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Different levels of Ammensalism in a Mathematical Model with Harvesting at Variable Rate for Enmity Species

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ABSTRACT

The purpose of this study is to explore an ecological Ammensal model with four species of Species-1 i.e Ammensal Prey (P_A), Species-2 i.e Ammensal Predator (Q_A), Species-3 i.e Ammensal Enemy (E_A), and Species-4 i.e Enmity(N) with harvesting at variable rate(χ) . The System mainly consists of an Ammensal Prey (P_A), a Ammensal Predator (Q_A) that survives on an Ammensal –prey(P_A), Ammensal Enemy (E_A) and Enmity(N) for which P_A, Q_A are Ammensal, i.e., E_A and N have an undesirable effect on P_A and Q_A without being affected in any way. Furthermore, E_A is enormous for N, and N is harmful to E_A. First level Ammensalism is represented by the pair P_A & E_A , the pair of second level Ammensalism is Q_A & N and third level Ammensalism by the pair E_A & N. The system's model equations form a set of four non-linear ordinary differential coupled equations of first order. There are sixteen equilibrium points found. The interactions between the four species are explored in light of different changes in various species' growth rate.

Keywords: Ammensal, Enemy, Enmity, Prey, Predator, Stable

1. Introduction:

The environment of ecosystems is deeply influenced by many physical and biotic factors. The physical environment, which includes abiotic components such as temperature, radiation, light, chemistry, climate, and geology, is external to the level of biological structure under consideration. Genes, cells, organisms, members of the same species, and other species that share a habitat make up the biotic environment. The distinction between exterior and internal environments is an abstraction that breaks down life and environment into discrete pieces or facts that are in reality interrelated. The environment and life are intertwined in terms of cause and effect. Lotka [10] and Volterra [12] pioneered ecosystem mathematical modelling in 1925 and 1931, respectively. Meyer [11], Kapur [8,9], and other authors have offered general modelling notions in their treatises. N.C. Srinivas [13] investigated two and three-species competitive ecosystems with limited and infinite resources. Acharyulu et al. [1-7] just published some fascinating findings "on the stability of an enemy-Ammensal species pair under a variety of situations."

In this paper, A set of four first order non-linear ordinary differential coupled equations compose the model equations. There are sixteen equilibrium points found. The interactions between the four species are explored in light of changes in Ammensal-Prey(P) natural growth rate.

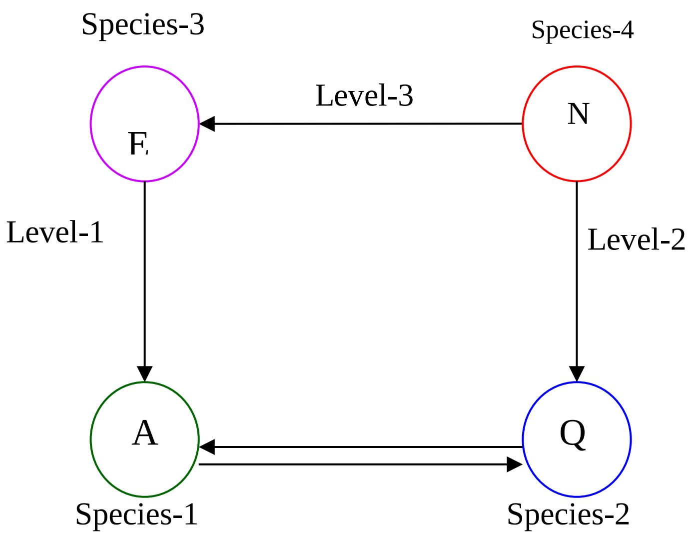

Fig. 1 Four Species Eco- System

1.1 Notations:

Here $\mathbf{P_A}$ stands for Ammensal Prey i.e Prey for Q_A and Ammensal for E_A,Q_A represents as Predator-Ammensal i.e Predator surviving upon P_A and Ammensal for N.E_A stands for Enemy-Ammensal i.e Enemy for the Prey Ammensal (P_A) and Ammensal for N.N represents as Enmity of the Ammensal Predator(Q_A) and also having with harvesting at variable rate(**χ)**

