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Rough Q-Neutrosophic ideals of Semiring

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Abstract:

The goal of this work is to present and investigate various features of semiring ideals using the Q-neutrosophic set notion. Also various operations such as intersection, composition, cartesian product, and so on are studied.

Keywords:

rough neutrosophic set, semiring, rough Q-neutrosophic ideal, cartesian product, composition.

1. Introduction

The notion of fuzzy sets was introduced by Zadeh [15] in 1965 in order to overcome the uncertainties associated with various problems in the environment, the economy, technology, etc. Atanassov [2] introduced an intuitionistic fuzzy set in 1986, according to which a degree of non-membership was taken into account in addition to the degree of membership of each element with [(membership value + non-membership value) $\leq$ 1].

Various generalizations such as vague sets, rough sets, vague sets, interval-valued sets etc., are the mathematical tools to deal with uncertainties. F. Sarandache[14] in 2005 introduced the Neutrosophic set in which he introduced indeterminacy in intuitionistic fuzzy sets. Now, several researchers have applied this concept in many practical areas such asmulti-criteria decision making,disaster management, signal processing, etc. Their applications can be found in [6,7,8].

Majumder [11] investigated the concept of Q-fuzzification of ideals of the gammasemigroup in 2011. Akram [1], Lekkoksung [9,10], Mandal [4,5] et al. extended this concept to gammasemigroups, ordered semigroups [10], Γ ordered half rings, soft fields, group theory and examined some important properties. In 2020 Debabratamandal[4] discussed Some Properties of Q-Neutrosophic Ideals of Semirings.

Motivated by this idea and the combination of the concept with the neutrosophic set, we investigated rough Q-Neutrosophic ideals of Semiring. Also some of the theorems and its properties were discussed in this paper.

2. Preliminaries

In this section we can see some of the definitions which will be used in the discussion of the paper.

Definition 2.1:[4]

A semiring is a nonempty set S_R on which two operations + and · have been defined such that (S, +) and (S, ·) form monoid where · distributes over + from any side.

Definition 2.2:[4]

A nonempty subset X ($\neq S_R$) of semiring S is said to be an ideal if for all

a,b∈ X and r∈ S_R, a+b∈ X, ra∈ X. Similarly we can define a right ideal also. An ideal

of S is a nonempty subset which satisfies both properties of left ideal and right ideal.

Definition 2.3:[4]

A neutrosophic set κ on the universe U is defined as

$\kappa = \{< x, \kappa_t(x), \kappa_i(x), \kappa_f(x) >, x \in U\}$,

where $\kappa_t, \kappa_i, \kappa_f : U \to [0,1]$ and $0 \le \kappa_t(x) + \kappa_i(x) + \kappa_f(x) \le 3$.

Definition 2.4:[4]

For a non-empty set Q, a mapping $\phi : S_R \times Q \to [0,1]$ is said to be a Q-fuzzy

subset of S and $\phi_l = \{(r,q) \in S_R \times Q \mid \phi(r,q) \ge l\}$ where $l \in [0,1]$ is its level subset.

3. Rough Q-neutrosophic ideals of semiring.

In this section rough Q-neutrosophic ideals of semiring is introduced and related results are investigated.

Definition 3.1:

A neutrosophic set κ is said to be rough Q-neutrosophic left ideal of semiring S_R if it is both lower rough neutrosophic left ideal and upper rough neutrosophic left ideal of S_R.

Definition 3.2:

A neutrosophic set κ is said to be lower(upper) rough Q-neutrosophic left ideal of S_R if its lower(upper) approximation is also an rough Q-neutrosophic left ideal of S_R.

