Turkish Online Journal of Qualitative Inquiry (TOJQI)
Volume 12, Issue 8, July 2021: 7894-7902

A Harvested Commensal-Host Mathematical Model -Global Stability Analysis

K.V.L.N.Acharyulu

Associate Professor, Department of Mathematics,
Bapatla Engineering College,Bapatla-522101, India,
Email: kvlna@yahoo.com

Abstract

This present study successfully constructed a specific commensal model of two species (Commensal-Host) in a syn eco-system with harvesting in both ways (i.e constant and variable rates). The nine equilibrium points are obtained with the essential conditions for their existence and also categorized under Interdependent and non Interdependent cases. The global stability of this ecological model is examined with Liapunov Technique.

Key words: Commensal, Host, Stability, Harvesting.

1. INTRODUCTION

Stability analysis of linear, time-invariant systems can be done using a variety of methods. Stability analysis for non-linear systems which may be extremely difficult or impossible in some cases. For non-linear systems, Liapunov Stability Analysis is one of the effective methods that can be used effectively and frequently. A.M. Liapunov established the direct approach to explore global equilibrium state stability in 1892.His method is based on Liapunov's function, which is the most important feature of building a scalar function. That is, we may observe the stability of any ecological or mathematical model not solving directly but examining it by using Liapunov's method. This is useful to solve non-linear and/or time-invariant state equations. Furthermore, the approaches in this concept used there necessitate explicit knowledge of matching linear system solutions.

In a standard ecological system, the predator consumes the prey in order to survive, while the prey adds members to the system as they are hunted down to keep the system in balance. Primitive humans used to hunt animals or gather crops, leaving behind a group of persons to raise for future generations. Only when humans are involved in the interaction does the concept of harvesting come into play, as we kill or harvest a proportion of the population, unlike a typical predator. Harvesting encompasses everyday human activities such as agriculture, cattle, sheep, and hen rearing, among others. Agriculture was learned by humans, and they harvested the product when it was ripe. We learned how to make use of these items and harvest them for beneficial purposes during development. Growing cattle and sheep for milk and wool, keeping poultry farms for meat, sustaining fisheries for fish, and so on are all examples of harvesting phenomena. Another key area where harvesting is done with extraordinary caution is the fishing industry. Actually the maximum number of individuals that can be harvested from a population equals the population's recruitment rate is referred to as Maximum sustainable yield. The number of new people added to the existing population is referred to as the recruitment rate in general.

Commensalism is a sort of living-organism relationship in which one organism benefits from the other without damaging it. A commensal species obtains mobility, shelter, food, or support from the host species, which (for the most part) neither benefits nor is damaged. Commensalism can range from brief interspecies interactions to long-term symbiosis.Simply, Commensalism is a symbiotic connection in which one species benefits while the other is not damaged or aided.The species that benefits is referred to as a commensal. The host species refers to the other species.A golden jackal follows a tiger to eat on leftovers from the tiger's kills is an example. Here golden jackal acts as a commensal and a tiger as a host.

In multicultural perspectives, K.V.L.N.Acharyulu et.al [2-9] investigated the local and global stability of numerous mathematical models of ecology. Kapur J.N [1] and several mathematicians [10–18] have developed key viewpoints and ideas in Mathematical Ecology, as well as intriguing applications for understanding the behaviours of various ecological models. Local stability indicates that the system is stable in the face of modest disruptions, whereas global stability indicates that the system is resistant to changes in its dynamics and external influences. In this model, the authors considered a commensal model with limited resources and involving harvesting in two ways(constant rate and variable rate).All nine equilibrium points are classified separately Interdependent and Non Interdependent cases. Global stability of this model is examined and established by designing an appropriate liapunov function.

