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Fundamental Algebraic Properties on $\kappa - Q -$ Anti Fuzzy Normed Prime Ideal and $\kappa - Q -$ Anti Fuzzy Normed Maximal Ideal

M. Premkumar[*1], M. Bhuvaneswari[2], E. Manikandan[3] and Dr. A. Prasanna[4]

[*1]Assistant Professor, Department of Mathematics, Kongunadu College of Engineering and Technology (Autonomous), Tiruchirappalli-620020, Tamilnadu, India.

Email: [*1]**mprem.maths3033@gmail.com**

[2]Assistant Professor, Department of Mathematics, Mahendra Institute of Technology (Autonomous), Namakkal-637503, Tamilnadu, India.

Email: [2]**bhuvanam.eswari0@gmail.com**

[3]Assistant Professor, Department of Mathematics, Kongunadu College of Engineering and Technology (Autonomous), Tiruchirappalli-620020, Tamilnadu, India.

Email: [3]**emanikncet@gmail.com**

[4] Assistant Professor, PG and Research Department of Mathematics, Jamal Mohamed College (Autonomous), (Affiliated to Bharathidasan University), Tiruchirappalli-620020, Tamilnadu, India.

Email: [4]**apj_jmc@yahoo.co.in**

Abstract:

In this paper, the concept of $\kappa - Q -$Anti fuzzy normed ring is introduced and some basic properties related to it are established. That our definition of a algebraic structure which we call a $\kappa - Q -$ Anti Fuzzy Normed Rings. We also defined $\kappa - Q -$ Anti Fuzzy Normed Rings homomorphism, $\kappa - Q -$ Anti Fuzzy Normed Subring, $\kappa - Q -$ Fuzzy Normed Ideal, $\kappa - Q -$ Fuzzy Normed Prime Ideal and $\kappa - Q -$ Anti Fuzzy Normed Maximal Ideal of a Normed ring respectively.

Keywords:

Fuzzy Sets, Normed Space, $\kappa - Q -$ Anti fuzzy Sub set, $\kappa - Q -$ Anti fuzzy normed ring and $\kappa - Q -$ Anti fuzzy normed ideal.

1. Introduction

Zahed L A[19] , published his maiden well acknowledged research paper about fuzzy sets in 1965. In 1952, described the new notion of generalization of normed rings by Arens R[1]. Sahin M[13] &[14] et.al., developed the new concept of Soft Normed Rings in 1950, and also established the Normed Quotient Rings in 2018. Swamy U M and Swamy K L N[16], commenced the idea of Fuzzy Peime Ideals in 1988. In 2017, started the conception of Normed Z-Modules by Ulucay V[17], et.al. Aykut Emniyet and Memet Sahin[2], initiated the concept of Fuzzy Normed Rings in 2018. Solairaju A and Nagarajan R[15],developed a new structure of Q- fuzzy subgroup groups discussed some elementary properties of these groups in 2009. In Commutative Normed Rings, will be the concept developed by Gel'fand I[3] et.al. in 1964. Rasuli R[12], explained the notion of Characterization of Q-fuzzy subrings (Anti Q-fuzzy subrings) with respect to a T-norm (T-conorm) in 2018. Theory of T-norms and Fuzzy Inference Methods, started the new work by Gupta M M and Qi J[4], in 1991. In 2011, introduced the concept of

Normed Rings-Algebraic Patching by Jarden M[5]. Komogorov A N[6] et.al. writen the book as Introductory Real Analysis in 1970. In 1982, studied the Fuzzy Invariant Subgroups and Fuzzy ideals by Liu W[7]. Mordeson J N[8] et.al. developed a new structure of fuzzy group theory in 2005. Mukherjee N P and Bhattacharya P[9], initiated the concept of Fuzzy Normed subgroups and Fuzzy cosets in 1984. Prasanna A , Premkumar M[11], et.al. defined the new concept on $\kappa - Q -$Fuzzy Orders Relative to $\kappa - Q -$Fuzzy Subgroups and Cyclic group on various fundamental aspects in 2020. In 1964, published his maiden well acknowledged research paper in Normed Rings by Naimark M A[10]. In 2009, explored a Mathematics of Fuzziness by Wang X[18] et.al. Zimmermann H J[20], portrayed the Fuzzy set Theory and its applications in 1991.

