorphically correspondence([6],[13]) and is given by $P_{kji}{}^h = R_{kji}{}^h + \frac{1}{n+2}[\delta_j^h R_{ki} - \delta_k^h R_{ji} + F_j^h S_{ki} - F_k^h S_{ji} + 2F_i^h S_{kj}]$

and satisfies the following identities

(1.5) (a) $P_{kji}{}^{r}F_{r}{}^{h} = P_{kjr}{}^{h}F_{i}{}^{r}$

(b) $P_{rji}{}^{h}F_{k}{}^{r} = P_{rki}{}^{h}F_{j}{}^{r}$

(1.6) (a) $P_{kjir}F_{h}{}^{r} = -P_{kjrh}F_{i}{}^{r}$

(b) $P_{rjih}F_{k}{}^{r} = P_{rkih}F_{j}{}^{r}$

(1.7) (a) $P_{kjih}g^{ih} = 0,$

(b) $P_{kjih}g^{ki} = -A_{jh}$

(c) $P_{kjih}g^{ji} = A_{kh},$

(d) $P_{kjih}F^{ki} = F_{h}{}^{m}A_{mj}$

(e) $P_{kjih}F^{ji} = F_{k}{}^{m}A_{mh},$

(f) $P_{kjih}F^{ih} = 0$

where

$$A_{kh} = \frac{1}{(n+2)}[nR_{kh} - Rg_{kh}]$$

Further, Bochner curvature tensor given by

$$B_{kji}{}^{h} = R_{kji}{}^{h} + \delta_{k}{}^{h}L_{ji} - \delta_{j}{}^{h}L_{ki} + g_{ji}L_{k}{}^{h} - g_{ki}L_{j}{}^{h} + M_{ji}F_{k}{}^{h} - M_{ki}F_{j}{}^{h} + F_{ji}M_{k}{}^{h} - F_{ki}M_{j}{}^{h} - 2(M_{kj}F_{i}{}^{h} + F_{kj}M_{i}{}^{h})$$

where $L_{ji} = -\frac{1}{n+4}R_{ji} + \frac{1}{2(n+2)(n+4)}Rg_{ji}$

and $M_{ji} = -L_{jt}F_{i}{}^{t}$

andH-conhormonic curvature tensor given by

$$C_{kjih} = R_{kjih} + \frac{1}{n+4}[g_{jh}R_{ki} - g_{kh}R_{ji} + R_{jh}g_{ki} - R_{kh}g_{ji} + F_{jh}S_{ki} - F_{kh}S_{ji} + S_{jh}F_{ki} - S_{kh}F_{ji} + 2(S_{kj}F_{ih} + F_{kj}S_{ih})]$$

along with the tensor S_{kji} of type (0,3) and given by

$$S_{kji} = (\nabla_{k}R_{ji} - \nabla_{j}R_{ki}) + \frac{1}{n}(g_{ki}\nabla_{t}R_{j}{}^{t} - g_{ji}\nabla_{t}R_{k}{}^{t} + F_{ki}\nabla_{t}S_{j}{}^{t} - F_{ji}\nabla_{t}S_{k}{}^{t} + 2F_{kj}\nabla_{t}S_{i}{}^{t})$$

are known to us in Kaehlerian spaces.

2. Generalised Recurrent Kaehlerian space:

A Riemannian manifold V_n whose curvature tensor satisfies the relation

$$\nabla_{l}R_{kji}{}^{h} = a_{l}R_{kji}{}^{h} + b_{l}(\delta_{k}{}^{h}g_{ji} - \delta_{j}{}^{h}g_{ki})$$

has been called generalized recurrent Riemannian space by U. C. de and N. Guha [1]. Following the above we will introduce

Definition:A Kaehler space whose curvature tensor satisfies the relation

(2.1) $\nabla_l R_{kji}{}^h = a_l R_{kji}{}^h + b_l(\delta_k{}^h g_{ji} - \delta_j{}^h g_{ki} + F_k{}^h F_{ji} - F_j{}^h F_{ki} - 2F_{kj} F_i{}^h)$

Will be called generalized recuurentKaehler space or briefly GRK_n.