& Ammensal Enemy (E_A).$\mathbf{P_A}$, $\mathbf{Q_A}$, $\mathbf{E_A}$, **N stand for** The Population growth rates of $\mathbf{P_A}$, $\mathbf{Q_A}$, $\mathbf{E_A}$, **N** at time t.a_i is Natural growth rates of $\mathbf{P_A}$, $\mathbf{Q_A}$, $\mathbf{E_A}$, **N**, where i= a for $\mathbf{P_A}$ for ,p for $\mathbf{Q_A}$,e for $\mathbf{E_A}$ & m for **N** respectively.a_{ii} represents as Self-inhibition coefficient of $\mathbf{P_A}$, $\mathbf{Q_A}$, $\mathbf{E_A}$, **N**, i= a for $\mathbf{P_A}$ for ,p for $\mathbf{Q_A}$,e for $\mathbf{E_A}$ & m for **N** respectively. ψ_{ap} and ψ_{pa} represent as Interaction coefficients of $\mathbf{P_A}$ due to $\mathbf{Q_A}$ and $\mathbf{Q_A}$ due to $\mathbf{P_A}$. ψ_{ae}, ψ_{pm} , ψ_{em} stand for Inhibition coefficients for $\mathbf{P_A}$ due to the enemy $\mathbf{E_A}$, $\mathbf{Q_A}$ due to the malice N, $\mathbf{E_A}$ due to malice N . $K_i = \frac{\psi_i}{\psi_{ii}}$ is Carrying capacities of $\mathbf{P_A}$, $\mathbf{Q_A}$, $\mathbf{E_A}$, **N**, i= a for $\mathbf{P_A}$ for ,p for $\mathbf{Q_A}$,e for $\mathbf{E_A}$ & m for **N**. $\alpha_{ij} = \frac{\psi_{ij}}{\psi_{ii}}$ be Ammensal coeficient of $\mathbf{P_A}$, $\mathbf{Q_A}$, $\mathbf{E_A}$, **N**, i= a for $\mathbf{P_A}$ for ,p for $\mathbf{Q_A}$,e for $\mathbf{E_A}$ & m for **N** (i≠j)

β_{ab}, β_{ba} stand for Interaction (A-P) coefficients of $\mathbf{P_A}$ due to $\mathbf{Q_A}$ and $\mathbf{Q_A}$ due to $\mathbf{P_A}$.The variables $\mathbf{P_A}$, $\mathbf{Q_A}$, $\mathbf{E_A}$, **N** are non-negative and the parameters ψ_a, ψ_p, ψ_e, ψ_m; ψ_{aa}, ψ_{pp}, ψ_{ee}, ψ_{mm}; ψ_{ap}, ψ_{pa}, ψ_{ae}, ψ_{pm}, ψ_{em} are considered as non-negative constants.

2. Basic Equations:

The first order non-linear ordinary differential equations gives the present model equations for an ecological Ammensal model with considered four species in Syn Eco-System at three Ammensal levels.

The equations for the natural growth rates of **P_A, Q_A, E_A, N** are

$$(i)\quad \frac{dN}{dt} = \psi_{mm}((1-\chi)K_m N - N^2) \tag{1}$$

$$(ii)\quad \frac{dP_A}{dt} = \psi_{aa}(K_a P_A - P_A{}^2 - \beta_{ap} P_A Q_A - \alpha_{ae} P_A E_A) \tag{2}$$

$$(iii)\quad \frac{dQ_A}{dt} = \psi_{pp}(K_p Q_A - Q_A{}^2 + \beta_{pa} Q_A P_A - \alpha_{pm} Q_A N) \tag{3}$$

$$(iv)\quad \frac{dE_A}{dt} = \psi_{ee}(K_e E_A - E_A{}^2 - \alpha_{em} E_A N) \tag{4}$$