Definition 3.3:

Let κ be a subset of S_R, then κ is called a lower (upper) Q-neutrosophic left ideal of S_R if

$$(i)\underline{\varphi}(\kappa_t(\alpha+\beta,q)) \geq \underset{\alpha,\beta\in S_R,q\in Q}{\wedge} \min\{\underline{\varphi}(\kappa_t(\alpha,q)),\underline{\varphi}(\kappa_t(\beta,q))\}, \underline{\varphi}(\kappa_t(\alpha\beta,q)) \geq \underline{\varphi}(\kappa_t(\beta,q))$$
$$(ii)\underline{\varphi}(\kappa_i(\alpha+\beta,q)) \leq \underset{\alpha,\beta\in S_R,q\in Q}{\vee} \max\left\{\frac{\underline{\varphi}(\kappa_i(\alpha,q))+\underline{\varphi}(\kappa_t(\beta,q))}{2}, \underline{\varphi}(\kappa_i(\alpha\beta,q))\right\} \leq \underline{\varphi}(\kappa_i(\beta,q))$$
$$(iii)\underline{\varphi}(\kappa_f(\alpha+\beta,q)) \leq \underset{\alpha,\beta\in S_R,q\in Q}{\vee} \max\{\underline{\varphi}(\kappa_f(\alpha,q)),\underline{\varphi}(\kappa_f(\beta,q))\}, \underline{\varphi}(\kappa_f(\alpha\beta,q)) \leq \underline{\varphi}(\kappa_f(\beta,q))$$
$$\forall \alpha,\beta\in S_R, q\in Q.$$

Definition 3.4:

The intersection of any two lower(upper) QNS κ and $\kappa^{'}$ of $S_R\times Q$, is defined by,

$$(i)\underline{\varphi}(\kappa_t\cap\kappa_t^{'})(\alpha,q) = \underset{\alpha\in S_R,q\in Q}{\wedge} \min\{\underline{\varphi}((\kappa_t(\alpha,q)),\underline{\varphi}((\kappa_t^{'})(\alpha,q))\}$$

$$(ii)\underline{\varphi}(\kappa_i\cap\kappa_i^{'})(\alpha,q) = \underset{\alpha\in S_R,q\in Q}{\vee} \max\{\underline{\varphi}((\kappa_i(\alpha,q)),\underline{\varphi}((\kappa_i^{'})(\alpha,q))\}$$
$$(iii)\underline{\varphi}(\kappa_f\cap\kappa_f^{'})(\alpha,q) = \underset{\alpha\in S_R,q\in Q}{\vee} \max\{\underline{\varphi}((\kappa_f(\alpha,q)),\underline{\varphi}((\kappa_f^{'})(\alpha,q))\}$$
$$\forall \alpha\in S_R, q\in Q.$$

Proposition 3.5:

Intersection of any number of lower(upper) rough Q-neutrosophic left ideal of S_R is also a lower(upper) rough Q-neutrosophic left ideal.

Proof:

Let $\{\varphi(\kappa^c):c\in C\}$ be a collection of rough Q-neutrosophic left ideal of S_R and $\alpha,\beta\in S_R, q\in Q$. Then

$$\begin{aligned}\underline{\varphi}(\bigcap_{c\in C}\kappa_t^c)(\alpha+\beta,q) &= \underset{\alpha,\beta\in S_R,q\in Q}{\wedge} \inf_{c\in C}\{\underline{\varphi}(\kappa_t^c)(\alpha+\beta,q)\}\} \geq \underset{\alpha,\beta\in S_R,q\in Q}{\wedge} \inf_{c\in C}\{\min\{\underline{\varphi}(\kappa_t^c)(\alpha,q),\underline{\varphi}(\kappa_t^c)(\beta,q)\}\} \\ &= \underset{\alpha,\beta\in S_R,q\in Q}{\wedge} \min\{\inf_{c\in C}\{\underline{\varphi}(\kappa_t^c)(\alpha,q),\inf_{c\in C}\underline{\varphi}(\kappa_t^c)(\beta,q)\}\} \\ &= \underset{\alpha,\beta\in S_R,q\in Q}{\wedge} \min\{\bigcap_{c\in C}\{\underline{\varphi}(\kappa_t^c)(\alpha,q),\bigcap_{c\in C}\underline{\varphi}(\kappa_t^c)(\beta,q)\}\}\end{aligned}$$