It is clear from (2.1) that when$b_l = 0$, a GRK_n reduces to a recurrent Kaehler space. Further we know if a Kaehler space is of constant holomorphic sectional curvature K, then its curvature tensor $R_{kji}{}^h$ takes the form

(2.2) $R_{kji}{}^h = \frac{K}{4}(\delta_k{}^h g_{ji} - \delta_j{}^h g_{ki} + F_k{}^h F_{ji} - F_j{}^h F_{ki} - 2F_{kj} F_i{}^h)$

[See Fukami([4],[5]) and Yano and Mogi([18],[19])].

and so, for a Kaehler space of constant holomorphic sectional curvature K, from (2.1) and (2.2) we have

$$\nabla_{lR_{kji}}{}^h = (a_l + \frac{4}{k} b_l) R_{kji}{}^h$$

Showing that the space is always recurrent and thus, we have

<u>Theorom 2.1</u> A generalised recurrent kaehler space of constant holomorphic sectional curvature is identical with a recurrent Kaehler space.

Keeping the above into mind, now onward, we shall assume that our Kaehlerian space is not a space of constant holomorphic sectional curvature.

As an immediate consequence of (2.1) with the help of

$\nabla_k g_{ij} = 0$, $\nabla_k g^{ij} = 0$, $\nabla_k \delta_i{}^j = 0$ and

$R_{ijk}{}^l = \frac{\partial}{\partial x^j}\left\{{l \atop ik}\right\} - \frac{\partial}{\partial x^k}\left\{{l \atop ij}\right\} + \left\{{m \atop ik}\right\}\left\{{l \atop mj}\right\} - \left\{{m \atop ij}\right\}\left\{{l \atop mk}\right\}andF_{ji} = -F_{ji}$

together with $F_i{}^i=0$, $\delta_i{}^i=n$, we get (2.3)$\nabla_l R_{kjih} = a_l R_{kjih} + b_l(g_{kh} g_{ji} - g_{jh} g_{ki} + F_{kh} F_{ji} - F_{jh} F_{ki} - 2F_{kj} F_{ih})$

(2.4) $\nabla_l R_{ji} = a_l R_{ji} + (n+2) b_l g_{ji}$

and

(2.5) $\nabla_l R = a_l R + n(n+2) b_l$

Now, if R=0,the equation (2.5) yields

$b_l = 0$

and hence from (2.1) we have

$$\nabla_l R_{kji}{}^h = a_l R_{kji}{}^h$$

which shows that space again reduces to a recurrent Kaehler space and thus we have

<u>Theorem 2.2</u> A generalised recurrent Kaehler space if vanishing Riemannian curvature is a recurrent Kaehlerspace.

The above theorem suggests us that a generalisedrecurrentKaehler space must not have zero Riemannian curvature.

Again, if R is a covariant constant

i.e.$\nabla_l R = 0$

then from (2.5) we have

(2.6) $a_l + n(n+2)b_l = 0$

Which shows that vectors $a_l$and $b_l$are not linearly independent and so we can state

Theorem2.3 IfRiemannian curvature of a generalised recurrent Kaehler space is covariant constant then the vectors a_l and b_l are linearly dependent.

In a Kaehler space with vanishing Bochner curvature tensor, Matsumoto [8] has found the identity

(2.7) $4(n+1)\nabla_k R_{ji} = g_{ki}\nabla_j R + g_{kj}\nabla_l R + 2g_{ji}\nabla_k R - F_{kj}F_i{}^r\nabla_r R - F_{ki}F_j{}^r\nabla_r R$

Which on multiplication with g^{ji} and using the identities

$g_{ij}g^{ik} = \delta_j^k$ and $\nabla_k g_{ij} = 0$, $\nabla_k g^{ij} = 0$, $\nabla_k \delta_i{}^j = 0$and $F_j^i F_i^h = -\delta_j^h$& $F_{ji} = -F_{ji}$

yields (2.8) $\nabla_k R = 0$

On using this in (2.5), we have

Theorem 2.4 In a generalised recurrent Kaehler space with vanishing Bochner curvature tensor, the vectors a_l and b_l appearing in (2.1) are not linearly independent.

