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I_n- Mean Cordial Graphs

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Abstract

A graph labeling is an assignment of integers to the vertices or edges or both depending on certain conditions. A graph G with p vertices and q edges is said to admit I_n-mean cordial labeling if the vertex labeling h from $V(G)$to I_n, where $I_n = \{0, \pm 1, \pm 2, \ldots, \pm(n-1)\}$ induces the function h^* from $E(G)$ to I_n as$h^*(xy) = \left\lceil \frac{h(x)+h(y)}{2} \right\rceil$ with the condition that $|v_h(i) - v_h(j)| \leq 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \leq 1$ for all $i, j \in I_n$. A graph with I_n - mean cordial labeling is called an I_n - mean cordial graph. In this paper, we prove that path graphs, cycle graphs, complete graphs, wheel graphs and friendship graphs are I_2 - mean cordial graphs. Further, we prove that path graphs, cycle graphs are I_3 - mean cordial graphs.

Keywords: *labeling,cordial labeling, mean cordial labeling.*

1.Introduction

All graphs considered here are finite, simple, connected and undirected. A graph labeling is a function that carries the graph elements to a set of numbers. The labeling is called a vertex labeling(or an edge labeling) when the function's domain is a set of vertices(or edges). Cahit proposed the notion of cordial labeling in 1987 as a weaker version of graceful and harmonious labeling [2]. Several authors investigated the cordiality behaviour of various graphs. Some cordial labeling variations have been introduced such as prime cordial labeling, edge cordial labeling, product cordial labeling, integer cordial labeling, mean cordial labeling, k -cordial labeling and so on. The idea of mean cordial labeling was introduced by R. Ponraj, M. Sivakumar and M. Sundaram [5]. Let h be a function from V(G) to {0, 1, 2}. For each edge $xy$of G, assign the label $\left\lceil \frac{h(x)+h(y)}{2} \right\rceil$. The function h is called mean cordial labeling of G if $|v_h(i) - v_h(j)| \leq 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \leq 1$ for all $i, j \in I_n$, where $v_h(i), e_{h^*}(i)$ denote the number of vertices and edges labeled with i respectively. Motivated by this concept we introduce a new labeling called I_n – mean cordial labeling and we investigate I_2, I_3 - mean cordiality of some standard graphs. In this section we provide a summary of definitions required for our investigation.

Definition 1.1. Let $G = ((V(G), E(G))$ be a simple graph and let $h: V(G) \to I_n$ be a function, where $I_n = \{0, \pm 1, \pm 2, \ldots, \pm(n-1)\}$. For each edge xy, assign the label $h^*(xy) = \left\lceil \frac{h(x)+h(y)}{2} \right\rceil$. The function h is called I_n- mean cordial labeling if $|v_h(i) - v_h(j)| \leq 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \leq 1$ for all $i, j \in I_n$. A graph with I_n- mean cordial labeling is called an I_n- mean cordial graph.

Definition 1.2. The wheel graph W_n is defined to be the join $K_1 + C_n$. The vertex corresponding to K_1 is known as apex and the vertices corresponding to the cycle C_n are known as rim vertices while the edges corresponding to the cycle are known as rim edges.

Definition 1.3. The friendship graph F_n is a graph which consists of n triangles with a common vertex.

2. Main Results

Theorem 2.1. The path graph P_n admits I_2 - mean cordial labeling.

Proof. Let $G = P_n$. Let x_i be the vertices of G and let $E(G) = \{x_i x_{i+1}: 1 \leq i \leq n-1\}$ be the edge set of G. Define $h : V(G) \to I_2$ as follows:

Case(i): $n \equiv 0 \ (mod\ 3)$.

$$h(x_i) = \begin{cases} 1 & if\ 1 \leq i \leq \frac{n}{3} \\ -1 & if\ \frac{n}{3} + 1 \leq i \leq \frac{2n}{3} \\ 0 & if\ \frac{2n}{3} + 1 \leq i \leq n \end{cases}$$

In this case, h will induce the map $h^*: E(G) \to I_2$ and we get $v_h(i) = \frac{n}{3}$ for all $i \in I_2$; $e_{h^*}(0) = e_{h^*}(-1) = \frac{n}{3}$ and $e_{h^*}(1) = \frac{n}{3} - 1$.