3. Equilibrium States:

There are a maximum of sixteen possible equilibrium states for the system being studied and defined by

$$\frac{dS}{dt} = 0,\ where\ S = \mathbf{P_A, Q_A, E_A, N}$$

3A. State of Fully washed out :

$$(i)\ \overline{P_A} = 0, \overline{Q_A} = 0, \overline{E_A} = 0, \overline{N} = 0$$

3B. In three of the four states, the species have been washed away, but in the fourth, they have not:

$$(ii)\ \overline{P_A} = 0, \overline{Q_A} = 0, \overline{E_A} = 0, \overline{N} = \frac{(1-\chi)\psi_m}{\psi_{mm}} : Only\ the\ N\ of\ \mathrm{Q_A}\ and\ E_A\ survives$$

$$(iii).\ \overline{P_A} = \frac{\psi_a}{\psi_{aa}}, \overline{Q_A} = 0, \overline{E_A} = 0, \overline{N} = 0 : Only\ \mathrm{P_A}\ survives$$

$$(iv).\overline{P_A} = 0, \overline{Q_A} = \frac{\psi_p}{\psi_{pp}}, \overline{E_A} = 0, \overline{N} = 0 : Only\ Q_A\ endures$$

$$(v).\overline{P_A} = 0, \overline{Q_A} = 0, \overline{E_A} = \frac{a_e}{a_{ee}}, \overline{N} = 0 : Only\ the\ E_A\ of\ \mathrm{P}_A\ exists$$

3C. Only two of the four species have been washed away in these states

$$(vi).\overline{P_A}=0,\overline{Q_A}=0,\overline{E}=\frac{\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em}}{\psi_{ee}\psi_{mm}},\overline{N}=\frac{(1-\chi)\psi_m}{\psi_{mm}}:P_A\ and\ Q_A\ washed\ out$$

It should only exist if $\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em}>0$ is occurred

$$(vii).\ \overline{P_A}=0,\overline{Q_A}=\frac{\psi_p\psi_{mm}-(1-\chi)\psi_m\psi_{pm}}{\psi_{pp}\psi_{mm}},\overline{E_A}=0,\overline{N}=\frac{(1-\chi)\psi_m}{\psi_{mm}}\ :P_A\ and\ E_A\,washed\ out$$

It should only exist if $\psi_p\psi_{mm}-(1-\chi)\psi_m\psi_{pm}>0$ is occurred

$$(viii).\ \overline{P_A}=0,\overline{Q_A}=\frac{\psi_p}{\psi_{pp}},\overline{E_A}=\frac{\psi_e}{\psi_{ee}},\overline{N}=0:\ P_A\ and\ N\ washed\ out$$

$$(ix).\ \overline{P_A}=\frac{n_a}{n_{aa}},\overline{Q_A}=0,\overline{E_A}=0,\overline{N}=\frac{(1-\chi)\psi_m}{\psi_{mm}}:P_A\ and\ N\ washed\ out$$

$$(x).\ \overline{P_A}=\frac{\psi_a\psi_{ee}-\psi_e\psi_{ae}}{\psi_{aa}\psi_{ee}},\overline{Q_A}=0,\overline{E_A}=\frac{\psi_e}{\psi_{ee}},\overline{N}=0:Q_A\ and\ N\ washed\ out$$

It should only exist if $\psi_a\psi_{ee}-\psi_e\psi_{ae}>0$ is occurred

$$(xi).\ \overline{P_A}=\frac{\psi_a\psi_{pp}-\psi_p\psi_{ap}}{\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}},\ \overline{Q_A}=\frac{\psi_a\psi_{pa}+\psi_p\psi_{aa}}{\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}},\overline{E_A}=0,\overline{N}=0:P_A\ and\ \mathrm{Q_A}\,survive$$