On contracting (2.1) with respect to h and *l* and using (1.2) and $F_k^t\nabla_t S_{ji} = \nabla_i R_{jk} - \nabla_j R_{ik}$, we find

$$\nabla_k R_{ji} - \nabla_j R_{ki} = a_l R_{kji}{}^l + b_k g_{ji} - b_j g_{ki} - \tilde{b}_k F_{ji} + \tilde{b}_j F_{ki} + 2F_{kj}\tilde{b}_i$$

Where $\tilde{b}_k = -F_k{}^r b_r$ and a similar relation holds for$\tilde{a}_i$ also.

Further, from (2.4) and $\nabla_j F_i^h = 0$&$S_{ji=} R_{rj}F_i^r$, we get

(2.9) $F_i^l\nabla_l S_{jk} = \tilde{a}_i S_{kj} + (n+2)\tilde{b}_i F_{kj}$

From, these two equations by a direct calculation we have

(2.10) $2a_t R_i{}^t - a_i R = (n+2)(n-2)b_i$

Thus, we have

Theorem 2.5 In a generalised recurrent Kaehlerian spaces the identity (2.10) holds good.

A direct calculation based on HP-curvature tensor, (2.1) and the other identities obtained in this section yields the following

$$\begin{aligned}\nabla_l P_{kjih} = a_l R_{kjih} + \ & b_l\big(g_{kh}g_{ji} - g_{jh}g_{ki} + F_{kh}F_{ji} - F_{jh}F_{ki} - 2F_{kj}F_{ih}\big) \\ & + \frac{1}{(n+2)}\big[g_{jh}(a_l R_{ki} + (n+2)b_l g_{ki}) + g_{kh}\big(a_l R_{ji} + (n+2)b_l g_{ji}\big) \\ & + F_{jh}(a_l R_{mi} + (n+2)b_l g_{mi})F_k^m - F_{kh}(a_l R_{mi} + (n+2)b_l g_{mi})F_j^m \\ & + 2F_{ih}\big(a_l R_{mj} + (n+2)b_l g_{mj}\big)F_k^m\big]\end{aligned}$$

Which on multiplication by F^{ih} in view of$\nabla_j F_i^h = 0$ and (1.7(f)) gives

(2.11) $(\nabla_l P_{kjih})F^{ih} = a_l \frac{(n+3)}{(n+2)} S_{jk} + b_l F_{kj}$

Now we suppose that the Kaehlerian space is holomorphically symmetric [6]. Then from (2.11) we immediately have

$$a_l \frac{(n+3)}{(n+2)} S_{jk} + b_l F_{kj} = 0$$

Or,

$$0 = a_l \frac{(n+3)}{(n+2)} R_{rk} - b_l g_{rk}$$

Showing that

(2.12) $R_{rk} = \frac{(n+2) b_l a^l}{(n+3)|a|^2} g_{jk}$

And so, if vectors a and b are not orthogonal, we find that

$$R_{ik} \propto g_{ik}$$

Theorem 2.6 If a generalised recurrent Kaehlerian spaces is holomorphically symmetric also and the associated vectors a_1 and b_1 are not orthogonal, then it reduces to a KaehlerEinstein space.

Further if a_1 and b_1 are orthogonal, from (2.11) we find that

$$R_{ik}=0$$

And consequently, R=0. Thus

Theorem 2.7 The generalised recurrent Kaehlerianspaces with mutually orthogonal vectors is if associated holomorphically projective symmetric also then its scalar curvature vanishes identically.