2. BASIC EQUATIONS OF THE MODEL

A harvested commensal- host mathematical model at constant rates and variable rates are formed by

$$\frac{dN_1}{dt} = (1-\lambda_1)a_1N_1 - a_{11}N_1^{\,2} + (1-\lambda)a_{12}N_1N_2 - a_{11}H_1$$

$$\frac{dN_2}{dt} = (1-\lambda_2)a_2N_2 - a_{22}N_2^{\,2} - a_{22}H_2 \qquad (2.1)$$

Here $N_1(t)$ and $N_2(t)$ stand for Commensal and Host growth rates respectively. The natural growth rates of Commensal and Host species are referred as a_1, a_2. a_{12} is the Commensal species flourish rate . a_{11}, a_{22} are the two species' decline rates due to natural resource restrictions. Ki=$\frac{a_i}{a_{ii}}$, i =1,2 stands for the N_i' s carrying capacities.C=$\frac{a_{12}}{a_{11}}$ represents coefficient of commensalism. $h_1 = a_{11} H_1$ denotes a constant harvesting rate of the Commensal. $h_2 = a_{22} H_2$ denotes a constant harvesting rate of the Host. $\lambda_1=h_1/a_1$ is decease in Commensal due to harvesting. $\lambda_2=h_2/a_2$ is decease in host due to harvesting and m is the constant characterized by the cover which is provided for the commensal species with the conditions ($o<\lambda_i<1$).

3. THE EQULIBRIUM STATES

Nine equilibrium points are obtained and categorized as two types based on Interdependent Harvesting rates and Non Interdependent Harvesting rates

Type (1): In the case of Interdependent Harvesting rates

Case (a): When $H_1 < \frac{1}{4}\left[(1-\lambda_1)K_1 + C\left((1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)^2\right];\ H_2 < \frac{(1-\lambda_2)K_2^{\,2}}{4}$

E1 : $\overline{N_1} = \left((1-\lambda_1)K_1 + (1-\lambda)\ C\ \left((1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)\right) - \frac{H_1}{K_1 + (1-\lambda)C\left((1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)}$;

$$\overline{N_2} = (1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}$$

E2 : $\overline{N_1} = \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\left((1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)}$ **;** $\overline{N_2} = (1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}$

Here E_1 and E_2 exist only when $\left((1-\lambda_2)K_2\right)^2 > H_2$

and $\left[(1-\lambda_1)K_1 + (1-\lambda)C\left((1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)\right]^2 > H_1$

Type (2): In the case of Non-Interdependent Harvesting rates

Case (b): When $H_1 > \frac{1}{4}\left[(1-\lambda_1)K_1 + \frac{3(1-\lambda)(1-\lambda_2)CK_2}{4}\right]^2;\ H_2 < \frac{\left((1-\lambda_2)K_2\right)^2}{4}$

E3 : $\overline{N_1} = \frac{(1-\lambda_1)K_1 + (1-\lambda)C\left((1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)}{2}$ **;** $\overline{N_2} = (1-\lambda_2)K_2 - \frac{H_2}{(1-\lambda_2)K_2}$,

this would exists only when $\left((1-\lambda_2)K_2\right)^2 > H_2$

Case (c): When $H_1 < \frac{1}{4}\left[(1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{4}\right]^2;\ H_2 < \frac{\left[1-\lambda_2)K_2\right]^2}{4}$

E4 : $\overline{N_1} = \left((1-\lambda_1)K_1 + \frac{(1-\lambda)CH_2}{(1-\lambda_2)K_2}\right) - \frac{H_1}{(1-\lambda_1)K_1 + \frac{(1-\lambda\ )CH_2}{(1-\lambda 2)K_2}}$ **;** $\overline{N_2} = \frac{H_2}{(1-\lambda_2)K_2}$

this would exists only when $\left((1-\lambda_1)K_1 + \frac{(1-\lambda)CH_2}{(1-\lambda_2)K_2}\right)^2 > H_1$

E5 : $\overline{N_1} = \dfrac{H_1}{(1-\lambda_1)K_1 + \frac{(1-\lambda)CH_2}{(1-\lambda_2)K_2}}$ **;** $\overline{N_2} = \dfrac{H_2}{(1-\lambda_2)K_2}$

