Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 10, October 2021: 4325-4332

Research Article

2-Dominating Sets And 2-Domination Polynomials of Cycles

P. C. Priyanka Nair[1] and T. Anitha Baby[2]

[1]Research Scholar,
Department of Mathematics, Women's Christian College, Nagercoil, Tamilnadu, India.
[2]Assistant Professor,
Department of Mathematics, Women's Christian College, Nagercoil, Tamilnadu, India.

E-mail: priyanka86nair@gmail.com

E-mail: anithasteve@gmail.com

[Affiliated to Manonmaniam Sundaranar University, Abishekapatti,Tirunelveli-627 012, Tamil Nadu, India.]

ABSTRACT

Let G be a simple connected graph of order m. Let $D_2(G, i)$ be the family of 2-dominating sets in G with cardinality i. The polynomial $D_2(G, x) = \sum_{i=\gamma_2(G)}^{m} d_2(G, i)x^i$ is called the 2-domination polynomial of G. In this paper we obtain a recursive formula for $d_2(C_m, i)$. Using this recursive formula we construct the 2-domination polynomial, $D_2(C_m, x) = \sum_{i=\left\lceil\frac{m+1}{2}\right\rceil}^{m} d_2(C_m, i)x^i$, where $d_2(C_m, i)$ is the number of 2-dominating sets of C_m of cardinality i and some properties of this polynomial have been studied.

Keywords: Cycle, 2-dominating set, 2-domination number, 2-domination polynomial.

I. INTRODUCTION

Let G = (V, E) be a simple graph of order m. For any vertex v∈V, the open neighbourhood of V is the set N(v) = {u∈V / uv∈E} and the closed neighbourhood of V is the set N[v] = N(v) ∪ {v}. For a set S ⊆V, the open neighbourhood of S is $N(S) = \bigcup_{v\in S} N(v)$ and the closed neighbourhood of S is N[S] = N(S) ∪ S.

A set D ⊆V is a dominating set of G if N[D] = V or equivalently, every vertex in V − D is adjacent to at least one vertex in D.

The domination number of a graph G is defined as the minimum cardinality taken over all dominating sets D of vertices in G and is denoted by γ(G).

We use the notation $\lceil x\rceil$ for the smallest integer greater than or equal to x and $\lfloor x\rfloor$ for the largest integer less than or equal to x. Also, we denote the set {1, 2, 3 … m} by [m], throughout this paper.

II. 2-DOMINATING SETS OF CYCLES

In this section, we state the 2-domination number of cycle and some of its properties

Definition 2.1:

Let G be a simple graph of order m with no isolated vertices. A subset $D\subseteq V$ is a 2-dominating set of the graph G, if every vertex $v\in V-D$ is adjacent to at least 2 vertices in D. The minimum cardinality taken over all 2-dominating sets of G is called the 2-domination number and is denoted by $\gamma_2(G)$.

Lemma 2.2:

Let C_m be the cycle with m vertices, then its 2-domination number is $\gamma_2(C_m)=\left\lceil\frac{m}{2}\right\rceil$.

Lemma 2.3:

Let C_m, $m\geq 5$ be the cycle with $|V(C_m)| = m$. Then, $d_2(C_m, i) = 0$ if $i < \left\lceil\frac{m}{2}\right\rceil$ or $i > m$ and $d_2(C_m, i) > 0$ if $\left\lceil\frac{m}{2}\right\rceil \leq i \leq m$.

Proof:

If $i < \left\lceil\frac{m}{2}\right\rceil$ or $i > m$, then there is no 2-dominating set of cardinality i.

Therefore, $D_2(C_m, i) = \phi$.

By Lemma 2.2, the cardinality of the minimum 2-dominating set is $\left\lceil\frac{m}{2}\right\rceil$.

Therefore, $d_2(C_m, i) > 0$ if $i \geq \left\lceil\frac{m}{2}\right\rceil$ and $i \leq m$.

Hence, we have, $d_2(C_m, i) = 0$ if $i < \left\lceil\frac{m}{2}\right\rceil$ or $i > m$ and $d_2(C_m, i) > 0$ if $\left\lceil\frac{m}{2}\right\rceil \leq i \leq m$.

Lemma 2.4 :

Let C_m, $m \geq 5$ be the cycle with m vertices.

(i) If $D_2(C_{m-1}, i-1) = \phi$ and $D_2(C_{m-3}, i-1) = \phi$, then $D_2(C_{m-2}, i-1) = \phi$.