$$\begin{aligned}\underline{\varphi}(\bigcap_{c\in C}\kappa_i^c)(\alpha+\beta,q) &= \underset{\alpha,\beta\in S_R,q\in Q}{\vee} \sup_{c\in C}\{\underline{\varphi}(\kappa_i^c)(\alpha+\beta,q)\}\} \leq \underset{\alpha,\beta\in S_R,q\in Q}{\vee} \sup_{c\in C}\frac{\underline{\varphi}(\kappa_i^c)(\alpha,q)+\underline{\varphi}(\kappa_i^c)(\beta,q)}{2} \\ &= \underset{\alpha,\beta\in S_R,q\in Q}{\vee} \frac{\sup_{c\in C}\underline{\varphi}(\kappa_i^c)(\alpha,q)+\sup_{c\in C}\underline{\varphi}(\kappa_i^c)(\beta,q)}{2} \\ &= \underset{\alpha,\beta\in S_R,q\in Q}{\vee} \frac{\bigcap_{c\in C}\underline{\varphi}(\kappa_i^c)(\alpha,q)+\bigcap_{c\in C}\underline{\varphi}(\kappa_i^c)(\beta,q)}{2}\end{aligned}$$

$$\underline{\varphi}(\bigcap_{c\in C}\kappa_f^c)(\alpha+\beta,q)=\bigvee_{\alpha,\beta\in S_R,q\in Q}\sup_{c\in C}\{\underline{\varphi}(\kappa_f^c)(\alpha+\beta,q)\}\}\leq\bigvee_{\alpha,\beta\in S_R,q\in Q}\sup_{c\in C}\{\min\{\underline{\varphi}(\kappa_f^c)(\alpha,q),\underline{\varphi}(\kappa_f^c)(\beta,q)\}\}$$
$$=\bigvee_{\alpha,\beta\in S_R,q\in Q}\max\{\sup_{c\in C}\{\underline{\varphi}(\kappa_f^c)(\alpha,q),\sup_{c\in C}\underline{\varphi}(\kappa_f^c)(\beta,q)\}\}$$
$$=\bigvee_{\alpha,\beta\in S_R,q\in Q}\min\{\bigcap_{c\in C}\{\underline{\varphi}(\kappa_f^c)(\alpha,q),\bigcap_{c\in C}\underline{\varphi}(\kappa_f^c)(\beta,q)\}\}$$

$$\underline{\varphi}(\bigcap_{c\in C}\kappa_t^c)(\alpha\beta,q)=\bigwedge_{\alpha\beta\in S_R,q\in Q}\inf_{c\in C}\{\underline{\varphi}(\kappa_t^c)(\alpha\beta,q)\}\}\geq\bigwedge_{\alpha\beta\in S_R,q\in Q}\inf_{c\in C}\{\underline{\varphi}(\kappa_t^c)(\beta,q)\}=\bigwedge_{\alpha\beta\in S_R,q\in Q}\bigcap_{c\in C}\{\underline{\varphi}(\kappa_t^c)(\beta,q)\}$$
$$\underline{\varphi}(\bigcap_{c\in C}\kappa_i^c)(\alpha\beta,q)=\bigvee_{\alpha\beta\in S_R,q\in Q}\sup_{c\in C}\{\underline{\varphi}(\kappa_i^c)(\alpha\beta,q)\}\}\leq\bigvee_{\alpha\beta\in S_R,q\in Q}\sup_{c\in C}\{\underline{\varphi}(\kappa_i^c)(\beta,q)\}=\bigvee_{\alpha\beta\in S_R,q\in Q}\bigcap_{c\in C}\{\underline{\varphi}(\kappa_i^c)(\beta,q)\}$$
$$\underline{\varphi}(\bigcap_{c\in C}\kappa_f^c)(\alpha\beta,q)=\bigvee_{\alpha\beta\in S_R,q\in Q}\sup_{c\in C}\{\underline{\varphi}(\kappa_f^c)(\alpha\beta,q)\}\}\leq\bigvee_{\alpha\beta\in S_R,q\in Q}\sup_{c\in C}\{\underline{\varphi}(\kappa_f^c)(\beta,q)\}=\bigvee_{\alpha\beta\in S_R,q\in Q}\bigcap_{c\in C}\{\underline{\varphi}(\kappa_f^c)(\beta,q)\}$$

Hence $\underline{\varphi}(\bigcap_{c\in C}\kappa_f^c)$ is a lower(upper) rough QNLIof S_R.