E6 : $\overline{N_1} = \dfrac{(1-\lambda_1)K_1 + \frac{(1-\lambda)CH_2}{(1-\lambda_2)K_2}}{2}$ **;** $\overline{N_2} = \dfrac{H_2}{(1-\lambda_2)K_2}$

Case (d): When $H_1 < \frac{1}{4}\left[(1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{2}\right]^2; H_2 = \frac{K_2 2}{4}$

E7 : $\overline{N_1} = \left((1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{2}\right) - \dfrac{H_1}{(1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{2}}$ **;** $\overline{N_2} = \frac{(1-\lambda_2)K_2}{2}$

this would exists only when $\left((1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{2}\right)^2 > H_1$

E8 : $\overline{N_1} = \dfrac{H_1}{(1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{2}}$; $\overline{N_2} = \frac{(1-\lambda_2)K_2}{2}$

Case (e): When $H_1 = \frac{1}{4}\left[(1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{2}\right]^2; H_2 = \frac{((1-\lambda_2)K_2)2}{4}$

E9 : $\overline{N_1} = \frac{1}{2}\left[(1-\lambda_1)K_1 + \frac{(1-\lambda)(1-\lambda_2)CK_2}{2}\right]; \overline{N_2} = \frac{(1-\lambda_2)K_2}{2}$

4. LOCAL STABILITY

The authors invented local stability in many ecological models and also observed in this model at

E1 $: \overline{N_1} = \left((1-\lambda_2)K_1 + (1-\lambda)C\left((1-\lambda_2)K_2 - \dfrac{H_2}{(1-\lambda_2)K_2}\right)\right) - \dfrac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\left((1-\lambda_2)K_2 - \dfrac{H_2}{(1-\lambda_2)K_2}\right)}$**;**

$\overline{N_2} = (1-\lambda_2)K_2 - \dfrac{H_2}{(1-\lambda_2)K_2}$ is stable and also identified that all others are unstable.

5. GLOBAL STABILITY

The equations of system over the perturbations (µ1, µ2) are

$$\frac{d\mu_1}{dt} = -a_{11}\left(\overline{N_1} - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{N_2}}\right)\mu_1 + (1-\lambda)Ca_{11}\overline{N_1}\mu_2 \qquad (5.1)$$

$$\frac{d\mu_2}{dt} = -a_{22}\left(\overline{N_2} - \frac{H_2}{(1-\lambda_2)K_2}\right)\mu_2 \qquad (5.2)$$

The characteristic equation is $\left(\alpha + a_{11}\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)\right)\left(\alpha + a_{22}\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_1)K_2}\right)\right) = 0$

i.e., $\alpha^2 + \left[a_{22}\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right) + a_{11}\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)\right]\alpha$

$$+a_{22}a_{11}\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right) = 0 \quad (5.3)$$

equation (5.3) is in the form of $\alpha^2 + P\alpha + Q = 0$

where P = $a_{22}\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right) + a_{11}\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)$

$where\ Q = a_{22}a_{11}\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)$

By using

$$\left[a_{22}\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right) + a_{11}\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)\right]^2$$

$$> 4a_{22}a_{11}\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_1)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)$$

Thus, P and Q > 0.

$\therefore$ Hence, the necessary conditions of Liapunov are satisfied.