This paper is formed because the section 2 contains the fundamental definitions of Normed Rings, $\kappa - Q -$fuzzy set and related results which are thoroughly crucial to know the novelty of this paper. In section 3, we utilize the study of this phenomena to Fundamental Algebraic properties on $\kappa - Q -$ Anti Fuzzy Normed Prime Ideal and $\kappa - Q -$ Anti Fuzzy Normed Maximal Ideal and their theorems and also established the examples.

2. Preliminaries

In this section, definition of normed linear space, normed ring, Archimedean strict T-norm and concept of fuzzy sets are outlined.

Definition 2.1 [6]:

A function $\|\|$ defined on a linear space L is said to be a norm if it has the following properties:

(i) $\|x\| \geq 0, for\ all\ x \in L, where\ \|x\| = 0$ if and only if $x = 0$;

(ii) $\|\alpha . x\| = |\alpha| . \|x\|; (and\ hence\ \|x\| = \|-x\|), for\ all\ x \in L$ and for all α;

(iii) $Tringle\ inequality$: $\|x + y\| \leq \|x\| + \|y\|, for\ all\ x, y \in L.$

A linear space L, equipped with a norm is called linear space.

Definition 2.2 [4]:

A ring A is said to be a normed ring if A possesses a norm $\|\|$, that is a non-negative real-valued function $\|\|: A \to R$ such that for any $a, b \in A$,

(i) $\|a\| = 0 \Leftrightarrow a = 0$,

(ii) $\|a + b\| \leq \|a\| + \|b\|$,

(iii) $\|-a\| = \|a\|$,

(iv) $\|ab\| \leq \|a\|\|b\|$.

Definition 2.3 [5]:

Let $*: [0,1] \times [0,1] \to [0,1]$. $*$ is an Archimedean strict T-norm iff for all $x, y, z \in [0,1]$:

(i) $*$ is commutative and associative, that is, $* (x, y) =* (y, x) and\ * \left(x * (y, z)\right) =* (* (x, y), z)$,

(ii) $*$ is continuous,

(iii) $* (x, 1) = x$,

(iv) $*$ is monotone, which means $* (x, y) \leq* (x, z) if y \leq z$,

(v) $* (x, x) < x for\ x \in (0,1)$, and

(vi) $when\ x < z\ and\ y < t, * (x, y) <* (z, t) for\ all\ x, y, z\ , t \in (0,1)$.

For convenience, we use the word $t - norm$ shortly and show it as $x * y$ instead of $* (x, y)$. Some examples of $t - norm$ are $x * y = min\{x, y\}, x * y = max\{x + y - 1, 0\}\ and\ x * y = x. y$.

Definition 2.4 [5]:

Let $\diamondsuit: [0,1] \times [0,1] \to [0,1]$. $*$ is an Archimedean strict $T -$conorm iff for all $x, y, z \in [0,1]$:

(i) $\diamondsuit$ is commutative and associative that is $\diamondsuit(x, y) = \diamondsuit(y, x) and\ \diamondsuit\left(x, \diamondsuit(y, z)\right) = \diamondsuit(\diamondsuit(x, y), z)$,

(ii) $\diamondsuit$ is continuous

(iii) $\diamond(x, 0) = x$,

(iv) $\diamond$ is monotone, which means $\diamond\,(x, y) \leq \diamond(x, z) if y \leq z$,

(v) $\diamond\,(x, x) > x for\, x \in (0,1), and$

(vi) When $x < z\; and\; y < t, \diamond\,(z, t), \diamond(z, t) < \diamond(x, y) forall\, x, y, z, t \in (0,1)$.

For convenience we use the word $s - norm$ shortly and show it as $x \diamond y$ instead of $\diamond(x, y)$. Some examples of $s - norm$ are $x \diamond y = max\{x, y\}, x * y = min\{x + y, 1\}\; and\; x \diamond y = x + y - x.y$.