It should only exist if $\psi_a\psi_{pp}>\psi_p\psi_{ap}$ is occurred

3D. A state in which only one of the four species is washed away:

$$(xii).\ \overline{P_A}=0,\overline{Q_A}=\frac{\psi_p\psi_{mm}-(1-\chi)\psi_{pm}\psi_m}{\psi_{pp}\psi_{mm}},\overline{E_A}=\frac{\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em,}}{\psi_{ee}\psi_{mm}},$$

$$\overline{N}=\frac{(1-\chi)\psi_m}{\psi_{mm}}:Only\ \mathrm{P_A}\ washed\ out$$

It should only exist if $\psi_p\psi_{mm}>(1-\chi)\psi_{pm}\psi_m,\psi_e\psi_{mm}>(1-\chi)\psi_m\psi_{em}$ is occurred .

$$(xiii).\ \overline{P_A}=\frac{\psi_a\psi_{ee}\psi_{mm}+\psi_{ae}(\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em})}{\psi_{aa}\psi_{ee}\psi_{mm}},\overline{P_A}=0:Only\ \mathrm{P_A}\ washed\ out$$

$$\overline{E_A}=\frac{\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em,}}{\psi_{ee}\psi_{mm}}and\ \ \overline{N}=\frac{(1-\chi)\psi_m}{\psi_{mm}}$$

It should only exist if $\psi_e\psi_{mm}>(1-\chi)\psi_m\psi_{em}$ is occurred .

$(xiv).\ \overline{P_A}=\frac{\psi_a\psi_{pp}\psi_{mm}-\psi_{ap}(\psi_p\psi_{mm}-(1-\chi)\psi_m\psi_{pm})}{\psi_{mm}(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa})},\overline{Q_A}=\frac{\psi_a\psi_{pa}\psi_{ee}+\psi_{aa}(\psi_p\psi_{mm}-(1-\chi)\psi_m\psi_{pm})}{\psi_{mm}(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa})},$

$\overline{E_A}=0,\overline{N}=\frac{(1-\chi)\psi_m}{\psi_{mm}}:\mathrm{E_A}\ \ washed\ \ out:$

$when\ \ only\ \ \psi_a\psi_{pp}\psi_{mm}>\psi_{ap}(\psi_p\psi_{mm}-(1-\chi)\psi_m\psi_{pm})$

$and\ \psi_a\psi_{pa}\psi_{ee}>\psi_{aa}(\psi_p\psi_{mm}-(1-\chi)\psi_m\psi_{pm})$

$(xv).\ \overline{P_A}=\frac{\psi_{pp}\left(\psi_a\psi_{ee}-\psi_e\psi_{ae}\right)-\psi_p\psi_{ap}\psi_{ee}}{\psi_{ee}\left(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}\right)},\overline{Q_A}=\frac{\psi_{pa}\left(\psi_a\psi_{ee}-\psi_e\psi_{ae}\right)+\psi_p\psi_{aa}\psi_{ee}}{\psi_{ee}\left(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}\right)},$

$\overline{E_A}=\frac{\psi_e}{\psi_{ee}},\overline{N}=0:N\ washed\ out$

3E. A state in which none of the four species have been washed out:

(xvi) The state of coexistence:

$$\overline{P_A}=\psi_{pp}\frac{\left(\psi_a\psi_{ee}\psi_{mm}+\psi_{ae}(\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em,})\right)}{\psi_{ee}\psi_{mm}\left(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}\right)}-\psi_{ap}\frac{\left(\psi_p\psi_{mm}+(1-\chi)\psi_{pm}\psi_m\right)}{\psi_{mm}\left(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}\right)},$$

$$\overline{Q_A}=\psi_{aa}\frac{\left(\psi_p\psi_{mm}-(1-\chi)\psi_{pm}\psi_m\right)}{\psi_{mm}\left(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}\right)}+\psi_{pa}\frac{(\psi_{ee}\psi_{mm}-\psi_{ae}(\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em}))}{\psi_{ee}\psi_{mm}\left(\psi_{aa}\psi_{pp}+\psi_{ap}\psi_{pa}\right)},$$

$$\overline{EA}=\frac{\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em}}{\psi_{ee}\psi_{mm}}\ and\ \overline{N}=\frac{(1-\chi)\psi_m}{\psi_{mm}}$$