Case(ii): $n \equiv 1 \ (mod\ 3)$.

$$h(x_i) = \begin{cases} 1 & if\ 1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil \\ -1 & if\ \left\lceil \frac{n}{3} \right\rceil + 1 \leq i \leq \left\lceil \frac{2n}{3} \right\rceil \\ 0 & if\ \left\lceil \frac{2n}{3} \right\rceil + 1 \leq i \leq n \end{cases}$$

In this case, we have $v_h(1) = \left\lceil \frac{n}{3} \right\rceil, v_h(0) = v_h(-1) = \left\lfloor \frac{n}{3} \right\rfloor$; $e_{h^*}(i) = \frac{n-1}{3}$ for all $i \in I_2$.

Case(iii): $n \equiv 2 \ (mod\ 3)$.

Here, label the vertices of G as in case(ii). Then we have $v_h(1) = v_h(-1) = \left\lceil \frac{n}{3} \right\rceil$ and $v_h(0) = \left\lfloor \frac{n}{3} \right\rfloor$; $e_{h^*}(0) = e_{h^*}(1) = \left\lfloor \frac{n-1}{3} \right\rfloor$ and $e_{h^*}(-1) = \left\lceil \frac{n-1}{3} \right\rceil$.

Thus in each case, we have $|v_h(i) - v_h(j)| \leq 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \leq 1$ for all $i, j \in I_2$. Hence, P_n is an I_2 - mean cordial graph.

Theorem 2.2. The cycle graph C_n admits I_2 - mean cordial labeling.

Proof. Let G be the cycle graph C_n. Let $x_i (1 \leq i \leq n)$ be the vertices of G and let $E(G) = \{x_i x_{i+1}: 1 \leq i \leq n-1\} \cup \{x_n x_1\}$ be the edge set of G. Now, label the vertices of G as in Theorem 2.1. Then the labeling h will induce the map $h^* : E(G) \to I_2$ and we get $v_h(i) = e_{h^*}(i) = \frac{n}{3} (i \in I_2)$ for $n \equiv 0 \ (mod\ 3)$; $v_h(1) = e_{h^*}(1) = \left\lceil \frac{n}{3} \right\rceil$ and $v_h(0) = v_h(-1) = e_{h^*}(0) = e_{h^*}(-1) = \left\lfloor \frac{n}{3} \right\rfloor$ for $n \equiv 1 \ (mod\ 3)$. Also $v_h(1) = v_h(-1) = e_{h^*}(1) = e_{h^*}(-1) = \left\lceil \frac{n}{3} \right\rceil$ and $v_h(0) = e_{h^*}(0) = \left\lfloor \frac{n}{3} \right\rfloor$ for $n \equiv 2 \ (mod\ 3)$.

Thus in each case, we have $|v_h(i) - v_h(j)| \leq 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \leq 1$ for all $i, j \in I_2$.Hence, C_n is an I_2 - mean cordial graph.

Theorem 2.3. The complete graph K_n admits I_2 - mean cordial labeling.

*Proof.*Let $G = K_n$. Let $x_i (1 \leq i \leq n)$ be the vertices of G. Define $h : V(G) \to I_2$ as follows:

Case(i):$n \equiv 0 \ (mod\ 3)$.

In this case, label the vertices of G in any order in such a way that $v_h(i) = \frac{n}{3}$ for all $i \in I_2$.Then we have $e_{h^*}(i) = \frac{n(n-1)}{6}$ for all $i \in I_2$.

Case(ii):$n \equiv 1 \ (mod\ 3)$.

Here, label the vertices of G in any order with $v_h(0) = \left\lceil \frac{n}{3} \right\rceil, v_h(1) = v_h(-1) = \left\lfloor \frac{n}{3} \right\rfloor$.Then the number of edges labeled with $i \in I_2$ are $e_{h^*}(i) = \frac{n(n-1)}{6}$.

Case(iii):$n \equiv 2 \ (mod\ 3)$.