Definition 3.6:

The Cartesian product of any two lower(upper) QNS κ and $\kappa^{'}$ of $S_R\times Q,$ is defined by,

$$(i)\underline{\varphi}(\kappa_t\times\kappa_t^{'})((\alpha,\beta),q)=\bigwedge_{(\alpha,\beta)\in S_R,q\in Q}\min\{\underline{\varphi}((\kappa_t(\alpha,q)),\underline{\varphi}((\kappa_t^{'})(\beta,q))\}$$
$$(ii)\underline{\varphi}(\kappa_i\times\kappa_i^{'})((\alpha,\beta),q)=\bigvee_{(\alpha,\beta)\in S_R,q\in Q}\max\frac{\{\underline{\varphi}((\kappa_i(\alpha,q))+\underline{\varphi}((\kappa_i^{'})(\beta,q))\}}{2}$$
$$(iii)\underline{\varphi}(\kappa_f\times\kappa_f^{'})((\alpha,\beta),q)=\bigvee_{(\alpha,\beta)\in S_R,q\in Q}\max\{\underline{\varphi}((\kappa_f(\alpha,q)),\underline{\varphi}((\kappa_f^{'})(\beta,q))\}$$
$$\forall\alpha,\beta\in S_R,q\in Q.$$

Theorem 3.7:

For any two lower(upper) QNLI κ and $\kappa^{'}$ of $S_R,\kappa\times\kappa^{'}$ is also a upper(lower) rough QNLIof $S_R\times S_R$.

Proof:

$\forall(s,t),(u,v)\in S_R\times S_R;q\in Q$

Now

$$\underline{\varphi}\{(\kappa_t\times\kappa_t^{'})((s,t)+(u,v),q)=\underline{\varphi}\{(\kappa_t\times\kappa_t^{'})((s+u,t+v),q)\}$$

$$=\bigwedge_{(s,t,u,v)\in S_R,q\in Q}\min\{\underline{\varphi}(\kappa_t(s+u),q),\underline{\varphi}(\kappa_t^{'}(t+v),q)\}$$
$$\geq\bigwedge_{(s,t,u,v)\in S_R,q\in Q}\min\{\min\{\underline{\varphi}(\kappa_t(s,q),\underline{\varphi}(\kappa_t(u,q)\},\min\{\underline{\varphi}(\kappa_t^{'}(t,q),\underline{\varphi}(\kappa_t^{'}(v,q)\}$$
$$=\bigwedge_{(s,t,u,v)\in S_R,q\in Q}\min\{\min\{\underline{\varphi}(\kappa_t(s,q),\underline{\varphi}(\kappa_t^{'}(t,q)\},\min\{\underline{\varphi}(\kappa_t(u,q),\underline{\varphi}(\kappa_t^{'}(v,q)\}$$
$$=\bigwedge_{(s,t,u,v)\in S_R,q\in Q}\min\{\underline{\varphi}(\kappa_t\times\kappa_t^{'})((s,t),q)\},\min\{\underline{\varphi}(\kappa_t\times\kappa_t^{'})((u,v),q)\}$$

$$\underline{\varphi}\{(\kappa_i\times\kappa_i^{'})((s,t)+(u,v),q)=\underline{\varphi}\{(\kappa_i\times\kappa_i^{'})((s+u),(t+v),q)\}$$