It should only exist if $\psi_e\psi_{mm}>(1-\chi)\psi_m\psi_{em,}\psi_p\psi_{mm}>(1-\chi)\psi_{pm}\psi_m$,
$and\ \psi_{ee}\psi_{mm}>\psi_{ae}(\psi_e\psi_{mm}-(1-\chi)\psi_m\psi_{em})$ are occurred

4. The reversal times of t_{ap}*,t_{ae}*,t_{am}*,t_{pe}*,t_{pm}* are affected by changes in the growth rate of P_A, which is regarded to be a finite interval:

To maintain dominance over Ammensal Predator and Ammensal Enemy, Ammensal-Prey uses a sustainable growth rate to attack both of the aforementioned species. In addition, When the Ammensal Enemy-(E_A) attacks, the Ammensal Predator (Q_P) grows rapidly to counter the threat (E_A). One species dominates the others, Enmity (N), with no change in behaviour.

Between Ammensal-Prey(P_A) and Ammensal Predator(Q_A), a shift in power takes place (Q_A). After the time instinct (t_{ap}*), Ammensal-prey(P_A) overtakes Ammensal predator. As Ammensal-Predator (Q_A) grows in power, Ammensal-Prey (P_A)prepares to attack the enemy with noteworthy durability.

For a short period of time, Ammensal-Prey (P_A) grows at a steeper rate than the other three species (Q_A, N, and E_A), but then settles into a steady distance from them. Between the pairs (Q_A,N) and (Q_A,E_A), two dominance-reversal times, t_{pm}* and t_{pe}*, occur. In terms of equilibrium, enmity (N) is a stable state. The decay rate of Ammensal Enemy (E_A) decreases, bringing it closer to equilibrium.

There are three dominance reversal times between the pairs (P_A,N;Q_A,N;Q_A,E_A): t_{am}*, t_{pm}*, and t_{pe}*. After the time instinct t_{am}*, Ammensal Predator (Q_A) commands and keeps a constant distance from Enmity (N). Ammensal-Prey (P_A) and Enmity (N) will no longer participate in the dominance reversal time tam*.

During the period of instinct t_{am}*, Ammensal-Prey (P_A) takes over Enmity (N) and also other species (Q_A&E_A). Throughout the period, Ammensal Enemy (E_A) declines and Ammensal Predator (Q_A) takes over. The time instincts t_{am}* and t_{pe}* have been shown to be declining over time. Anger (N) and resentment (Q_A) prepare to influence one another.

Between Ammensal Predator (Q_A) and Enemy (E_A), there is only one dominance reversal time (N). Prey(P_A) has a steady growth rate up to a certain point and then maintains an invariable distance from the equilibrium point. The growth rate of Enemy-Ammensal decreases step by step. Even though all four species have the same initial population strength, Ammensal-prey is found to be in the zenithal position and to be dominating all other species in the natural growth rate in this final case.

Enmity (N) is the most dominant of the three species (Q_A, E_A and N) over the time period. Ammensal Predator (Q_A) dominates only Ammensal–Prey(P_A), while the Ammensal Enemy (E_A) rules over two species (Q_A&N) (A). It has been observed that there has been no reversal of dominance between any two species over time. Only one of the four species (E_A) grows exponentially, whereas the other three (Q_A, E_A, N) all decline at an exponential rate.

5. Conclusions: When Ecologic Ammensalism with four species at 3 levels rises naturally as Ammensal-Prey(A) species grow:

(i).After a certain point, P_A stabilizes at a constant distance from each other.

(ii).E_A diminishes at an exponential rate.

(iii).P_A and E_A dominance time t^*_{ae} gradually decreases.

(iv).Q_A and E_A have dominance reversal time t^*_{pe} that decays stepwise.

(v).t^*_{am} between N and P_A is steadily decreasing.

(vi).P_A's capacity increases.

(vii).P_A versus Q_A dominance reversal time t^*_{ap} shrinks over time.

(viii).Q_A and E_A dominance reversal time t^*_{pm} decreases over time.

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