Definition 2.5 [19]:

The fuzzy set B on a universal set X is a set of ordered pairs $B = \{(x, \mu_B(x): x \in X)\}$ here, $\mu_B(x)$ is the membership function or membership grade of x in B. for all $x \in X$, we have $0 \leq \mu_B(x) \leq 1$. If $x \notin B$, $\mu_B(x) = 0$, and if x is entirely contained in B, $\mu_B(x) = 1$. The membership grade of x in B is shown as $B(x)$ in the rest of this paper.

Definition 2.6 [19]:

For the fuzzy sets $A\; and\; B$, the membership functions of the intersection, unjon and complement are defined point wise as follows respectively:

(i) $(A \cap B)(x) = min\{A(x), B(x)\}$,

(ii) $(A \cup B)(x) = max\{A(x), B(x)\}$,

(iii) $\bar{A}(x) = 1 - A(x)$.

Definition 2.7 [17]:

Let $(R, +, \cdot)$ be a ring and $F(R)$ be the set of all fuzzy subsets of R. As $A \in F(R)$, $\wedge$ is the fuzzy intersection and $\vee$ is the fuzzy union functions, for all $x, y \in R$, if A satisfies

(i) $A(x - y) \geq A(x) \wedge A(y)$

(ii) $A(x \cdot y) \geq A(x) \wedge A(y)$

then A is called a fuzzy subring of R. If A is a subring of R for all $a \in A$, then A is itself a fuzzy ring.

Definition 2.8 [15]:

A non-empty fuzzy subset A of R is said to be an ideal(in fact a fuzzy ideal) if and only if, for any $x, y \in R, A(x - y) \geq A(x) \wedge A(y)$ and $A(x \cdot y) \geq A(x) \vee A(y)$.

Remark 2.9:

The fuzzy operations of fuzzy subsets $A, B \in F(R)$ on the ring R can be extended to the operations below by $t - norms\; and\; s - norms$:

For all $z \in R$,

(i) $(A + B)(z) = \underset{x + y = z}{\diamond}\big(A(x) * B(y)\big)$;

(ii) $(A - B)(z) = \underset{x - y = z}{\diamond}\big(A(x) * B(y)\big)$;

(iii) $(A \cdot B)(z) = \underset{x.y = z}{\diamond}\big(A(x) * B(y)\big)$.

Definition 2.10 [14]:

Let Q and S be any two sets. Then the mapping $A: S \times Q \to [0,1]$ is called a Q-Fuzzy set in S

Definition 2.11 [11]:

A Q-Fuzzy set A of ring S is said to be Q-Fuzzy Subring if the following properties hold,

(i) $A(m - n, q) \geq \min\{A(m, q), A(n, q)\}$, for all $m, n \in S$ and $q \in Q$.

$(ii) A(mn, q) \geq \min\{A(m, q), A(n, q)\}$, for all $m, n \in S$ and $q \in Q$

Definition 2.12 [11]:

Let the mapping $f: S_1 \to S_2$ be a homomorphism. Let A and B be two Q-Fuzzy Subrings of S_1 and S_2 respectively, then $f(A)$ and $f^{-1}(B)$ are image of A and the inverse image of B respectively, defined as

(i) $f(A)(n,q) = \begin{cases} \sup\{A(m,q): m \in f^{-1}(n)\}, & \text{if } f^{-1}(n) \neq \emptyset \\ 0, & \text{if } f^{-1}(n) = \emptyset \end{cases}$, for every $n \in S_2$ and $q \in Q$

(ii) $f^{-1}(B)(m,q) = B(f(m),q)$, for every $m \in S_1$ and $q \in Q$.

Definition 2.13[11]

Let G and Q be any two nonempty sets and $\kappa \in [0,1]$ and A be a $Q -$ Fuzzy Subset ($Q - FSb$) of a set G. The fuzzy set A^κ of G is called the $\kappa - Q -$ Fuzzy Subset ($\kappa - Q - FSb$) of G is defined by

$$A^\kappa(x,q) = (A(x,q),\kappa), \forall\, x \in G \text{ and } q \in Q.$$

3. Fundamental Algebraic properties on $\kappa - Q -$ Anti Fuzzy Normed Prime Ideal and $\kappa - Q -$ Anti Fuzzy Normed Maximal Ideal

In this section, $\kappa - q -$Anti fuzzy normed prime ideal and $\kappa - q -$ Anti fuzzy normed maximal ideal are outlined.