$$= \vee_{(s,t,u,v)\in S_R, q\in Q} \max \frac{\{\underline{\varphi}(\kappa_i(s+u),q)+\underline{\varphi}(\kappa_i^{'}(t+v),q)\}}{2}$$
$$\leq \vee_{(s,t,u,v)\in S_R, q\in Q} \frac{1}{2}\max\left\{\frac{\underline{\varphi}(\kappa_i(s,q)+\underline{\varphi}(\kappa_i(u,q)}{2}+\frac{\underline{\varphi}(\kappa_i^{'}(t,q)+\underline{\varphi}(\kappa_i^{'}(v,q)}{2}\right\}$$
$$=\frac{1}{2} \vee_{(s,t,u,v)\in S_R, q\in Q} \max\left\{\frac{\{\underline{\varphi}(\kappa_i(s,q),+(\kappa_i^{'}(t,q)\}}{2}+\frac{\{\underline{\varphi}(\kappa_i(u,q)+\underline{\varphi}(\kappa_i^{'}(v,q)\}}{2}\right\}$$
$$=\frac{1}{2} \vee_{(s,t,u,v)\in S_R, q\in Q} \max\{\underline{\varphi}(\kappa_i\times\kappa_i^{'})((s,t),q)\}+\{\underline{\varphi}(\kappa_i\times\kappa_i^{'})((u,v),q)\}$$

$$\underline{\varphi}\{(\kappa_f\times\kappa_f^{'})((s,t)+(u,v),q)=\underline{\varphi}\{(\kappa_f\times\kappa_f^{'})((s+u,t+v),q)\}$$

$$= \vee_{(s,t,u,v)\in S_R, q\in Q} \max\{\underline{\varphi}(\kappa_f(s+u),q),\underline{\varphi}(\kappa_f^{'}(t+v),q)\}$$
$$\leq \vee_{(s,t,u,v)\in S_R, q\in Q} \max\{\max\{\underline{\varphi}(\kappa_f(s,q),\underline{\varphi}(\kappa_f(u,q)\},\max\{\underline{\varphi}(\kappa_f^{'}(t,q),\underline{\varphi}(\kappa_f^{'}(v,q)\}$$
$$= \vee_{(s,t,u,v)\in S_R, q\in Q} \max\{\max\{\underline{\varphi}(\kappa_f(s,q),\underline{\varphi}(\kappa_f^{'}(t,q)\},\max\{\underline{\varphi}(\kappa_f(u,q),\underline{\varphi}(\kappa_f^{'}(v,q)\}$$
$$= \vee_{(s,t,u,v)\in S_R, q\in Q} \max\{\underline{\varphi}(\kappa_f\times\kappa_f^{'})((s,t),q)\},\max\{\underline{\varphi}(\kappa_f\times\kappa_f^{'})((u,v),q)\}$$

$$\underline{\varphi}\{(\kappa_t\times\kappa_t^{'})((s,t)(u,v),q)=\underline{\varphi}\{(\kappa_t\times\kappa_t^{'})((su,tv),q)\}$$

$$= \wedge_{(su,tv)\in S_R, q\in Q} \min\{\underline{\varphi}(\kappa_t(su),q),\underline{\varphi}(\kappa_t^{'}(tv),q)\}$$
$$\geq \wedge_{(u,v)\in S_R, q\in Q} \min\{\underline{\varphi}(\kappa_t(u,q)),\underline{\varphi}(\kappa_t^{'}(v,q))\}$$
$$= \wedge_{(u,v)\in S_R, q\in Q} \underline{\varphi}(\kappa_t\times\kappa_t^{'})((u,v),q)$$

$$\underline{\varphi}\{(\kappa_i\times\kappa_i^{'})((s,t)(u,v),q)=\underline{\varphi}\{(\kappa_i\times\kappa_i^{'})(su,tv,q)\}$$

$$= \vee_{(s,t,u,v)\in S_R, q\in Q} \max \frac{\{\underline{\varphi}(\kappa_i(su),q)+\underline{\varphi}(\kappa_i^{'}(tv),q)\}}{2}$$
$$\leq \vee_{(u,v)\in S_R, q\in Q} \max\left\{\frac{\underline{\varphi}(\kappa_i(u,q)+\underline{\varphi}(\kappa_i^{'}(v,q)}{2}\right\}$$
$$= \vee_{(u,v)\in S_R, q\in Q} \underline{\varphi}(\kappa_i\times\kappa_i^{'})((u,v),q)$$