With the help of $F_k \nabla_t R = 2\nabla_t S_k^t$, $F_k^t \nabla_t S_{ji} = \nabla_i R_{jk} - \nabla_j R_{ik}$ and (1.3) we find that the tensor field S_{kji} reduces into the form

(2.13) $S_{kji} = F_i{}^t \nabla_t S_{jk} + \frac{1}{2}\{(g_{ki}\nabla_j R - g_{ji}\nabla_k R) + (F_{ki}F_j{}^t - F_{jk}F_k{}^t + 2F_{kj}F_i{}^t)\nabla_t R\}$

And so the value of this tensor is generalized recurrent Kaehlerian spaces in view of (2.5) and (2.9) will be

$$S_{kji} = \tilde{a}_i S_{kj} + (n+2)\tilde{b}_i F_{kj} + \frac{1}{2n}\{(g_{ki}a_j - g_{ji}a_k) - (F_{ki}\tilde{a}_j + F_{ji}\tilde{a}_k - 2F_{jk}\tilde{a}_i)\}R + \left(\frac{n+2}{2}\right)\{g_{ki}b_j - g_{ji}b_k - F_{ki}\tilde{b}_j + F_{ji}\tilde{b}_k - 2F_{jk}\tilde{b}_i\}R$$

Since,

$$S_{kji} + S_{jik} + S_{ikj} = 0$$

We have from the above equation

$$\left(S_{kj} + \frac{2R}{n}F_{kj}\right)\tilde{a}_i + \left(S_{ji} + \frac{2R}{n}F_{ji}\right)\tilde{a}_k + \left(S_{ik} + \frac{2R}{n}F_{ik}\right)\tilde{a}_j + (n+2)R\{\tilde{b}_i F_{kj} + \tilde{b}_k F_{ji} + \tilde{b}_j F_{ik}\} = 0$$

Which after multiplying by $a^i a^j a^k$ and using $\tilde{a}_i \tilde{a}^i = 0$ gives

(2.14) $R(n+2)\tilde{b}_i a^i = 0$

and consequently, we have

Either(i) R=0 or(ii) $\tilde{b}_i a^i = 0$

Therefore, we find

Theorem 2.8 A generalised recurrent Kaehler space satisfies one of the following

(i) It is a space of zero Reimann curvature.
(ii) The associated vector a and b satisfy

$$a^i \tilde{b}^i = 0$$

3. Generalised Ricci-recurrent Kaehlerian Manifolds:

Generalised Ricci-recurrent Riemannian spaces have been introduced and studied by De, U. C., Guha, N. and Kamilya, D. [2]. Following them we shall call a Kaehlerian space to be generalised Ricci recurrent provided the Ricci tensor satisfies

(3.1) $\nabla_h R_{ji} = a_h R_{ji} + b_h g_{ji}$

When b_h vanishes identically, it is clear that

$$\nabla_h R_{ji} = a_h R_{ji}$$

and so, the space is Ricci recurrent and when a_h and b_h both vanish simultaneously, then it is Ricci symmetric. Thus, for the existence of a generalised Ricci recurrent space, it is essential that these vector field are always non vanishing.

As an immediate consequence of (3.1) after multiplication by g^{ji} is

(3.2) $\nabla_h R = a_h R + n b_h$

Thus, if R=0, we find that

$$b_h=0$$

and consequently $\tilde{b}_h = 0$ and so the genralised Ricci recurrent Kaehlerian space reduces into a Ricci recurrent space. Thus, we have

Theorem 3.1 The Riemannian curvature of a generalised Ricci recurrent space cannot be zero.

Or

Generalised Ricci recurrent Kaehler space of zero Riemann curvature does not exist.

Again, if R is covariant constant then from (3.2) we find that

$$a_h R+nb_h= 0$$

which shows that a_h and b_h are linearly dependent. Conversely if $a_h R+nb_h = 0$, then (3.2)it is evident that

$\nabla_l R = 0$ and therefore, we have

Theorem 3.2 The Riemann curvature of a generalised Ricci recurrent Kaehler space is constant if and only if vectors a_h and b_h are related by (3.3)

