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Research Article

Coincidence Point and Fixed-Point Theorem in Partially Ordered Metric Spaces

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Abstract

In this paper, we prove a coincidence point and fixed point result in partially ordered metric spaces. The proved result generalizes and extends some Known results in the literature.

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1. Introduction and preliminaries

The Banach contraction principle plays a vital role to obtain an unique solution of the results. There are a lot of generalization of the Banach contraction principle in the literature (see [1]-[8] and references cited therein.) Several research work has been obtained on various spaces such as quasi metric spaces, probabilistic metric spaces, D-metric spaces, fuzzy metric spaces, G-metric spaces, cone metric spaces, complex valued metric spaces, and so on to prove the existing results. Recently , many authors have obtained fixed point, common fixed point and coincidence point results in partially ordered metric spaces (see [9, 10, 11, 12, 13, 14, 15, 16, 17,]).

The aim of this paper is to prove some coincidence point and common fixed point results in partially ordered metric spaces for a pair of self-mappings satisfying a generalized contractive condition of rational type. Our results generalize and extend the results of Rao et al.[14] and Chandok et al.[15] in ordered metric space.

The following definitions are frequently used in results given in upcoming sections.

Definition 1. The triple $(X, d, \preceq)$ is called a partially ordered metric space, if $(X, \preceq)$ is partially ordered set together with (X, d) is a metric space.

Definition 2. If (X, d) is a complete metric space, then the triple $(X, d, \preceq)$ is called a partially ordered complete metric space.

Definition 3. Let $(X, \preceq)$ be partially ordered set. A self-mapping f:X → X is said to be strictly increasing, if $f(x) \prec f(y)$, for all x, y ∈ X with $x \prec y$ and is also said to be strictly decreasing, if $f(x) \succ f(y)$, for all x, y ∈ X with $x \prec y$.

Definition 4. A point $x \in$ A, where A is a non-empty subset of metric space (X, d) is called a common fixed (coincidence) point of two self-mappings f *and* T if $fx = \mathrm{T}x = x(fx = Tx)$.

Definition 5. The two self-mappings f *and* T defined over a subset A of a metric space (X, d) are called commuting if $f\mathrm{T}x = \mathrm{T}fx$ for all $x \in$ A.

Definition 6. Two self-mappings f *and* T defined over $A \subset X$ are compatible, if for any sequence $\{x_n\}$ with $\lim_{n\to+\infty} f x_n = \lim_{n\to+\infty} Tx_n = u$, for some $u \in A$ then $\lim_{n\to+\infty} d(T(f x_n), f(Tx_n)) = 0$.

Definition 7. Two self-mappings f and T defined over $A \subset X$ are said to be weakly compatible, if they commute at their coincidence points. i.e., if $fx = Tx$ then $fTx = Tfx$.

Definition 8. Let f *and* T be two self-mappings defined over a partially ordered set $(X, \preceq)$. A mapping T is called a monotone f non-decreasing if

$$fx \preceq fy \text{ implies } Tx \preceq Ty, \text{ for all, } x, y \in X.$$

Definition 9. Let A be a non-empty subset of a partially ordered set $(X, \preceq)$ If any two elements of A are comparable then it is called well ordered set.

Definition 10. A partially ordered metric space $(X, d, \preceq)$ is called an ordered complete, if for each convergent sequence $\{x_n\}_{n=0}^{+\infty} \subset X$, one of the following condition holds

- If $\{x_n\}$ is a nondecreasing sequence in X such that $x_n \to x$ implies $x_n \preceq x$, for all $n \in \mathbb{N}$ that is, $x = \sup\{x_n\}$ or
- If $\{x_n\}$ is a nonincreasing sequence in X such that $x_n \to x$ implies $x \preceq x_n$, for all $n \in \mathbb{N}$ that is, $x = \inf\{x_n\}$.

2. Main Results

In this section, we prove some coincidence point theorem in the context of ordered metric space.