Definition 3.1:

Let A^κ and B^κ be two $\kappa - q -$ Anti fuzzy subsets of the normed ring NR. We define the operation $A^\kappa \circ B^\kappa$ as follows:

$$A^\kappa \circ B^\kappa(x,q) = \begin{cases} \underset{x=yz}{\diamondsuit}\left(A^\kappa(y,q) * B^\kappa(z,q)\right), & If\ x\ can\ be\ defined\ as\ x = yz\ and\ q \in Q \\ 0, & otherwise. \end{cases}$$

If the normed ring NR has a multiplicative inverse, namely if $NR\,.\,NR = NR$, then the second case does not occur.

Definition 3.2:

Let A^κ and B^κ be $\kappa - q -$ Anti fuzzy normed ideals of a normed ring NR and let FNP be a non-constant function, which is not an ideal of NR. If

$$A^\kappa \circ B^\kappa \subseteq FNP \Rightarrow A^\kappa \subseteq FNP \text{ or } B^\kappa \subseteq FNP,$$

then FNP is called a $\kappa - q -$ Anti fuzzy normed prime ideal of NR.

Example 3.2.1:

Show that if the $\kappa - q -$ Anti fuzzy normed ideal I ($I \neq NR$) is a $\kappa - q -$ Anti fuzzy normed prime ideal of NR, then the characteristic function λ_I is also a $\kappa - q -$ Anti fuzzy normed prime ideal.

Solution:

As $I \neq NR, \lambda_I$ is a non-constant function on NR. LetA^κ and B^κ be two $\kappa - q -$ Anti fuzzy normed ideals on NR such that $A^\kappa \circ B^\kappa \subseteq \lambda_I$, but $A^\kappa \lambda_I$ and $B^\kappa \lambda_I$.

There exists $x, y \in NR$ and $q \in Q$ such that $A^\kappa(x,q) \leq \lambda_I(x,q)$ and $B^\kappa(x,q) \leq \lambda_I(y,q)$.

In this case,

$$A^\kappa(x,q) \neq 0 \text{ and } B^\kappa(y,q) \neq 0, but\ \lambda_I(x,q) = 0 \text{ and } \lambda_I(y,q) = 0.$$

Therefore, $x \notin I, y \notin I$.

As I is a $\kappa - q -$ Anti fuzzy normed prime ideal, there exists an $r \in NR$ and $q \in Q$, such that $xry \notin I$.

This is obvious, because if I is $\kappa - q -$ Anti fuzzy normed prime,

$A^\kappa \circ B^\kappa(xry,q) \subseteq I \Rightarrow or\ B^\kappa(ry,q) \subseteq I$ and therefore as $(NRxNR,q)(NRryNR,q) = (NRxNR,q)(NRyNR,q) \subseteq I$, we have either $(NRxNR,q) \subseteq I$ or $(NRyNR,q) \subseteq I$.

Assume $(NRxNR,q) \subseteq I$. Then $((x\,x\,x),q) = (x,q)^3 \in I \Rightarrow (x,q) \subseteq I$, but this contradicts with the fact that $\lambda_I(x,q) = 0$.

Now let $a = xry. \lambda_I(a,q) = 0$.

Thus $A^\kappa \circ B^\kappa(a,q) = 0$. On the other hand,

$$\begin{aligned} A^\kappa \circ B^\kappa(a,q) &= \underset{a=cd}{\diamondsuit}(A^\kappa(c,q) * B^\kappa(d,q)) \\ &\leq A^\kappa(x,q) * B^\kappa(ry,q) \\ &\leq A^\kappa(x,q) * B^\kappa(y,q) \end{aligned}$$

≤ 0 ($as\ A^{\kappa}(x,q) \neq 0\ and\ B^{\kappa}(y,q) \neq 0$).