(ii) If $D_2(C_{m-1}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) \neq \phi$, then $D_2(C_{m-2}, i-1) \neq \phi$.

(iii) If $D_2(C_{m-1}, i-1) = \phi$ and $D_2(C_{m-2}, i-1) = \phi$, then $D_2(C_m, i) = \phi$.

(iv) If $D_2(C_{m-1}, i-1) \neq \phi$ and $D_2(C_{m-2}, i-1) \neq \phi$, then $D_2(C_m, i) \neq \phi$.

Proof:

(i) Let $D_2(C_{m-1}, i-1) = \phi$ and $D_2(C_{m-3}, i-1) = \phi$.

Then by Lemma 2.3,

$$i-1 > m-1 \text{ or } i-1 < \left\lceil\frac{m-1}{2}\right\rceil \text{ and } i-1 > m-3 \text{ or } i-1 < \left\lceil\frac{m-3}{2}\right\rceil.$$

Therefore, $i-1 > m-1$ or $i-1 < \left\lceil\frac{m-3}{2}\right\rceil$.

Hence, $i-1 > m-2$ or $i-1 < \left\lceil\frac{m-2}{2}\right\rceil$ holds.

Hence, $D_2(C_{m-2}, i-1) = \phi$.

(ii) Suppose that $D_2(C_{m-2}, i-1) = \phi$.

by Lemma 2.3, then

we have, $i-1 > m-2$ or $i-1 < \left\lceil\frac{m-2}{2}\right\rceil$.

If $i-1 > m-2$, then $i-1 > m-3$.

Therefore, $D_2(C_{m-3}, i-1) = \phi$, a contradiction.

If $i-1 < \left\lceil\frac{m-2}{2}\right\rceil$, then $i-1 < \left\lceil\frac{m-1}{2}\right\rceil$ holds.

Therefore, $d_2(C_{m-1}, i-1) = \phi$, a contradiction.

Hence, $D_2(C_{m-2}, i-1) \neq \phi$.
(iii) Since $D_2(C_{m-1}, i-1) = \phi$ and $D_2(C_{m-2}, i-1) = \phi$,
by Lemma 2.3,

$$i-1 > m-1 \text{ or } i-1 < \left\lceil \frac{m-1}{2} \right\rceil \text{ and } i-1 > m-2 \text{ or } i-1 < \left\lceil \frac{m-2}{2} \right\rceil.$$

Therefore, $i-1 > m-1$ or $i-1 < \left\lceil \frac{m-2}{2} \right\rceil$.

Therefore, $i > m$ or $i < \left\lceil \frac{m}{2} \right\rceil$.

Hence, $d_2(C_m, i) = \phi$.
(iv) By hypothesis,

$$\left\lceil \frac{m-1}{2} \right\rceil \leq i-1 \leq m-1 \text{ and } \left\lceil \frac{m-2}{2} \right\rceil \leq i-1 \leq m-2.$$

Therefore, $\left\lceil \frac{m-2}{2} \right\rceil \leq i-1 \leq m-1$.

Therefore, $\left\lceil \frac{m}{2} \right\rceil \leq i \leq m$ holds.

Hence, $D_2(C_m, i) \neq \phi$.

Lemma 2.5 :
If $D_2(C_m, i) \neq \phi$, then for every $m \geq 5$, we have,
(i) $D_2(C_{m-1}, i-1) = \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) \neq \phi$ iff $m = 2k-1$ and $i = k$ for some $k \geq 3$.
(ii) $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) = \phi$ and $D_2(C_{m-3}, i-1) = \phi$ iff $i = m$.
(iii) $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-3},i-1) = \phi$ iff $i = m-1$.
(iv) $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) \neq \phi$ iff $\left\lceil \frac{m-1}{2} \right\rceil+1 \leq i \leq m-2$

Proof:
(i) Assume $D_2(C_{m-1}, i-1) = \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-2}, i-1) \neq \phi$.
Since, $D_2(C_m, i-1) = \phi$,
by Lemma 2.3, $i-1 > m-1$ or $i-1 < \left\lceil \frac{m-1}{2} \right\rceil$.
If $i-1 > m-1$, then $i > m$, which implies $D_2(C_m, i) = \phi$, which is a contradiction.
Therefore, $i-1 < \left\lceil \frac{m-1}{2} \right\rceil$.
That is, $i \leq \left\lceil \frac{m-1}{2} \right\rceil$ ----------------- (1)
Since, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) \neq \phi$, we have

$$\left\lceil \frac{m-2}{2} \right\rceil \leq i-1 \leq m-2 \text{ and } \left\lceil \frac{m-3}{2} \right\rceil \leq i-1 \leq m-3.$$

Therefore, $\left\lceil \frac{m-2}{2} \right\rceil \leq i-1 \leq m-3$.