Theorem 1. *Let $(X, d, \preceq)$ be a complete partially ordered metric space. Suppose that the self-mappings f and* T *on* X *are continuous,* T *is a monotone f-nondecreasing.*$T(X) \subseteq f(X)$ *and satisfying the condition:*

$$d(Tx,Ty) \le \alpha\left(\frac{d(fx,Tx)d(fy,Ty)}{d(fx,fy)}\right) + \beta[d(fx, fy)] + \gamma[d(fx,Tx) + d(fy,Ty)]$$
$$+ \delta[d(fx, Ty) + d(fy,Tx)] \qquad \ldots\ldots(2.1)$$

for all x,y in X *with* $f(x) \neq f(y)$ are comparable, where $\alpha, \beta, \gamma, \delta \in [0,1)$ with $0 \le \alpha + \beta + 2\gamma + 2\delta < 1$. *If there exists a point $x_0 \in X$ such that $f(x_0) \preceq T(x_0)$ and the mappings* T *and* f *are compatible, then* T *and* f *have a coincidence point in* X.

Proof. Let $x_0 \in X$ such that $f(x_0) \preceq T(x_0)$. Since from hypotheses, we have $T(X) \subseteq f(X)$ then, we can choose a point $x_1 \in X$ such that $fx_1 = Tx_0$. But $Tx_1 \in f(X)$ then, again there exists another point $x_2 \in X$ such that $fx_2 = Tx_1$. By continuing the same way, we can construct a sequence $\{x_n\}$ in X such that $fx_{n+1} = Tx_n$. for all n.

Again, by hypotheses, we have $f(x_0) \preceq T(x_0) = f(x_1)$ and T is a monotone f – nondecreasing mapping then, we get $T(x_0) \preceq T(x_1)$. Similarly, we obtain $T(x_1) \preceq T(x_2)$, *since* $f(x_1) \preceq f(x_2)$ and then by continuing the same procedure, we obtain that

$$T(x_0) \preceq T(x_1) \preceq T(x_2) \preceq \ldots\ldots.. T(x_n) \preceq T(x_{n+1}) \preceq \ldots\ldots$$

The equality $T(x_{n+1}) = T(x_n)$ is impossible because $f(x_{n+2}) \neq f(x_{n+1})$ for all $n \in \mathbb{N}$. Thus

$d(T(x_n), T(x_{n+1})) > 0$ for all $n \geq 0$ therefore, from contraction condition *(2.1)*, we have

$$d(Tx_{n+1},Tx_n) \le \alpha\left(\frac{d(fx_{n+1}, Tx_{n+1})\, d(fx_n, Tx_n)}{d(fx_{n+1}, fx_n)}\right) + \beta[d(fx_{n+1}, fx_n)]$$
$$+\gamma[d(fx_{n+1}, Tx_{n+1}) + d(fx_n, Tx_n)] + \delta[d(fx_{n+1}, Tx_n) + d(fx_n, Tx_{n+1})]$$

$$= \alpha\left(\frac{d(Tx_n, Tx_{n+1})\, d(Tx_{n-1}, Tx_n)}{d(Tx_n, Tx_{n-1})}\right) + \beta[d(Tx_n, Tx_{n-1})] +$$
$$+ \gamma[d(Tx_n, Tx_{n+1}) + d(Tx_{n-1}, Tx_n)] + \delta[d(Tx_n, Tx_n) + d(Tx_{n-1}, Tx_{n+1})]$$

$$= \alpha[d(Tx_n, Tx_{n+1})] + \beta[d(Tx_n, Tx_{n-1})] + \gamma[d(Tx_n, Tx_{n+1}) + d(Tx_{n-1}, Tx_n)]$$
$$+ \delta[d(Tx_{n-1}, Tx_{n+1})]$$
$$\le \alpha[d(Tx_n, Tx_{n+1})] + \beta[d(Tx_n, Tx_{n-1})] + \gamma[d(Tx_n, Tx_{n+1}) + d(Tx_{n-1}, Tx_n)]$$

$$+\ \delta[d(Tx_{n-1}, Tx_n) + d(Tx_n, Tx_{n+1})]$$
$$= (\alpha+\delta+\gamma)\, d(Tx_n, Tx_{n+1}) + (\beta+\gamma+\delta) d(Tx_{n-1}, Tx_n)$$

which implies that

$$d(Tx_{n+1}, Tx_n) \le \left(\frac{\beta+\gamma+\delta}{1-(\alpha+\gamma+\delta)}\right) \mathrm{d}(Tx_n, Tx_{n-1})$$

Continuing the same process up to (n-1) times, we get

$$d(Tx_{n+1}, Tx_n) \le \left(\frac{\beta+\gamma+\delta}{1-(\alpha+\gamma+\delta)}\right)^n \mathrm{d}(Tx_1, Tx_0)$$