Define $L(\mu_1, \mu_2) = \frac{1}{2}\left[A\,\mu_1^{\,2} + 2B\,\mu_1\,\mu_2 + c\,\mu_2^{\,2}\right]$

Where

$$A = \frac{a_{22}^{\;2}\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)^2 + a_{11}a_{22}\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)\left(\overline{N}_1 - \frac{H_1}{(1-\lambda_2)K_1 + (1-\lambda)C\overline{\overline{N_2}}}\right)}{D} \quad (5.4)$$

$$B = \frac{(1-\lambda)C\,a_{11}\,a_{22}\overline{\overline{N_1}}\left(\overline{N}_2 - \frac{H_2}{(1-\lambda_2)K_2}\right)}{D} \quad (5.5)$$

$$c=\frac{a_{11}{}^{2}\left(\overline{N}_1-\dfrac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{\overline{N_2}}}\right)^2+\left[(1-\lambda)C\right]^2 a_{11}{}^{2}\overline{N}_1{}^{2}}{D}$$

$$+\frac{a_{11}a_{22}\left(\overline{N}_2-\dfrac{H_2}{(1-\lambda_2)K_2}\right)\left(\overline{N}_1-\dfrac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{\overline{N_2}}}\right)}{D} \tag{5.6}$$

Where

$$D=PQ=\left[a_{22}\left(\overline{N}_2-\frac{H_2}{(1-\lambda_2)K_2}\right)+a_{11}\left(\overline{N}_1-\frac{H_1}{(1-\lambda_1)K_1+(1-\lambda)\,C\,\overline{\overline{N_2}}}\right)\right]$$

$$\times\left[a_{22}a_{11}\left(\overline{N}_1-\frac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{\overline{N_2}}}\right)\left(\overline{N}_2-\frac{H_2}{(1-\lambda_2)K_2}\right)\right]$$

Now

$$D^2\left(Ac-B^2\right)\;=$$

$$D^2\{\left(\frac{a_{22}{}^{2}\left(\overline{N}_2-\dfrac{H_2}{(1-\lambda_2)K_2}\right)^2+a_{11}a_{22}\left(\overline{N}_2-\dfrac{H_2}{(1-\lambda_2)K_2}\right)\left(\overline{N}_1-\dfrac{H_1}{(1-\lambda_2)K_1+(1-\lambda)C\overline{\overline{N_2}}}\right)}{D}\right)$$

$$\times\left[\frac{a_{11}{}^{2}\left(\overline{N}_1-\dfrac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{\overline{N_2}}}\right)^2+\left((1-\lambda_1)\mathrm{C}\right)^2 a_{11}{}^{2}\overline{N}_1{}^{2}}{D}\right.$$

$$\left.+\frac{a_{11}a_{22}\left(\overline{N}_2-\dfrac{H_2}{(1-\lambda_2)K_2}\right)\left(\overline{N}_1-\dfrac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{\overline{N_2}}}\right)}{D}\right]$$

$$-\left(\frac{(1-\lambda)Ca_{11}a_{22}\overline{N}_1\left(\overline{N}_2-\dfrac{H_2}{(1-\lambda_2)K_2}\right)}{D}\right)^2\}$$

$$\Rightarrow D^2\left(Ac-B^2\right)>0\;\; i.e.,\; B^2-Ac\;<\;0$$

The function $L\left(\mu_1, \mu_2\right)$ is positive definite.

Further

$$\frac{\partial L}{\partial \mu_1}\frac{d\mu_1}{dt}+\frac{\partial L}{\partial \mu_2}\frac{d\mu_2}{dt}=\left(A\mu_1+B\mu_2\right)\left(-a_{11}\left(\overline{N_1}-\frac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{N_2}}\right)\mu_1+(1-\lambda)Ca_{11}\overline{N_1}\mu_2\right)$$

$$+\left(B\mu_1+c\mu_2\right)\left(-a_{22}\left(\overline{N_2}-\frac{H_2}{(1-\lambda_2)K_2}\right)\mu_2\right)$$

$$\frac{\partial L}{\partial \mu_1}\frac{d\mu_1}{dt}+\frac{\partial L}{\partial \mu_2}\frac{d\mu_2}{dt} =$$

$$\left(A(1-\lambda)Ca_{11}\overline{N_1}-Ba_{11}\left(\overline{N_1}-\frac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{N_2}}\right)-Ba_{22}\left(\overline{N_2}-\frac{H_2}{(1-\lambda_2)K_2}\right)\right)\mu_1\,\mu_2$$

$$+\left(B(1-\lambda)Ca_{11}\overline{N_1}-ca_{22}\left(\overline{N_2}-\frac{H_2}{(1-\lambda_2)K_2}\right)\right)\mu_2^{\,2} \qquad (5.7)$$