Therefore, $\left\lceil \frac{m}{2} \right\rceil \leq i$ ---------------- (1)

From (1) and (2)
We get $\left\lceil \frac{m}{2} \right\rceil \leq i \leq \left\lceil \frac{m-1}{2} \right\rceil$
This inequality is true only when $m = 2k-1$ and $i = k$ for some $k \in N$ and $k \geq 3$.
Conversely, assume that $m = 2k-1$ and $i = k$
Therefore, $m-1 = 2k-2$ and $i-1 = k-1$.
Therefore, $k = \frac{m+1}{2}$
We have, $i-1 = k-1$

$$= \frac{m-1}{2} < \left\lceil \frac{m-1}{2} \right\rceil$$

Therefore, $D_2(C_{m-1}, i-1) = \phi$.

Also, $D_2(C_{m-2}, i-1) = D_2(C_{2k-3}, k-1) \neq \phi$, since, $\left\lceil\frac{2k-3}{2}\right\rceil = \lceil k-3/2 \rceil = k-1$.

$D_2(C_{m-3}, i-1) = D_2(C_{2k-5}, k-1) \neq \phi$, since, $\left\lceil\frac{2k-5}{2}\right\rceil = \left\lceil k-{}^{5}\!/_{2}\right\rceil = k-2$.

(ii) Assume $D_2(C_{m-2}, i-1) = \phi$ and $D_2(C_{m-3}, i-1) = \phi$ and $D_2(C_{m-1}, i-1) \neq \phi$.
Since, $D_2(C_{m-2}, i-1) = \phi$ and $D_2(C_{m-3}, i-1) = \phi$,
Then by Lemma 2.3,
We have $i-1 > m-2$ or $i-1 < \left\lceil\frac{m-2}{2}\right\rceil$ and $i-1 > m-3$ or $i-1 < \left\lceil\frac{m-3}{2}\right\rceil$
Therefore, $i-1 > m-2$ or $i-1 < \left\lceil\frac{m-3}{2}\right\rceil$.
If $i-1 < \left\lceil\frac{m-3}{2}\right\rceil$, then $i-1 < \left\lceil\frac{m}{2}\right\rceil$ holds.
Therefore, by Lemma 2.3,
$D_2(C_m, i) = \phi$, which is a contradiction.
So, we have, $i-1 > m-2$
Therefore, $i \geq m$ ----------------(1)
Also, since $D_2(C_{n-1}, i-1) \neq \phi$,
We have $\left\lceil\frac{m-1}{2}\right\rceil \leq i-1 \leq m-1$
Therefore, $i \leq m$ ----------------(2)
From (1) and (2) we get, $i = m$
Conversely, if $i = m$,
$D_2(C_{m-2}, i-1) = D_2(C_{m-2}, m-1) = \phi$
$D_2(C_{m-3}, i-1) = D_2(C_{m-3}, m-1) = \phi$
$D_2(C_{m-1}, i-1) = D_2(C_{m-1}, m-1) \neq \phi$

(iii) Assume that, $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) = \phi$.
Since, $D_2(C_{m-3}, i-1) = \phi$,
then by Lemma 2.3, $i-1 > m-3$ or $i-1 < \left\lceil\frac{m-3}{2}\right\rceil$.
If $i-1 < \left\lceil\frac{m-3}{2}\right\rceil$, then $i < \left\lceil\frac{m}{2}\right\rceil$ holds.
Therefore, $D_2(C_m, i) = \phi$, which is a contradiction.
Therefore, $i-1 > m-3$.
Therefore, $i \geq m-1$ ---------------(1)
Since, $D_2(C_{m-1}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) \neq \phi$,
We have $\left\lceil\frac{m-1}{2}\right\rceil \leq i-1 \leq m-1$ and $\left\lceil\frac{m-2}{2}\right\rceil \leq i-1 \leq m-2$.
Therefore, $\left\lceil\frac{m-1}{2}\right\rceil \leq i-1 \leq m-2$.
Therefore, $i \leq m-1$ ---------------(2).
From (1) and (2) we get,
$i = m-1$.
Conversely,
suppose $i = m-1$,
Then, $D_2(C_{m-1}, i-1) = D_2(C_{m-1}, m-2) \neq \phi$,
$D_2(C_{m-2}, i-1) = D_2(C_{m-2}, m-2) \neq \phi$ and
$D_2(C_{m-3}, i-1) = D_2(C_{m-3}, m-2) = \phi$.