Here, label the vertices of G in any order with $v_h(0) = \left\lfloor \frac{n}{3} \right\rfloor, v_h(1) = v_h(-1) = \left\lceil \frac{n}{3} \right\rceil$. Then the number of edges labeled with $i \in I_2$are $e_{h^*}(1) = e_{h^*}(-1) = \left\lfloor \frac{n(n-1)}{6} \right\rfloor$, $e_{h^*}(0) = \left\lceil \frac{n(n-1)}{6} \right\rceil$.

Thus in each case, we have $|v_h(i) - v_h(j)| \leq 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \leq 1$ for all $i, j \in I_2$.Hence, K_n is an I_2 - mean cordial graph.

Theorem 2.4. The star graph $K_{1,n}$ admits I_2 - mean cordial labeling.

Proof. Let G be the star graph $K_{1,n}$.Let$x, x_i \ (1 \leq i \leq n)$ be the vertices of G. Then $E(G) = \{xx_i : 1 \leq i \leq n\}$is the edge set of G. Now, label the vertex x by 0 and the vertices $x_i (1 \leq i \leq n)$ as follows:

Case(i):$n \equiv 0 \ (mod\ 3)$

In this case, label the vertices $x_i (1 \leq i \leq n)$of G in any order in such a way that $v_h(0) = \frac{n}{3} + 1$, $v_h(1) = v_h(-1) = \frac{n}{3}$ for all $i \in I_2$. Then the number of edges labeled with $i \in I_2$ are$e_{h^*}(i) = \frac{n}{3}$.

Case(ii):$n \equiv 1 \ (mod\ 3)$

Here, label the vertices $x_i \ (1 \leq i \leq n)$of G in any order with $v_h(0) = v_h(1) = \left\lceil \frac{n}{3} \right\rceil, v_h(-1) = \left\lfloor \frac{n}{3} \right\rfloor$. Then we have $e_{h^*}(1) = \left\lceil \frac{n}{3} \right\rceil, e_{h^*}(0) = e_{h^*}(-1) = \left\lfloor \frac{n}{3} \right\rfloor$.

Case(iii):$n \equiv 2 \ (mod\ 3)$

In this case, label the vertices of G in any order with $v_h(i) = \left\lceil \frac{n}{3} \right\rceil$ for all $i \in I_2$. Then the number of edges labeled with $i \in I_2$ are $e_{h^*}(1) = e_{h^*}(-1) = \left\lceil \frac{n}{3} \right\rceil$, $e_{h^*}(0) = \left\lfloor \frac{n}{3} \right\rfloor$.

Thus in each case, we have $|v_h(i) - v_h(j)| \leq 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \leq 1$ for all $i, j \in I_2$.Hence, $K_{1,n}$ is an I_2 - mean cordial graph.

Theorem 2.5. The wheel graph W_n admits I_2 - mean cordial labeling.

*Proof.*Let $G = W_n$. Let x be the apex vertex and let $x_i \ (i \leq i \leq n)$ be the rim vertices of G and let $E(G) = \{xx_i : 1 \leq i \leq n\} \cup \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_n x_1\}$ be the edge set of G. Define $h : V(G) \to I_2$ as follows:

Case(i):$n \equiv 0 \ (mod\ 3)$

In this case, label the apex vertex by 0 and the rim vertices as in Theorem 2.1. Then we have $v_h(0) = \frac{n}{3} + 1, v_h(1) = v_h(-1) = \frac{n}{3}$; $e_{h^*}(i) = \frac{2n}{3}$ for all$i \in I_2$.

Case(ii): $n \equiv 1 \ (mod \quad 3)$

$$h(x) = 0, h(x_n) = -1 \text{ and } h(x_i) = \begin{cases} 1 & if \quad 1 \le i \le \left\lceil \frac{n}{3} \right\rceil \\ -1 & if \quad \left\lceil \frac{n}{3} \right\rceil + 1 \le i \le \left\lfloor \frac{2n}{3} \right\rfloor \\ 0 & if \quad \left\lceil \frac{2n}{3} \right\rceil \le i \le n-1 \end{cases}.$$

Here, we have $v_h(0) = v_h(1) = \left\lceil \frac{n}{3} \right\rceil, v_h(-1) = \left\lfloor \frac{n}{3} \right\rfloor$; $e_{h^*}(0) = e_{h^*}(1) = \left\lceil \frac{2n}{3} \right\rceil$ and $e_{h^*}(-1) = \left\lfloor \frac{2n}{3} \right\rfloor$.