$$\underline{\varphi}\{(\kappa_f\times\kappa_f^{'})((s,t)(u,v),q)=\underline{\varphi}\{(\kappa_f\times\kappa_f^{'})((su,tv),q)\}$$

$$= \wedge_{(su,tv)\in S_R, q\in Q} \max\{\underline{\varphi}(\kappa_f(su),q),\underline{\varphi}(\kappa_f^{'}(tv),q)\}$$
$$\leq \wedge_{(u,v)\in S_R, q\in Q} \max\{\underline{\varphi}(\kappa_f(u,q)),\underline{\varphi}(\kappa_f^{'}(v,q))\}$$
$$= \wedge_{(u,v)\in S_R, q\in Q} \underline{\varphi}(\kappa_f\times\kappa_f^{'})((u,v),q)$$

Hence $\kappa\times\kappa^{'}$ is a lower(upper) rough QNLIof $S_R\times S_R$.

Definition 3.8:

Composition for any two lower (upper) QN set κ and κ' of S_R is defined by,

$$(i)\underline{\varphi}((\kappa_t \circ \kappa_t')(\alpha,q)) = \begin{cases} \bigwedge_{(\alpha_c,\beta_c)\in S_R, p\in Q} \sup_{\alpha=\sum_{c=1}^{n}\alpha_c,\beta_c} \min_c\{\underline{\varphi}((\kappa_t(\alpha_c,q)),\underline{\varphi}((\kappa_t')(\beta_c,q))\} \\ 0, otherwise. \end{cases}$$

$$(ii)\underline{\varphi}((\kappa_i \circ \kappa_i')(\alpha,q)) = \begin{cases} \bigvee_{(\alpha_c,\beta_c)\in S_R, q\in Q} \inf_{\alpha=\sum_{c=1}^{n}\alpha_c,\beta_c} \max \sum_{c=1}^{n} \frac{\{\underline{\varphi}((\kappa_i(\alpha_c,q)) + \underline{\varphi}((\kappa_i')(\beta_c,q))\}}{2n} \\ 0, otherwise. \end{cases}$$

$$(iii)\underline{\varphi}((\kappa_f \circ \kappa_f')(\alpha,q)) = \begin{cases} \bigvee_{(\alpha_c,\beta_c)\in S_R, q\in Q} \inf_{\alpha=\sum_{c=1}^{n}\alpha_c,\beta_c} \max_c\{\underline{\varphi}((\kappa_f(\alpha_c,q)),\underline{\varphi}((\kappa_f')(\beta_c,q))\} \\ 0, otherwise. \end{cases}$$

$\forall \alpha_c, \beta_c \in S_R, q \in Q, n \in N.$

Theorem 3.9:

For any two lower (upper) QNLI κ and κ' of S_R, is also a upper (lower) rough QNLI of S_R.

Proof:

Assume that $(\alpha+\beta) = (\sum_{c=1}^{n}\alpha_c\beta_c, q)$, Then

$$\underline{\varphi}((\kappa_t \circ \kappa_t')(\alpha+\beta,q)) = \bigwedge_{(\alpha_c,\beta_c)\in S_R, p\in Q} \sup_{\alpha+\beta=\sum_{c=1}^{n}\alpha_c\beta_c} \min_c\{\underline{\varphi}((\kappa_t(\alpha_c,q)),\underline{\varphi}((\kappa_t'(\beta_c,q))\}$$

$$\geq \bigwedge_{(\alpha_c,\beta_c)\in S_R, p\in Q} \sup_{\alpha=\sum_{c=1}^{n}a_cb_c, \beta=\sum_{c=1}^{n}c_cd_c} \min_c\{\underline{\varphi}((\kappa_t(a_c,q)),\underline{\varphi}((\kappa_t'(b_c,q)),\underline{\varphi}((\kappa_t(c_c,q)),\underline{\varphi}((\kappa_t'(d_c,q))\}$$