(iv) Assume that, $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) \neq \phi$.
Then by Lemma 2.3, we have,
$\left\lceil\frac{m-1}{2}\right\rceil \leq i-1 \leq m-1$, $\left\lceil\frac{m-2}{2}\right\rceil \leq i-1 \leq m-2$ and $\left\lceil\frac{m-3}{2}\right\rceil \leq i-1 \leq m-3$.

Therefore, $\left\lceil\frac{m-1}{2}\right\rceil \leq i-1 \leq m-3$
Hence, $\left\lceil\frac{m-1}{2}\right\rceil + 1 \leq i \leq m-2$
Conversely,
suppose $\left\lceil\frac{m-1}{2}\right\rceil + 1 \leq i \leq m-2$.
Then, $\left\lceil\frac{m-1}{2}\right\rceil \leq i-1 \leq m-3$,
Therefore, $\left\lceil\frac{m-2}{2}\right\rceil \leq i-1 \leq m-2$, $\left\lceil\frac{m-3}{2}\right\rceil \leq i-1 \leq m-3$.
From these we obtain,
$D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$, $D_2(C_{m-3}, i-1) \neq \phi$.

Theorem 2.6:
For every $m \geq 5$ and $i > \left\lceil\frac{m}{2}\right\rceil$,
(i) $D_2(C_{2m}, m) = \{1,3,5,\ldots,2m-1\} \cup \{2,4,6,\ldots,2m\}$.
(ii) If $D_2(C_{m-2}, i-1) = \phi$, $D_2(C_{m-3}, i-1) = \phi$ and $D_2(C_{m-1}, i-1) \neq \phi$,
then $D_2(C_m, i) = D_2(C_m, m) = \{1,2,3,...,m\} = [m]$.
(iii) If $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$, and $D_2(C_{m-3}, i-1) = \phi$,
then $D_2(C_m, m-1) = \{[m]-\{x\} \,/\, x \in [m]\}$.
(iv) If $D_2(C_{m-1}, i-1) = \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ then,
$D_2(C_m, i) = \{\{X \cup \{m-1\} \text{ if } m-3 \in X\} \cup \{X \cup \{m\} \text{ if } m-2 \in X\} \,/\, X \in D_2(C_{m-2}, i-1)\}$
(v) If $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) = \phi$
then, $D_2(C_m, i) = \{Y \cup \{m\} \,/\, Y \in D_2(C_{m-1}, i-1)\}$
(vi) If $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$
then, $D_2(C_m, i) = \{\{X \cup \{m-1\} \text{ if } m-3 \in X\} \cup \{X \cup \{m\}$
$\text{if } m-2 \in X\} \cup \{Y \cup \{m-1\} \text{ if } m-2 \in Y\} \cup \{Y \cup \{m\} \text{ if } m-1 \in Y\}\}$
Where $X \in D_2(C_{m-2}, i-1)$ and $Y \in D_2(C_{m-1}, i-1)$.