Case(iii): $n \equiv 2 \ (mod \quad 3)$

Here, label the vertices of G as in case(ii). Then, we have $v_h(0) = v_h(1) = \left\lceil \frac{n}{3} \right\rceil$, $v_h(-1) = \left\lfloor \frac{n}{3} \right\rfloor$; $e_{h^*}(0) = e_{h^*}(1) = \left\lfloor \frac{2n}{3} \right\rfloor$ and $e_{h^*}(-1) = \left\lceil \frac{2n}{3} \right\rceil$.

Thus in each case, we have $|v_h(i) - v_h(j)| \le 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \le 1$ for all $i, j \in I_2$. Hence, W_n is an I_2 - mean cordial graph.

Theorem 2.6. The friendship graph F_n admits I_2 - mean cordial labeling.

Proof. Let G be the friendship graph F_n. Let $x, x_i, y_i \ (i \le i \le n)$ be the vertices of G and let $E(G) = \{x x_i, x y_i, x_i y_i : 1 \le i \le n\}$ be the edge set of G. Define $h: V(G) \to I_2$ as follows:

Case(i): $n \equiv 0 \ (mod \quad 3)$

$$h(x) = 0, \ h(x_i) = \begin{cases} 0 & if \quad 1 \le i \le \frac{2n}{3} \\ 1 & if \quad \frac{2n}{3} + 1 \le i \le n \end{cases} \text{ and } h(y_i) = \begin{cases} 1 & if \quad 1 \le i \le \frac{n}{3} \\ -1 & if \quad \frac{n}{3} + 1 \le i \le n \end{cases}$$

In this case, we have $v_h(1) = v_h(-1) = \left\lfloor \frac{2n+1}{3} \right\rfloor$ and $v_h(0) = \left\lceil \frac{2n+1}{3} \right\rceil$.

Case(ii): $n \equiv 1 \ (mod \quad 3)$

$$h(x) = 0, \qquad h(x_i) = \begin{cases} 0 & if \quad 1 \le i \le \frac{2(n-1)}{3} \\ 1 & if \quad \frac{2n+1}{3} \le i \le n-1 \end{cases}$$

$$h(y_i) = \begin{cases} 1 & if \quad 1 \le i \le \frac{n-1}{3} \\ -1 & if \quad \frac{n+2}{3} \le i \le n-1 \end{cases}, \quad h(x_n) = 1 \text{ and } h(y_n) = -1.$$

Here, we have $v_h(i) = \frac{2n+1}{3}$ for all $i \in I_2$.

Case(iii): $n \equiv 2 \ (mod \quad 3)$

Here, label the vertices of G as in case(i) for $1 \le i \le n-2$ and $h(x_{n-1}) = h(x_n) = 1, h(y_{n-1}) = h(y_n) = -1$. In this case, we have $v_h(0) = \left\lfloor \frac{2n+1}{3} \right\rfloor$ and $v_h(1) = v_h(-1) = \left\lceil \frac{2n+1}{3} \right\rceil$.

Also in each case, we get $e_{h^*}(i) = n$ for all $i \in I_2$. Thus in all the cases, we have $|v_h(i) - v_h(j)| \le 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \le 1$ for all $i, j \in I_2$. Hence, F_n is an I_2 - mean cordial graph.

Theorem 2.7. The path graph P_n admits I_3 - mean cordial labeling.