$$= \bigwedge_{(\alpha_c,\beta_c)\in S_R, p\in Q} \min\{ \sup_{\alpha=\sum_{c=1}^{n}a_cb_c} \{\min_c\{\underline{\varphi}((\kappa_t(a_c,q)),\underline{\varphi}((\kappa_t'(b_c,q))\}\}, \sup_{\beta=\sum_{c=1}^{n}c_cd_c} \{\min_c \underline{\varphi}((\kappa_t(c_c,q)),\underline{\varphi}((\kappa_t'(d_c,q))\}\}$$

$$= \bigwedge_{(\alpha_c,\beta_c)\in S_R, p\in Q} \min\{(\underline{\varphi}(\kappa_t)\circ\underline{\varphi}(\kappa_t'))(\alpha,q),(\underline{\varphi}(\kappa_t)\circ\underline{\varphi}(\kappa_t'))(\beta,q)\}$$

$$\underline{\varphi}((\kappa_i \circ \kappa_i')(\alpha+\beta,q)) = \bigvee_{(\alpha_c,\beta_c)\in S_R, p\in Q} \inf_{\alpha+\beta=\sum_{c=1}^{n}\alpha_c\beta_c} \max_c \sum_{c=1}^{n} \frac{\{\underline{\varphi}((\kappa_i(\alpha_c,q)) + \underline{\varphi}((\kappa_i')(\beta_c,q))\}}{2n}$$

$$\leq \bigvee_{(\alpha_c,\beta_c)\in S_R, p\in Q} \inf_{\alpha=\sum_{c=1}^{n}a_cb_c, \beta=\sum_{c=1}^{n}c_cd_c} \sum_{c=1}^{m} \frac{\{\underline{\varphi}((\kappa_t(a_c,q)) + \underline{\varphi}((\kappa_i'(b_c,q)) + \underline{\varphi}((\kappa_i(c_c,q)) + \underline{\varphi}((\kappa_i'(d_c,q))\}}{2m}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R, p\in Q} \{ \inf_{\alpha=\sum_{c=1}^{n}a_cb_c} \{\max_c \frac{\{\underline{\varphi}((\kappa_i(a_c,q)),\underline{\varphi}((\kappa_i'(b_c,q))\}}{2m}\}, \sup_{\beta=\sum_{c=1}^{n}c_cd_c} \{\frac{\min_c \underline{\varphi}((\kappa_i(c_c,q)),\underline{\varphi}((\kappa_i'(d_c,q))\}}{2m}\}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R, p\in Q} \frac{\{(\underline{\varphi}(\kappa_i)\circ\underline{\varphi}(\kappa_i'))(\alpha,q) + (\underline{\varphi}(\kappa_i)\circ\underline{\varphi}(\kappa_i'))(\beta,q)\}}{2}$$

$$\underline{\varphi}((\kappa_f \circ \kappa_f^{'})(\alpha+\beta,q)) = \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \inf_{\alpha+\beta=\sum_{c=1}^{n}\alpha_c\beta_c} \max_c\{\underline{\varphi}((\kappa_f(\alpha_c,q)),\underline{\varphi}((\kappa_f^{'}(\beta_c,q))\}$$

$$\leq \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \inf_{\alpha=\sum_{c=1}^{n}a_cb_c,\,\beta=\sum_{c=1}^{n}c_cd_c} \max_c\{\underline{\varphi}((\kappa_f(a_c,q)),\underline{\varphi}((\kappa_f^{'}(b_c,q)),\underline{\varphi}((\kappa_f(c_c,q)),\underline{\varphi}((\kappa_f^{'}(d_c,q))\}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \max\{ \inf_{\alpha=\sum_{c=1}^{n}a_cb_c} \{\max_c\{\underline{\varphi}((\kappa_f(a_c,q)),\underline{\varphi}((\kappa_f^{'}(b_c,q))\}\}, \inf_{\beta=\sum_{c=1}^{n}c_cd_c} \{\max_c \underline{\varphi}((\kappa_f(c_c,q)),\underline{\varphi}((\kappa_f^{'}(d_c,q))\}\}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \max\{(\underline{\varphi}(\kappa_f)\circ\underline{\varphi}(\kappa_f^{'}))(\alpha,q),(\underline{\varphi}(\kappa_f)\circ\underline{\varphi}(\kappa_f^{'}))(\beta,q)\}$$