Proof:
(i) For every $m \geq 5$, $D_2(C_{2m}, m) = \{1,3,5,\ldots,2m-1\} \cup \{2,4,6,\ldots,2m\}$.
(ii) Since $D_2(C_{m-2}, i-1) = \phi$, $D_2(C_{m-3}, i-1) = \phi$ and $D_2(C_{m-1}, i-1) \neq \phi$,
by Lemma 2.6 (ii), $i = m$.
Therefore, $D_2(C_m, i) = D_2(C_m, m) = [m]$.
(iii) If $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ and $D_2(C_{m-3}, i-1) = \phi$.
Then, $D_2(C_m, i) = D_2(C_m, m-1) = \{[m]-\{x\}/x \in [m]\}$.
(iv) Let X be a 2-dominating set of C_{m-2} with cardinality $i-1$. All the elements of $D_2(C_{m-2}, i-1)$ end with $m-3$ or $m-2$. Therefore, $m-3 \in X$, adjoin $m-1$ with X and when $m-2 \in X$ adjoin m with X. Hence, every X of $D_2(C_{m-2}, i-1)$ belongs to $D_2(C_m, i)$ by adjoining $m-1$ and m.
(v) Let Y be a 2-dominating set of C_{m-1} with cardinality $i-1$. All the elements of $D_2(C_{m-1}, i-1)$ end with $m-1$. Adjoin m with Y. Hence, every Y of $D_2(C_{m-1}, i-1)$ belongs to $D_2(C_m, i)$ by adjoining m only.
(vi) The construction of $D_2(C_m, i)$ from $D_2(C_{m-1}, i-1)$ and $D_2(C_{m-2}, i-1)$
Let X be a 2-dominating set of C_{m-2} with cardinality $i-1$. All the elements of $D_2(C_{m-2}, i-1)$ ends with $m-3$ or $m-2$. Therefore, $m-3 \in X$, adjoin $m-1$ with X and when $m-2 \in X$ adjoin m with X. Hence, every X of $D_2(C_{m-2}, i-1)$ belongs to $D_2(C_m, i)$ by adjoining $\{m-1\}$ or $\{m\}$ only. Now let us consider $D_2(C_{m-1}, i-1)$, Here all the elements of $D_2(C_{m-1}, i-1)$ end with $m-2$ or $m-1$. Now let Y be 2-dominating set of C_{m-1} with cardinality
$m-1$.
Here all the elements of $D_2(C_{m-1}, i-1)$ ends with $m-2$ or $m-1$. Therefore, when $m-2 \in Y$, adjoin $m-1$ with Y and when $m-1 \in Y$, adjoin m with Y. Hence, every Y of $D_2(C_{m-1}, i-1)$ belongs to $D_2(C_m, i)$ by adjoining $\{m-1\}$ or $\{m\}$.

Theorem 2.7:

If $D_2(C_m, i)$ is the family of the 2-dominating sets of C_m with cardinality i, where $i \geq \left\lceil \frac{m}{2} \right\rceil$ Then $d_2(C_m, i) = d_2(C_{m-1}, i-1) + d_2(C_{m-2}, i-1)$.

Proof:

From Theorem 2.6, we consider all the four cases as given below, where $i \geq \left\lceil \frac{m}{2} \right\rceil$.

(i) If $D_2(C_{m-1}, i-1) = \phi$ and $D_2(C_{m-2}, i-1) = \phi$, then $D_2(C_m, i) = \phi$.

(ii) If $D_2(C_{m-1}, i-1) = \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ then,

$$D_2(C_m, i) = \{\{X \cup \{m-1\} \text{ if } m-3 \in X\} \cup \{X \cup \{m\} \text{ if } m-2 \in X\} / X \in D_2(C_{m-2}, i-1)\}.$$

(iii) If $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) = \phi$ then, $D_2(C_m, i) = \{Y \cup \{m\} / Y \in D_2(C_{m-1}, i-1)\}$

(iv) If $D_2(C_{m-1}, i-1) \neq \phi$, $D_2(C_{m-2}, i-1) \neq \phi$ then

$$D_2(C_m, i) = \{\{X \cup \{m-1\} \text{ if } m-3 \in X\} \cup \{X \cup \{m\} \text{ if } m-2 \in X\} \cup \{Y \cup \{m-1\} \text{ if } m-2 \in Y\} \cup \{Y \cup \{m\} \text{ if } m-1 \in Y\}\}$$

Where $X \in D_2(C_{m-2}, i-1)$ and $Y \in D_2(C_{m-1}, i-1)$.

From the above construction we obtain that

$$d_2(C_m, i) = d_2(C_{m-1}, i-1) + d_2(C_{m-2}, i-1).$$

III. 2-DOMINATION POLYNOMIALS OF CYCLES

Definition 3.1 : Let $D_2(C_m, i)$ be the family of 2-dominating sets of C_m with cardinality i and let $d_2(C_m, i) = |D_2(C_m, i)|$. Then, the 2-domination polynomial $D_2(C_m, x)$ of C_m is defined as,

$$D_2(C_m, x) = \sum_{i=\gamma_2(C_m)}^{m} d_2(C_m, i)x^i$$

where $\gamma_2(C_m)$ is the 2-domination number of C_m.