This is a contraction, since $A^{\kappa} \circ B^{\kappa}(a,q) = 0$. Therefore if $A^{\kappa}\ and\ B^{\kappa}$ are $\kappa - q -$ Anti fuzzy normed ideals of a normed ring $NR, then\ A^{\kappa} \circ B^{\kappa} \subseteq \lambda_I \Rightarrow A \subseteq \lambda_I\ or\ B^{\kappa} \subseteq \lambda_I$. As a result, the characteristic function λ_I is a $\kappa - q -$ Anti fuzzy normed prime ideal.

Definition 3.3:

Let A^{κ} be a $\kappa - q -$ Anti fuzzy normed ideal of a normed ring NR. If A^{κ} is non-constant and for all $\kappa - q -$ Anti fuzzy normed ideals B^{κ} of $NR, A^{\kappa} \subseteq B^{\kappa}$ implies $A^{\kappa^0} = B^{\kappa^0}\ or\ B = \lambda_{NR}$, A^{κ} is called a $\kappa - q -$ Anti fuzzy normed maximal ideal of the normed ring NR. $\kappa - q -$ Anti Fuzzy normed maximal left (right) ideals are defined similarly.

Example 3.3.1:

Let A^{κ} be a $\kappa - q -$ Anti fuzzy normed maximal left (right) ideal of a normed ring NR. Then, $A^{\kappa^0} = \{x \in NR\ and\ q \in Q: A^{\kappa}(x,q) = A^{\kappa}(0,q),\}$ is a Anti fuzzy normed maximal left (right) ideal of .

Lemma 3.4:

If $A^{\kappa}\ and\ B^{\kappa}$ are a $\kappa - q -$ Anti fuzzy normed right and a $\kappa - q -$ Anti fuzzy normed left ideal of a normed ring NR, respectively, $A^{\kappa} \circ B^{\kappa} \subseteq A^{\kappa} \cap B^{\kappa}\ and\ hence\ (A^{\kappa} \circ B^{\kappa})(x,q) \leq (A^{\kappa} \cap B^{\kappa})(x,q), \forall\ x \in NR\ and\ q \in Q$.

Proof:

It is shown in Example 2 that if $A^{\kappa}\ and\ B^{\kappa}$ are $\kappa - q -$ Anti fuzzy normed left ideals of NR, then $A^{\kappa} \cap B^{\kappa}$ is also a $q -$ Anti fuzzy normed left ideal. Now, let $A^{\kappa}\ and\ B^{\kappa}$ be a $\kappa - q -$ Anti fuzzy normed right and a $\kappa - q -$ Anti fuzzy normed left ideal of NR, respectively. If $A^{\kappa} \circ B^{\kappa}(x,q) = 0$, the proof is trivial.

Let $A^{\kappa} \circ B^{\kappa}(x,q) = \underset{x = yz}{\diamond}(A^{\kappa}(y,q) * B^{\kappa}(z,q))$.

As A^{κ} is a $\kappa - q -$ Anti fuzzy normed right ideal and B^{κ} is a $\kappa - q -$ Anti fuzzy normed left ideal, we have

$A^{\kappa}(y,q) \leq A^{\kappa}(yz,q) = A^{\kappa}(x,q)$ and $B^{\kappa}(y,q) \leq B^{\kappa}(yz,q) = B^{\kappa}(x,q)$

Thus

$$\begin{aligned}(A^{\kappa} \circ B)(x,q) &= \underset{x = yz}{\diamond}(A^{\kappa}(y,q) * B^{\kappa}(z,q)) \\ &\leq max\{A^{\kappa}(x,q), B^{\kappa}(x,q)\} \\ &= (A^{\kappa} \cap B^{\kappa})(x,q).\end{aligned}$$

Theorem 3.5:

If A^{κ} is a $\kappa - q -$ Anti fuzzy normed left (right) maximal ideal of a normed ring NR, then $A^{\kappa}(0,q) = 1$.

Proof:

Assume $A^{\kappa}(0,q) \neq 1$.

Let $A^{\kappa}(0) < t < 1$ and let B^{κ} be a $\kappa - q -$ Anti fuzzy subset of NR such that $B^{\kappa}(x,q) = t$ for all $x \in NR\ and\ q \in Q$. B^{κ} is trivially an ideal of NR.

Also it is easy to verify that $A^{\kappa} \subset B^{\kappa}, B^{\kappa} \neq \lambda_{NR}$ and $B^{\kappa^o} = \{x \in NR\ and\ q \in Q: B^{\kappa}(x,q) = B^{\kappa}(0,q)\} = NR$.