$$\underline{\varphi}((\kappa_t \circ \kappa_t^{'})(\alpha\beta,q)) = \bigwedge_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \sup_{\alpha\beta=\sum_{c=1}^{n}\alpha_c\beta_c} \min_c\{\underline{\varphi}((\kappa_t(\alpha_c,q)),\underline{\varphi}((\kappa_t^{'}(\beta_c,q))\}$$

$$\geq \bigwedge_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \sup_{\alpha\beta=\sum_{c=1}^{n}\alpha a_cb_c} \min_c\{\underline{\varphi}((\kappa_t(\alpha a_c,q)),\underline{\varphi}((\kappa_t^{'}(b_c,q))\}$$

$$= \bigwedge_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \{ \sup_{\gamma=\sum_{c=1}^{n}a_cb_c} \{\min_c\{\underline{\varphi}((\kappa_t(a_c,q)),\underline{\varphi}((\kappa_t^{'}(b_c,q))\}\}$$

$$= \bigwedge_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \{(\underline{\varphi}(\kappa_t)\circ\underline{\varphi}(\kappa_t^{'}))(\gamma,q)\}$$

$$\underline{\varphi}((\kappa_i \circ \kappa_i^{'})(\alpha\beta,q)) = \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \inf_{\alpha\beta=\sum_{c=1}^{n}\alpha_c\beta_c} \max_c \sum_{c=1}^{n} \frac{\{\underline{\varphi}((\kappa_i(\alpha_c,q))+\underline{\varphi}((\kappa_i^{'})(\beta_c,q))\}}{2n}$$

$$\leq \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \inf_{\alpha\beta=\sum_{c=1}^{n}\alpha a_cb_c} \sum_{c=1}^{n}\left\{\frac{\{\underline{\varphi}(\kappa_i(\alpha a_c,q))+\underline{\varphi}(\kappa_i^{'}(b_c,q))\}}{2n}\right\}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \inf_{\gamma=\sum_{c=1}^{n}a_cb_c} \max_c\left\{\frac{\{\underline{\varphi}((\kappa_i(a_c,q)),\underline{\varphi}((\kappa_i^{'}(b_c,q))\}}{2n}\right\}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} (\underline{\varphi}(\kappa_i)\circ\underline{\varphi}(\kappa_i^{'}))(\gamma,q)$$

$$\underline{\varphi}((\kappa_f \circ \kappa_f^{'})(\alpha\beta,q)) = \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \inf_{\alpha\beta=\sum_{c=1}^{n}\alpha_c\beta_c} \max_c\{\underline{\varphi}((\kappa_f(\alpha_c,q)),\underline{\varphi}((\kappa_f^{'}(\beta_c,q))\}$$

$$\leq \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \inf_{\alpha\beta=\sum_{c=1}^{n}\alpha a_cb_c} \max_c\{\underline{\varphi}((\kappa_f(\alpha a_c,q)),\underline{\varphi}((\kappa_f^{'}(b_c,q))\}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \{ \inf_{\gamma=\sum_{c=1}^{n}a_cb_c} \{\max_c\{\underline{\varphi}((\kappa_f(a_c,q)),\underline{\varphi}((\kappa_f^{'}(b_c,q))\}\}$$

$$= \bigvee_{(\alpha_c,\beta_c)\in S_R,\, p\in Q} \{(\underline{\varphi}(\kappa_f)\circ\underline{\varphi}(\kappa_f^{'}))(\gamma,q)\}$$

Therefore, $\underline{\varphi}(\kappa_f \circ \kappa_f^{'})$ of S_R, is a upper (lower) rough QNLIof S_R.

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