*Proof.*Let $G = P_n$. Let x_i be the vertices of G and let $E(G) = \{x_i x_{i+1}: 1 \leq i \leq n-1\}$ be the edge set of G. Define $h: V(G) \rightarrow I_2$ as follows:

Case(i): $n \equiv 0, 1 \text{ or } 4 \pmod 5$.

$$h(x_i) = \begin{cases} 1 & \text{if } 1 \leq i \leq \left\lceil \frac{n}{5} \right\rceil \\ 2 & \text{if } \left\lceil \frac{n}{5} \right\rceil + 1 \leq i \leq \left\lceil \frac{2n}{5} \right\rceil \\ -2 & \text{if } \left\lceil \frac{2n}{5} \right\rceil + 1 \leq i \leq \left\lceil \frac{3n}{5} \right\rceil \\ -1 & \text{if } \left\lceil \frac{3n}{5} \right\rceil + 1 \leq i \leq \left\lceil \frac{4n}{5} \right\rceil \\ 0 & \text{if } \left\lceil \frac{4n}{5} \right\rceil + 1 \leq i \leq n \end{cases}$$

In this case, the number of vertices and edges labeled with $i \in I_3$ are as follows:

$v_h(i) = \frac{n}{5} (i \in I_3)$, $e_{h^*}(1) = \left\lfloor \frac{n-1}{5} \right\rfloor$, $e_{h^*}(i) = \left\lceil \frac{n-1}{5} \right\rceil$ $(i = 0, -1, \pm 2)$for $n \equiv 0 \pmod 5$;

$v_h(1) = \left\lceil \frac{n}{5} \right\rceil$, $v_h(i) = \left\lfloor \frac{n}{5} \right\rfloor (i = 0, -1, \pm 2)$, $e_{h^*}(i) = \frac{n-1}{5} (i \in I_3)forn \equiv 1 \pmod 5$;

$$v_h(i) = \left\lceil \frac{n}{5} \right\rceil (i = \pm 1, \pm 2), \quad v_h(0) = \left\lfloor \frac{n}{5} \right\rfloor, e_{h^*}(i) = \left\lceil \frac{n-1}{5} \right\rceil (i = -1, \pm 2),$$

$e_{h^*}(i) = \left\lfloor \frac{n-1}{5} \right\rfloor (i = 0,1)$ for$n \equiv 4 \pmod 5$.

Case(ii): $n \equiv 2 \pmod 5$.

$$h(x_i) = \begin{cases} 1 & \text{if } 1 \leq i \leq \left\lceil \frac{n}{5} \right\rceil \\ 2 & \text{if } \left\lceil \frac{n}{5} \right\rceil + 1 \leq i \leq \left\lceil \frac{2n}{5} \right\rceil + 1 \\ -2 & \text{if } \left\lceil \frac{2n}{5} \right\rceil + 2 \leq i \leq \left\lceil \frac{3n}{5} \right\rceil \\ -1 & \text{if } \left\lceil \frac{3n}{5} \right\rceil + 1 \leq i \leq \left\lceil \frac{4n}{5} \right\rceil \\ 0 & \text{if } \left\lceil \frac{4n}{5} \right\rceil + 1 \leq i \leq n \end{cases}$$

In this case, we have $v_h(i) = \left\lceil \frac{n}{5} \right\rceil (i = 1, 2)$, $v_h(i) = \left\lfloor \frac{n}{5} \right\rfloor (i = 0, -1, -2)$,

$$e_{h^*}(2) = \left\lceil \frac{n-1}{5} \right\rceil, e_{h^*}(i) = \left\lfloor \frac{n-1}{5} \right\rfloor (i = 0, \pm 1, -2).$$

Case(iii): $n \equiv 3 \pmod 5$.

$$h(x_i) = \begin{cases} 1 & if \quad 1 \le i \le \left\lceil \frac{n}{5} \right\rceil \\ 2 & if \quad \left\lceil \frac{n}{5} \right\rceil + 1 \le i \le \left\lceil \frac{2n}{5} \right\rceil \\ -2 & if \quad \left\lceil \frac{2n}{5} \right\rceil + 1 \le i \le \left\lceil \frac{3n}{5} \right\rceil + 1 \\ -1 & if \quad \left\lceil \frac{3n}{5} \right\rceil + 2 \le i \le \left\lceil \frac{4n}{5} \right\rceil \\ 0 & if \quad \left\lceil \frac{4n}{5} \right\rceil + 1 \le i \le n \end{cases}$$

In this case, we have

$$v_h(i) = \left\lceil \frac{n}{5} \right\rceil (i = 1, 2, -2), v_h(i) = \left\lfloor \frac{n}{5} \right\rfloor (i = 0, 2), e_{h^*}(i) = \left\lceil \frac{n-1}{5} \right\rceil (i = \pm 2), e_{h^*}(i) = \left\lfloor \frac{n-1}{5} \right\rfloor (i = 0, \pm 1).$$

Thus in each case, we have $|v_h(i) - v_h(j)| \le 1$ and $|e_{h^*}(i) - e_{h^*}(j)| \le 1$ for all $i, j \in I_2$.Hence, P_n is an I_3 - mean cordial graph.