Theorem 3.2 :

For every $m \geq 5$,

$D_2(C_m, x) = x\,[D_2(C_{m-1}, x) + D_2(C_{m-2}, x)]$ with the initial values

$D_2(C_3, x) = x^3+3x^2$

$D_2(C_4, x) = x^4+4x^3+2x^2$

Proof:

We have $d_2(C_m, i) = d_2(C_{m-1}, i-1) + d_2(C_{m-2}, i-1)$

Therefore,

$$d_2(C_m, i)\, x^i = d_2(C_{m-1}, i-1)\, x^i + d_2(C_{m-2}, i-1)\, x^i$$

$$\sum d_2(C_m, i)\, x^i = \sum d_2(C_{m-1}, i-1)\, x^i + \sum d_2(C_{m-2}, i-1)\, x^i$$

$$= x\sum d_2(C_{m-1}, i-1)\, x^{i-1} + x\sum d_2(C_{m-2}, i-1)\, x^{i-1}$$

$$D_2(C_m, x) = xD_2(C_{m-1}, x) + xD_2(C_{m-2}, x)$$

$$D_2(C_m, x) = x[D_2(C_{m-1}, x) + D_2(C_{m-2}, x)],$$ with the initial values

$$D_2(C_3, x) = x^3+3x^2$$

$$D_2(C_4, x) = x^4+4x^3+2x^2$$

We obtain $d_2(C_m, i)$, for $2 \leq m \leq 15$ as shown in table 1

TABLE 1

$d_2(C_m, i)$, the number of 2-dominating sets of C_m with cardinality i

m\i	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C_3	3	1												
C_4	2	4	1											
C_5	0	5	5	1										
C_6	0	2	9	6	1									
C_7	0	0	7	14	7	1								
C_8	0	0	2	16	20	8	1							
C_9	0	0	0	9	30	27	9	1						
C_{10}	0	0	0	2	25	50	35	10	1					
C_{11}	0	0	0	0	11	55	77	44	11	1				
C_{12}	0	0	0	0	2	36	105	112	54	12	1			
C_{13}	0	0	0	0	0	13	91	182	156	65	13	1		
C_{14}	0	0	0	0	0	2	49	196	294	210	77	14	1	
C_{15}	0	0	0	0	0	0	15	140	378	450	275	90	15	1

In the following Theorem, we obtain some properties of $d_2(C_m, i)$.

Theorem 3.3:

The following properties hold for the coefficients of $D_2(C_m, x)$.

(i) $d_2(C_m, m) = 1$, for every $m \geq 3$.

(ii) $d_2(C_m, m-1) = m$, for every $m \geq 3$.

(iii) $d_2(C_m, m-2) = \frac{1}{2}[m^2-3m]$, for every $m \geq 4$.

(iv) $d_2(C_m, m-3) = \frac{1}{6}[m^3-9m^2+20m]$, for every $m \geq 6$.

(v) $d_2(C_m, m-4) = \frac{1}{24}[m^4-18m^3+107m^2-210m]$, for every $m \geq 8$.

(vi) $d_2(C_{2m}, m) = 2$, for every $m \geq 4$.

(vii) $d_2(C_{2m-1}, m) = 2m-1$, for all $m \geq 5$.

(viii) $d_2(C_{2m}, m+1) = m^2$, $m \geq 2$.

CONCLUSION

In this paper, the 2-domination polynomials of cycle has been derived by identifying its 2-dominating sets. It also helps us to characterize the 2-dominating sets of cardinality i. We can generalize this study to any power of cycle and some interesting properties can be obtained via the roots of the 2-domination polynomial of C_m.

REFERENCES

[1] M. chellali, O.Favaron, A. Hansberg, L.Volkmann, "k-Domination and k-Independence in Graphs" A Survey, Received: 27 May 2009/ Revised: 27 February 2011 / Published online: 8 April 2011© Springer 2011.

[2] Adriana Hansberg, Lutz Volkmann, "On graphs with equal domination and 2- domination numbers", Discrete Mathematics, Vol.308,(2008), 2277 – 2281.

[3] Mateusz Miotk, Jerzy Topp and Pawel Zylinski, "Disjoint dominating and 2−dominating sets in graphs", University of Gdansk, 80-952 Gdansk, Poland. Article in Discrete Optimization, March 2019.

[4] S. Alikhani and Y.H. Peng, "Dominating sets and Domination Polynomials of cycles", arXiv.org, arXiv:0905.3268v1,20 May 2009, pp 1-13.

[5] A. Vijayan and T. Anitha Baby, “Connect Total Dominating sets and connected Total Domination Polynomials of square of paths”, International Journal of Mathematics Trends and Technology, Vol.11, no.1, Jul 2014, pp 56-63.

[6] A.Vijayan, K. Lal Gipson, “Dominating sets and Domination Polynomials of square of path”, International Journal of Discrete Mathematics- (2013),Vol. 3, 60-69.