Let $k = \frac{\beta+\gamma+\delta}{1-(\alpha+\gamma+\delta)} \in [0,1)$, then from triangular inequality for $m \ge n$, we have

$$d(Tx_m, Tx_n) \le \mathrm{d}(Tx_m, Tx_{m-1}) + \mathrm{d}(Tx_{m-1}, Tx_{m-2}) + \ldots\ldots\ldots + d(Tx_{n+1}, Tx_n)$$
$$\le (k^{m-1} + k^{m-2} + \ldots\ldots\ldots + k^n)\, \mathrm{d}(Tx_1, Tx_0)$$
$$\le \frac{k^n}{1-k} \mathrm{d}(Tx_1, Tx_0)$$

as $m, n \to +\infty$, $d(Tx_m, Tx_n) \to 0$, which shows that the sequence $\{Tx_n\}$ is a Cauchy sequence in X**.** So, by the completeness of X, there exists a point $u \in X$ such that $Tx_n \to u$ as $n \to +\infty$.

Again, by the continuity of T , we have

$$\lim_{n\to+\infty} \mathrm{T}(\mathrm{Tx_n}) = T(\lim_{n\to+\infty}(\mathrm{Tx_n})) = Tu.$$

But $f\mathrm{x}_{n+1} = \mathrm{Tx}_n$, then $f\mathrm{x}_{n+1} \to u$ as $n \to +\infty$ and from the compatibility for T and f , we have

$$\lim_{n\to+\infty} \mathrm{d}(\mathrm{T}(f\mathrm{x_n}), f(\mathrm{Tx_n})) = 0.$$

Further by triangular inequality , we have

$$d(Tu, fu) = d\big(Tu, T(f\mathrm{x_n})\big) + d\big(T(fx_n), f(\mathrm{Tx_n})\big) + d(f(\mathrm{Tx_n}), fu)$$

On taking limit as $n \to +\infty$ in both sides of the above equation and using the fact that T and f are continuous then, we get $d(Tu, fu) = 0$. Thus $Tu = fu$**.** Hence , u is a coincidence point of T and f in X**.**

Corollary 1. Let $(X, d, \preceq)$ be a complete partially ordered metric space. Suppose that the self-mappings f and T on X are continuous, T is a monotone f*-nondecreasing,* $\mathrm{T(X)} \subseteq f\mathrm{(X)}$ and satisfying the following condition

$$d(Tx, Ty) \le \alpha\left(\frac{d(fx, Tx)d(fy, Ty)}{d(fx, fy)}\right) + \gamma[d(fx, \mathrm{T}x) + d(fy, \mathrm{T}y)]$$

for all x , y in X with $f(x) \neq f(y)$ are comparable and for some $\alpha, \gamma \in [0,1)$ with $0 \le \alpha + 2\gamma < 1$.

If there exists a point $x_0 \in X$ such that $f(x_0) \preceq \mathrm{T}(x_0)$ and the mapping T and f are compatible, then T and f have a coincidence point in X**.**

Proof. Set $\beta = 0$, $\delta = 0$ in Theorem 1.

Corollary 2. Let $(X, d, \preceq)$ be a complete partially ordered metric space. Suppose that the self-mappings f and T on X are continuous, T is a monotone f*-nondecreasing,* $\mathrm{T(X)} \subseteq f\mathrm{(X)}$ and satisfying the following condition

$$d(Tx, Ty) \le \beta[d(fx, fy)] + \gamma[d(fx, \mathrm{T}x) + d(fy, \mathrm{T}y)]$$

for all x , y in X with $f(x) \neq f(y)$ are comparable and for some $\beta, \gamma \in [0,1)$ with $0 \le 2\gamma + \beta < 1$.

If there exists a point $x_0 \in X$ such that $f(x_0) \preceq \mathrm{T}(x_0)$ and the mapping T and f are compatible, then T and f have a coincidence point in X**.**

Proof. The proof can be obtain by setting $\alpha = 0$, $\delta = 0$ in Theorem 1.