Again from (3.1) we find

(3.4) $\nabla_h R_j{}^k = a_h R_j{}^k + b_h \delta_j^k$

Which on contraction with respect to h and k and on using the identity

$$\nabla_k R_j{}^k = \frac{1}{2}\nabla_j R$$

and (3.2) gives

(3.5) $2a_h R_j{}^h = a_j R + (n-2)b_j$

On differentiating $S_{ji} = R_{rj}F_i^r$ covariantly with respect to x^h and using $\nabla_j F_i^h = 0$ and (3.1), we get

(3.6) $\nabla_h S_{ji} = a_h S_{ji} + b_h F_{ji}$

On substituting from (3.6) and (3.2) in $F_k^t \nabla_t S_{ji} = \nabla_i R_{jk} - \nabla_j R_{ik}$ and on simplifying with the help of

$F_j^i F_i^h = -\delta_j^h$, we find

$$(\tilde{a}_k S_{ji} + \tilde{b}_k F_{ji}) + (a_i R_{jk} - a_j R_{ik}) + (b_i g_{jk} - b_j g_{ik}) = 0$$

Which on multiplication by g^{ik} yields

$$\tilde{a}_k S_j{}^k - nb_j + a_i R_j{}^i - a_j R = 0$$

On using (3.5) in it, we find

$$2\tilde{a}_k S_j{}^k - 2nb_j + a_j R + (n-2)b_j - 2a_j R = 0$$

(3.7) $2\tilde{a}_k S_j{}^k = a_j R + (n+2)b_j$

And consequently from (3.7) and (3.5), we find

(3.8) $b_j = \frac{1}{2}[\tilde{a}_k S_j{}^k - a_h R_j{}^h]$

Thus, we have got a form of b_j and therefore, we may state

Theorem 3.3 The vector b_h associated with a generalised Ricci recurrent Kaehlerian space has the form (3.8)

On substituting from (3.1) and (3.2) in (2.13), we find

$$S_{kji} = \{a_k R_{ji} + b_k g_{ji} - a_j R_{ki} - b_j g_{ki}\} + \frac{1}{2n}\{g_{ki}(a_j R + nb_j) - g_{ji}(a_k R + nb_k)\} + \frac{1}{2n}\{F_{ki}F_j^t - F_{ji}F_k^t + 2F_{kj}F_i^t\}(a_t R + nb_t)$$

Which on multiplying by g^{ji} and using (3.5) yields

(3.9) $a_k = -\frac{S_{kji}g^{ji}}{R}$

Thus, we have determined the form of a_k in terms of tensor field S_{kji} and so we state

Theorem 3.4 The vector field a_k of generalised Ricci recurrent Kaehlerian space is given by (3.9).

In a Kaehlerian space with vanishing Bochner curvature tensor, Mastumoto has proved that equation (2.7) holds good

On substituting from (3.1) and (3.2) in (2.7), we find

$$4(n+1)\{a_k R_{ji} + b_k g_{ji}\} - g_{ki}(a_j R + nb_j) - g_{kj}(a_i R + nb_i) - 2g_{ji}(a_k R + nb_k) + F_{kj}F_i{}^r(a_r R + nb_r) + F_{ki}F_j{}^r(a_r R + nb_r) = 0$$

Which after transvection with g^{ji} gives

$$a_k R + b_k n = 0$$

Which in view of (3.2) gives

$$\nabla_h R = 0$$

i.e. space is of constant Riemannian curvature and thus, we have

Theorem 3.5 The generalised Ricci recurrent Kaehlerian space with vanishing Bochner curvature is of constant curvature.

4.Generalised Ricci 2-recurrent Kaehlerian Space:

An n-dimensional Kaehlerian manifold is said to be generalized Ricci 2-recurrent Kaehlerian manifold if the Ricci tensor of the space satisfies

(4.1) $\nabla_l \nabla_k R_{ji} = a_l \nabla_k R_{ji} + b_{lk} R_{ji}$

Where a_l is a covariant vector and $b_{lk}$is covariant tensor of order 2 From the definition it is clear that when a_l vanishes identically, then the space reduces to a Ricci-2 recurrent space. From (4.1) by simple calculation, we find

(4.2) $\nabla_l \nabla_k R = a_l \nabla_k R + b_{lk} R$

(4.3) $\nabla_l \nabla_k R = a_l \nabla_k R + 2b_{lr} R_k{}^r$

Where the identity $\nabla_k R_j{}^k = \frac{1}{2}\nabla_j R$ has been used.