Theorem 2.8. The cycle graph C_n admits I_3 - mean cordial labeling.

Proof. Let G be the cycle graph C_n. Let $x_i\ (1 \le i \le n)$ be the vertices of G and let $E(G) = \{x_i x_{i+1} : 1 \le i \le n-1\} \cup \{x_n x_1\}$ be the edge set of G. Now, label the vertices of G as in Theorem 2.7. Then number of vertices and edges labeled with $i \in I_3$ are as follows:

$v_h(i) = e_{h^*}(i) = \frac{n}{5}$ for all $i \in I_3$, $n \equiv 0 \ (mod \ 5)$

$$v_h(i) = e_{h^*}(i) = \begin{cases} \left\lceil \frac{n}{5} \right\rceil & if\ i = 1 \\ \left\lfloor \frac{n}{5} \right\rfloor & if\ i = 0, -1, \pm 2 \end{cases} \quad \text{for } n \equiv 1 \ (mod \ 5)$$

$$v_h(i) = e_{h^*}(i) = \begin{cases} \left\lceil \frac{n}{5} \right\rceil & if\ i = 1, 2 \\ \left\lfloor \frac{n}{5} \right\rfloor & if\ i = 0, -1, -2 \end{cases} \quad \text{for } n \equiv 2 \ (mod \ 5)$$

$$v_h(i) = e_{h^*}(i) = \begin{cases} \left\lceil \frac{n}{5} \right\rceil & if\ i = 1, \pm 2 \\ \left\lfloor \frac{n}{5} \right\rfloor & if\ i = 0, -1 \end{cases} \quad \text{for } n \equiv 3 \ (mod \ 5)$$

$$v_h(i) = e_{h^*}(i) = \begin{cases} \left\lceil \frac{n}{5} \right\rceil & if\ i = \pm 1, \pm 2 \\ \left\lfloor \frac{n}{5} \right\rfloor & if\ i = 0 \end{cases} \quad \text{for } n \equiv 4 \ (mod \ 5)$$

Thus in each case, we have $|v_h(i) - v_h(j)| \le 1$and $|e_{h^*}(i) - e_{h^*}(j)| \le 1$ for all $i, j \in I_2$. Hence, C_n is an I_3 - mean cordial graph.

Theorem 2.9. The star graph $K_{1,n}$ admits I_3 - mean cordial labelling if and only if $n \le 4$.

Proof. Let G be the star graph $K_{1,n}$. Let $x, x_i\ (1 \le i \le n)$ be the vertices of G . Then $E(G) = \{x x_i : 1 \le i \le n\}$ is the edge set of G.

If $n \le 2$, the result is true by Theorem 2.7. If $n = 3$ or 4, label $h(x) = 1, h(x_1) = 0, h(x_2) = -1, h(x_3) = 2, h(x_4) = -2$. Here, G satisfies the vertex and edge conditions.

Suppose that G admits I_3 – mean cordial labeling h for all $n > 4$. Then $h(x) \ne 0$; otherwise $e_{h^*}(2) = e_{h^*}(-2) = 0$. Now, if $h(x) = 1$, $e_{h^*}(-2) = 0$; if $h(x) = -1$, $e_{h^*}(2) = 0$; if $h(x) = 2$, $e_{h^*}(-1) = 0$; if $h(x) = -2$, $e_{h^*}(1) = 0$.

Thus in each case, we have $|e_{h^*}(i) - e_{h^*}(j)| > 1$ for some $i, j \in I_3$. Hence, $K_{1,n}$ is an I_3 - mean cordial graph.

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