But, despite the fact that $A^{\kappa} \subset B^{\kappa}, A^{\kappa^0} \neq B^{\kappa^0}$ and $B^{\kappa} \neq \lambda_{NR}$ is a contradiction to the $\kappa - q -$ Anti fuzzy normed minimality of A^{κ}.

Thus $A^{\kappa}(0,q) = 1$.

4. Conclusion

In this paper, we defined a $\kappa - q -$ Anti fuzzy normed ring. Here we examine the algebraic properties of $\kappa - q -$ Anti fuzzy sets in ring structures. Some related notions, the $\kappa - q -$fuzzy normed ring homomorphism, $\kappa - q -$ Anti fuzzy normed subring, $\kappa - q -$ Anti fuzzy normed ideal , $\kappa - q -$ Anti fuzzy normed prime ideal and $\kappa - q -$ Anti fuzzy normed prime maximal ideal are proposed.

References

1. Arens R, A generalization of normed rings, Pac.J.Math. 1952, 2, 455-471.
2. Aykut Emniyet and Memet Sahin, Fuzzy Normed Rings, Symmetry, 2018.
3. Gel'fand I, Raikov M D A and Shilov G E, Commutative Normed Rings, Chelsea Publishing Company, New York, USA, 1964.
4. Gupta M M and Qi J, Theory of T-norms and Fuzzy Inference Methods, Fuzzy sets system, 1991, 40,431-450.
5. Jarden M, Normed Rings-Algebraic Patching, Springer, Berlin/Heidelberg, Germany, 2011.
6. Komogorov A N, Silverman R A and Fomin S V, Introductory Real Analysis, Dover Publications, Inc. New York, NY, USA, 1970.
7. Liu W, Fuzzy Invariant Subgroups and Fuzzy ideals, Fuzzy sets system 1982, 8,133-139.
8. Mordeson J N, Bhutani K R and Rosenfeld A, fuzzy group theory, Springer, Heidelberg, Germany, 2005.
9. Mukherjee N P and Bhattacharya P, Fuzzy Normed subgroups and Fuzzy cosets, Inf.Sci. 1984, 40,431-450.
10. Naimark M A, Normed Rings, Noordhoff: Groningen, The Netherlands, 1964.
11. Prasanna A , Premkumar M, Ismail Mohideen S and Dhirendra Kumar Shukla, $\kappa - Q -$Fuzzy Orders Relative to $\kappa - Q -$Fuzzy Subgroups and Cyclic group on various fundamental aspects, Materials Today: Proceedings, 2020, https://doi.org/10.1016/j.matpr.2020.12.1063, 1-4.
12. Rasuli R, Characterization of Q-fuzzy subrings (Anti Q-fuzzy subrings) with respect to a T-norm (T-conorm), Journal of Information and Optimization Sciences, vol.39, pp.$827 - 837,$ 2018.
13. Sahin M , Olgun N and Ulucay V, Soft Normed Rings, Springer Plus 2016, 5, 1950-1956.
14. Sahin M , Olgun N and Ulucay V, Normed Quotient Rings, New Trends Math. Sci. 2018, 6,52-58.
15. Solairaju A and Nagarajan R, A new structure and construction of Q- fuzzy groups. Advances in Fuzzy Mathematics, vol.4, no.1 pp.$23 - 29,$ 2009.
16. Swamy U M and Swamy K L N, Fuzzy Peime Ideals of Rings, J. Math. Anal. Appl. 1988, 134, 94-103.
17. Ulucay V, Sahin M and Olgun N, Normed Z-Modules, Int. J. Pure. Appl. Math. 2017, 112, 425-435.
18. Wang X, Ruan D and Kerre E E, Mathematics of Fuzziness, Bassic Issues, Springer, Berlin/ Heridelberg, Germany, 2009.
19. Zadeh L A, Fuzzy sets, Inf. Control 1965, 8,338-353.
20. Zimmermann H J, Fuzzy set Theory and its applications, 2nd ed; Kluwer Academic Publishers, Norwell, MA, USA, 1991