The above two equations show that

(4.4) $b_{lk} = \frac{2}{R} b_{lr} R_k{}^r$

Thus, the tensor of recurrences has got a specified form.

From (4.2) it is clear that when R is constant then

$$b_{lk}R=0$$

And thus, eitherR=0 orb_{lk}=0.

Now when b_{lk} vanishes the space loses its nature of being generalised Ricci 2 recurrent and therefore the only option is that R=0 and thus we have

Theorem 4.1 If the scalar curvature of a generalised Ricci 2-recurrent Kaehlerian space is a constant then it must be zero. Again since

$$\nabla_l \nabla_k R = \nabla_k \nabla_l R$$

From (4.2) we immediately have

$$a_l \nabla_k R - a_k \nabla_l R + R(b_{lk} - b_{kl}) = 0$$

And so that if the tensor b_{lk} of recurrence is a symmetric tensor, we find that

$$a_l \nabla_k R = a_k \nabla_l R$$

Which shows that $\nabla_k R$ must be a quantity proportional to a_k and hence we have

Theorem 4.2 If the tensor of recurrence of a generalised Ricci 2-recurrent Kaehlerian space is symmetric, then $\nabla_k R$ is a vector along the associated vector a_k.

As an immediate consequence the equation (4.1), we have

(4.5) $\nabla_l \nabla_k S_{ji} = a_l \nabla_k S_{ji} + b_{lk} S_{ji}$

Where$\nabla_j F_i^h = 0$ and$S_{ji} = R_{rj} F_i^r$ has been used.

On differentiating covariantly the HP-curvature tensor, first with respect to x^m and then with respect to $x^l$and using$\nabla_k g_{ij} = 0$,$\nabla_k g^{ij} = 0$, $\nabla_k \delta_i{}^j = 0$&$g_{ij} g^{ik} = \delta_j^k$, (4.1), (4.5) and the equation of HP-curvature tensor itself. We find

(4.6) $\nabla_l \nabla_m P_{kji}{}^h - a_l \nabla_m P_{kji}{}^h - b_{lm} P_{kji}{}^h = \nabla_l \nabla_m R_{kji}{}^h - a_l \nabla_m R_{kji}{}^h - b_{lm} R_{kji}{}^h$

Thus, we find that when the Kaehlerian space is generalised Ricci 2-recurrent then equation (4.6) holds good. Conversly if we assume that (4.6) holds good then on contracting (4.6) with respect to h andk, we find that

$$\nabla_l \nabla_m R_{ji} = a_l \nabla_m R_{ji} + b_{lm} R_{ji}$$

Where we have used $P_{kji}{}^h = 0$ and$R_{ij} = R_{ijk}^k$. The above equation is nothing but equation (4.1) and so our space is generalised Ricci 2-recurrent and thus we have

Theorem 4.3 A Kaehlerian space is genralised Ricci 2-recurrent if and only if the equation (4.6) holds.

By the same method adopted as above, we can get two more equations similar to (4.6), one for the Bochner curvature tensor and the other for the Conharmonic curvature tensor, and proceeding exactly as above one can get

Theorem 4.4 A Kaehlerian space is generalised Ricci 2-recurrent if and only if

$$\nabla_l \nabla_m B_{kji}{}^h - a_l \nabla_m B_{kji}{}^h - b_{lm} B_{kji}{}^h = \nabla_l \nabla_m R_{kji}{}^h - a_l \nabla_m R_{kji}{}^h - b_{lm} R_{kji}{}^h$$

holds.

Theorem 4.5 A Kaehlerian space is generalised Ricci 2-recurrent if and only if

$$\nabla_l \nabla_m C_{kji}{}^h - a_l \nabla_m C_{kji}{}^h - b_{lm} C_{kji}{}^h = \nabla_l \nabla_m R_{kji}{}^h - a_l \nabla_m R_{kji}{}^h - b_{lm} R_{kji}{}^h$$

holds.