Putting the values of A, B and c from (5.4), (5.5) and (5.6) in (5.7)

we get $\frac{\partial L}{\partial \mu_1}\frac{d\mu_1}{dt}+\frac{\partial L}{\partial \mu_2}\frac{d\mu_2}{dt}=-\left(\frac{D}{D}\mu_1^{\,2}+\frac{D}{D}\mu_2^{\,2}\right)$

$i.e\ \frac{\partial L}{\partial \mu_1}\frac{d\mu_1}{dt}+\frac{\partial L}{\partial \mu_2}\frac{d\mu_2}{dt}=-\left(\mu_1^{\,2}+\mu_2^{\,2}\right)$

It is observed that The function is negative definite.

Thus $L\left(\mu_1, \mu_2\right)$ is Liapunov function for the linear system.

Now we observe that $L\left(\mu_1, \mu_2\right)$ is also a Liapunov function to the considered system

Define $U\left(N_1,\ N_2\right)=(1-\lambda_1)a_1N_1-a_{11}N_1^{\,2}+(1-\lambda)a_{12}N_1N_2-a_{11}H_1$

$V\ \left(N_1,\ N_2\right)\ =(1-\lambda_2)a_2N_2-a_{22}N_2^{\,2}-a_{22}H_2$

By putting $N_1=\overline{N_1}+\mu_1$ *and* $N_2=\overline{N_1}+\mu_2$ *in* $\left(5.1\right)$ *and* $\left(5.2\right)$

$$U\left(N_1,\ N_2\right)=\frac{d\mu_1}{dt}=-a_{11}\left[\overline{N}_1-\frac{H_1}{(1-\lambda_1)K_1+(1-\lambda)C\overline{N_2}}\right]\mu_1+(1-\lambda)Ca_{11}\overline{N_1}\mu_2+u\left(\mu_1,\ \mu_2\right)$$

$$V\left(N_1,\ N_2\right)=\frac{d\mu_2}{dt}=-a_{22}\left[\overline{N}_2-\frac{H_2}{(1-\lambda_2)K_2}\right]\mu_2+v\left(\mu_1,\ \mu_2\right)$$

where $u(\mu_1,\ \mu_2) = -a_{11}\mu_1^{\ 2} + a_{12}\ \mu_1\mu_2$ and $v(\mu_1,\ \mu_2) = -a_{22}\mu_2^{\ 2}$

Now

$$\frac{\partial L}{\partial \mu_1}U + \frac{\partial L}{\partial \mu_2}V = -(\mu_1^{\ 2} + \mu_2^{\ 2}) + (A\mu_1 + B\mu_2)\ \mathrm{u}(\mu_1, \mu_2) + (B\mu_1 + c\mu_2)\ v(\mu_1, \mu_2)$$

From the polar co-ordinate system,

$$\frac{\partial L}{\partial \mu_1}U + \frac{\partial L}{\partial \mu_2}V = -R^2 + R\,[(a\cos\theta + b\sin\theta)\ u(\mu_1, \mu_2) + (b\cos\theta + c\sin\theta)\ \mathrm{v}(\mu_1, \mu_2)$$

The largest number among | a|, |b|, |c| can be taken as M, our assumptions meet that

$$|u(\mu_1, \mu_2)| < \frac{R}{6M} \text{ and } |\mathrm{v}(\mu_1, \mu_2)| < \frac{R}{6M}\ \ for\ small\ R > 0$$

$$So\ \ \frac{\partial L}{\partial \mu_1}U + \frac{\partial L}{\partial \mu_2}V < -r^2 + \frac{4Kr^2}{6K} = -\frac{r^2}{3} < 0$$