Theorem 2. Let $(X, d, \preceq)$ be a complete partially ordered metric space. Suppose that f *and* T are self-mappings on X, T is a monotone f- nondecreasing, $T(X) \subseteq f(X)$ and satisfying

$$d(Tx, Ty) \leq \alpha\left(\frac{d(fx, Tx)d(fy, Ty)}{d(fx, fy)}\right) + \beta[d(fx, fy)] + \gamma[d(fx, Tx) + d(fy, Ty)]$$
$$+ \delta[d(fx, Ty) + d(fy, Tx)] \quad \ldots\ldots\ldots\ldots\ldots(2.2)$$

for all x,y in X *with* $f(x) \neq f(y)$ *are comparable*, where $\alpha, \beta, \gamma, \delta \in [0, 1)$ with $0 \leq \alpha + \beta + 2\gamma + 2\delta < 1$. *If there exists a point* $x_0 \in X$ *such that* $f(x_0) \preceq T(x_0)$ *and* $\{x_n\}$ *is a nondecreasing sequence in* X *such that* $x_n \to x$, *then* $x_n \preceq x$ *for all* $n \in \mathbb{N}$.

If $f(X)$ is a complete subset of X, then T and f have a coincidence point in X. Further, if T and f are weakly compatible, then T and f have a common fixed point in X. moreover, the set of common fixed points of T and f is well ordered if and only if T and f have one and only one common fixed point in X.

Proof. Suppose $f(X)$ is a complete subset of X. As we know from the proof of Theorem 1, the sequence $\{Tx_n\}$ is a Cauchy sequence and hence $\{fx_n\}$ is also a Cauchy sequence in $(f(X), d)$ as $fx_{n+1} = Tx_n$ and $T(X) \subseteq f(X)$**.** Since $f(X)$ is complete then there exists some $fu \in f(X)$ such that

$$\lim_{n\to+\infty} T(x_n) = \lim_{n\to+\infty} f(x_n) = fu.$$

Also note that the sequence $\{Tx_n\}$ and $\{fx_n\}$ are nondecreasing and from hypotheses, we have $T(x_n) \preceq f(u)$ and $f(x_n) \preceq f(u)$ for all $n \in \mathbb{N}$. But T is a monotone f- nondecreasing then, we get $T(x_n) \preceq T(u)$ for all n. Letting $n \to +\infty$, we obtain that $f(u) \preceq T(u)$.

Suppose that $f(u) \prec T(u)$ then define a sequence $\{u_n\}$ by $u_0 = u$ and $fu_{n+1} = Tu_n$ for all $n \in \mathbb{N}$. An argument similar to that in the proof of theorem 1 yields that $\{fu_n\}$ is a nondecreasing sequence and

$\lim_{n\to+\infty} f(u_n) = \lim_{n\to+\infty} T(u_n) = f(v)$ for some $v \in X$. So from hypotheses, it is clear that sup $f(u_n) \preceq f(v)$ and sup $T(u_n) \preceq f(v)$, for all $n \in \mathbb{N}$. Notice that

$$f(x_n) \preceq f(u) \preceq f(u_1) \preceq \ldots\ldots\ldots \preceq f(u_n) \preceq \ldots\ldots \preceq f(v).$$

Case:1 Suppose if there exists some $n_0 \geq 1$. such that $f(x_{n_0}) = f(u_{n_0})$ then, we have

$$f(x_{n_0}) = f(u) = f(u_{n_0}) = f(u_1) = T(u).$$

Hence, u is a coincidence point of T and f in X.

Case:2 suppose that $f(x_{n_0}) \neq f(u_{n_0})$ for all n then, from (2.2), we have

$$d(fx_{n+1}, fu_{n+1}) = d(Tx_n, Tu_n)$$
$$\leq \alpha\left(\frac{d(fx_n, Tx_n)d(fu_n, Tu_n)}{d(fx_n, fu_n)}\right)$$
$$+ \beta[d(fx_n, fu_n)] + \gamma[d(fx_n, Tx_n) + d(fu_n, Tu_n)]$$
$$+ \delta[d(fx_n, Tu_n) + d(fu_n, Tx_n)]$$

Taking limit as $n \to +\infty$ on both sides of the above inequality, we get

$$d(fu, fv) \leq \beta[d(fu, fv)] + \delta[d(fu, fv) + d(fv, fu)]$$
$$= (\beta + 2\delta)\, d(fu, fv). \text{ since } (\beta + 2\delta) < 1.$$

Thus we have

$$f(u) = f(v) = f(u_1) = T(u).$$

Hence, we conclude that u is a coincidence point of T and f in X.