5. Vector fields in generalised recurrent Kaehlerian spaces:

In Kaehler space a parallel vector field is the vector field whose covariant derivative vanishes identically, i.e.,

(5.1) $\nabla_j v^i = 0$

If we put$\bar{v}^i = F_j{}^i v^j$ then in view of$\nabla_j F_i^h$= 0from (5.1) we have

On making the use of Ricci identity

(5.2) $\nabla_k \nabla_j v^h - \nabla_j \nabla_k v^h = R_{kji}{}^h v^i$

And the Bianchi identities, we have following results

(5.3) (a) $v^a R_{ajk} = 0, \quad v^a R_{aj} = 0$

(b) $\bar{v}^a R_{hajk} = 0, \quad \bar{v}^a R_{aj} = 0$

(5.4) (a) $v^a \nabla_l R_{hajk} = 0, v^a \nabla_l R_{aj} = 0$

(b) $\bar{v}^a \nabla_l R_{hajk} = 0, \bar{v}^a \nabla_l R_{aj} = 0$

(5.5) (a) $v^a \nabla_a R_{hjk} = 0,\ v^a \nabla_a R_{ij} = 0, \quad v^a \nabla_a R = 0$

(b) $\bar{v}^a \nabla_a R_{hijk} = 0, \bar{v}^a \nabla_a R_{ij} = 0, \quad \bar{v}^a \nabla_a R = 0$

Now on differentiating (5.3) covariantly with respect to $x^l$and using (2.1) (i.e. on supporting K_n to be generalised recurrent space) and (5.1), we find

$$a_l R_{kji}{}^h v^i + b_l\left(v_j \delta_k{}^h - \delta_j{}^h v_k - F_k{}^h \tilde{v}_j + F_j{}^h v_k\right) - 2F_{kj}\tilde{v}^h b_l = 0$$

On contracting the above equation with respect to h andk, we get

(5.6) $a_l R_{ji} v^i + (n+2) b_l v_j = 0$

Which after multiplication with $\tilde{a}^l$ and use of $a_l \tilde{a}^l = 0$ gives

$$b_l \tilde{a}^l = 0$$

and thus, we get

Theorem 5.1 If generalised recurrent Kaehlerian space admits a parallel vector field then its associated vectors a a_l and b_l satisfy $a_l \tilde{b}^l = 0$.

On the other hand, on multiplying (5.6) by a^l, we find

(5.7) $(R_{ji} - C g_{ji}) v^i = 0$

Where $C = \frac{-(n+2) b_l a^l}{|a|^2}$

Clearly C is a non-zero scalar, when vectors a_l and b_l are not orthogonal and becomes zero when they are orthogonal. Further as the (5.7) holds good for arbitrary vector field, we have

$$R_{ji} \propto g_{ji} \text{ when } \quad b_l a^l \neq 0$$

and

$$R_{ji} = 0 \text{ when } \quad b_l a^l = 0$$

Thus, we conclude that

Theorem 5.2 If generalised recurrent Kaehlerian space admits a parallel vector field and associated vectors a_l and b_l are orthogonal, then it space becomes a Kaehlerian Einstein space.

Theorem 5.3 If generalised recurrent Kaehlerian space admits a parallel vector field and associated vectorsa_l and b_l are orthogonal, then it reduces to a space of zero Riemannian curvature.

Now, in the following space will be assumed to be generalised Ricci recurrent, so that the identities of article 3 holds good.

On differentiating (5.3)(b) covariantly with respect to x^h and using (3.1), (5.1) and (5.3)(b) after simplification we get that

$$b_h v_j = 0$$

shows that there are two possibilities

(i) The parallel vector field is null vector field.
(ii) The vector field v_j is null vector field.

Since $b_h \neq 0$ we immediately find that $v_j = 0$ and therefore

Theorem 5.4 The parallel vector field in a generalised Ricci recurrent Kaehlerian spaces reduces to a null vector.

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