Hence, $\mathrm{L}(\mu_1, \mu_2)$ is a positive definite function by satisfying the property that

$$\frac{\partial L}{\partial \mu_1}U + \frac{\partial L}{\partial \mu_2}V \text{ is negative definite}$$

$\therefore$ Thus asymptotic stability is established at the equilibrium point E_1

6. Conclusions

In this research work, the authors effectively established a specific commensal model of two species (Commensal-Host) in syn eco-system with harvesting in both ways(constant rate and variable rate) .The nine equilibrium points are derived with necessary conditions for their existence and also characterized by Harvesting rates under Interdependent and Non Interdependent cases. The global stability of this ecological model is successfully established with Liapunov Technique.

7. References

1. Kapur J.N.,Mathematical Modelling, Wiley Eser,1985.
2. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Ammensal-Enemy Specie Pair With Limited And Unlimited Resources Respectively-A Numerical Approach, Int. J. Open Problems Compt. Math (IJOPCM), Vol. 3, No. 1, March 2010,73-91.
3. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Enemy- Ammensal Species Pair With Limited Resources –A Numerical Study, Int. J. Open Problems Compt. Math (IJOPCM), Vol. 3, No. 3, September 2010,339-356,
4. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; Mortal Ammensal and an Enemy Ecological Model with Immigration for Ammensal Species at a Constant Rate, International Journal of Bio-Science and Bio-Technology, Vol. 3, No.1, Marc 2011,39-48,

5. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Immigrated Ecological Ammensalism with Limited Resources"- International Journal of Advanced Science and Technology, Vol. 27 ,2011, 87-92.
6. K.V.L.N.Acharyulu and N.Ch.Pattabhi Ramacharyulu; A Numerical Study on an Ammensal -Enemy Species Pair with Unlimited Resources and Mortality Rate for Enemy Species"- International Journal of Advanced Science & Technology,Vol.30, May 2011,13-24.
7. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; An Ecological Ammensalism with Multifarious restraints- A Numerical Study" International Journal of Bio-Science and Bio-Technology, Vol. 3, No. 2, June 2011,1-12.
8. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; Multiple Constraints in Ecological Ammensalism- A Numerical Approach , Int. J. Advance. Soft Comput. Appl., Vol. 3, No. 2, July 2011,1-15.
9. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu; On the Carrying capacity of Enemy Species, Inhibition coefficient of Ammensal Species and Dominance reversal time in An Ecological Ammensalism - A Special case study with Numerical approach, International Journal of Advanced Science and Technology, Vol. 43, June,2012,49-58.
10. Lotka A.J.(1925). Elements of Physical Biology, Williams and willians, Baltimore,1925.
11. Lakshmi Narayan K.(2005).A Mathematical study of a prey – predator Ecological Model with a partial cover for the prey and alternative food for the predator, Ph.D. Thesis, JNTU.
12. Meyer WJ. Concepts of mathematical modelling. McGraw-Hill; 1985.
13. Mesterton-Gibbons Michael. A technique for finding optimal two species harvesting policies. Ecol Modell 1996;92:235–44.
14. Paul Colinvaux A. Ecology. New York: John Wiley; 1986.
15. Phani kumar N. Seshagiri Rao. N & Pattabhi Ramacharyulu N.Ch.,On the stability of a host -A flourishing commensal species pair with limited resources". International journal of logic based intelligent systems,3(1),2009,pp. 45-54.
16. PhanikumarN.,Pattabhi ramacharyulu N.Ch.,A three species eco-system consisting of a prey predator and host commensal to the prey" International journal of open problems compt.math, 3(1),2010,pp.92-113.
17. Srinivas NC. Some mathematical aspects of modelling in biomedical sciences. Ph.D. thesis. Kakatiya University; 1991.
18. Volterra V. Leconssen La Theorie Mathematique De La Leitte Pou Lavie. Paris: Gouthier-Villars; 1931.