Now, suppose that $T$and f are weakly compatible. Let ω be a coincidence point then,

$$T(\omega) = T\big(f(z)\big) = f\big(T(z)\big) = f(\omega), \text{ since } \omega = T(z) = f(z), \text{ for some } z \in \mathrm{X}.$$

Now by contraction condition, we have

$$\begin{aligned} d\big(T(z),T(\omega)\big) &\leq \alpha\left(\frac{d(fz,Tz)d(fw,Tw)}{d(fz,fw)}\right) + \beta[d(fz,fw)] + \gamma[d(fz,Tz) + d(fw,Tw)] \\ &\quad + \delta[d(fz,Tw) + d(fw,Tz)] \\ &\leq (\beta + 2\delta)\, d\big(T(z),T(\omega)\big) \end{aligned}$$

as $(\beta + 2\delta) < 1$, then $d\big(T(z),T(\omega)\big) = 0$. Therefore, $T(z) = T(\omega) = f(\omega) = \omega$. Hence, ω is a common fixed point of T and f in X.

Now suppose that the set of common fixed points of T and f is well ordered, we have to show that the common fixed point of T and f is unique. Let u and v be two common fixed points of T and f such that $u \neq v$ then from (2.2), we have

$$\begin{aligned} d(u,v) &\leq \alpha\left(\frac{d(fu,Tu)d(fv,Tv)}{d(fu,fv)}\right) + \beta[d(fu,fv)] + \gamma[d(fu,Tu) + d(fv,Tv)] \\ &\quad + \delta[d(fu,Tv) + d(fv,Tu)] \\ &\leq (\beta + 2\delta)\, d(u,v) \\ &< d(u,v), \text{ since } (\beta + 2\delta) < 1. \end{aligned}$$

Which is a contradiction. Thus, $u = v$. Conversely, suppose T and f have only one common fixed point then the set of common fixed points of T and f being a singleton is well ordered. This completes the proof.

Corollary 3. Let $(X, d, \preceq)$ be a complete partially ordered metric space. Suppose that f *and* T are self-mappings on X, T is a monotone f- nondecreasing, T(X) $\subseteq f$(X) and satisfying

$$d(Tx,Ty) \leq \alpha\left(\frac{d(fx,Tx)d(fy,Ty)}{d(fx,fy)}\right) + \gamma[d(fx,\mathrm{T}x) + d(fy,\mathrm{T}y)]$$

for all x,y in X *with* $f(x) \neq f(y)$ are comparable, where $\alpha, \gamma \in [0,1)$ with $0 \leq \alpha + 2\gamma < 1$. *If there exists a point* $x_0 \in$ X *such that* $f(x_0) \preceq \mathrm{T}(x_0)$ *and* $\{x_n\}$ *is a nondecreasing sequence in* X *such that* $x_n \to x$, *then* $x_n \preceq x$ *for all* $n \in \mathbb{N}$.

If $f(X)$ is a complete subset of X, then T and f have a coincidence point in X. Further, if T and f are weakly compatible, then T and f have a common fixed point in X. Moreover, the set of common fixed points of T and f is well ordered if and only if T and f have one and only one common fixed point in X.

Proof. Set $\beta = 0$, $\delta = 0$ in Theorem 2.

Corollary 4. Let $(X, d, \preceq)$ be a complete partially ordered metric space. Suppose that the self-mappings f and T on X , T is a monotone f*-nondecreasing*, T(X) $\subseteq f$(X) and satisfying

$$d(Tx,Ty) \leq \beta[d(fx,fy)] + \gamma[d(fx,\mathrm{T}x) + d(fy,\mathrm{T}y)]$$

for all x,y in X *with* $f(x) \neq f(y)$ are comparable, where $\beta, \gamma \in [0,1)$ with $0 \leq 2\gamma + \beta < 1$. *If there exists a point* $x_0 \in$ X *such that* $f(x_0) \preceq \mathrm{T}(x_0)$ *and* $\{x_n\}$ *is a nondecreasing sequence in* X *such that* $x_n \to x$, *then* $x_n \preceq x$ *for all* $n \in \mathbb{N}$.

If $f(X)$ is a complete subset of X, then T and f have a coincidence point in X. Further, if T and f are weakly compatible, then T and f have a common fixed point in X. Moreover, the set of common fixed points of T and f is well ordered if and only if T and f have one and only one common fixed point in X.

Proof. set $\alpha = 0$, $\delta = 0$ in Theorem 2.

Conclusion: In this paper, we prove a coincidence point and common fixed point results for compatible mappings in complete partially ordered metric space. Our results are generalizes and improve the results of Rao et al.[14] and chandok et al.[15].

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