

## Game Theory based Consensus Learning in Unidirectional Rings

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**Abstract:** This paper discusses the game theory based consensus learning in unidirectional rings. Distributed

system finds the ambiguous processors or rational agents in the same way as game theory finds ambiguous players. The rational agents are those processors which deviate from the protocol and instead they follow their own path in order to fulfil their utility. In a bidirectional rings, the rational agents can be detected easily by each neighbourhood agents. But, in unidirectional case, the rational agents cannot be easily detected. This limits the honest agent to detect incorrect IDs sent by adversarial agents. So, our main focus is on the asynchronous unidirectional ring. We study the shortcomings of the protocol presented in (Abraham *et al.* 2013) and (Afeke *et al.* 2014). In this paper, we present an improved version of the protocol based on Game Theory which is more resilient to the previous one, by applying invasion of adversarial processors and by proving it to be resilient in presence of adversaries.

**Keywords:** Consensus problem; Leader Election; Game Theory; Unidirectional Rings.

### 1. Introduction

Obtaining consensus in distributed computing on some data value is a fundamental problem to achieve overall reliability. In distributed system, the all components cannot be fault free. Therefore, all the components should agree on some data or state to guarantee the reliability of the system. Consensus can be understood as an agreement between the components of the distributed system on a data value or component. When the consensus is achieved on a component, then this mechanism is known as consensus mechanism based on leader election.

Consensus in distributed system is to elect a leader in an anonymous networks. Message passing in this network does not have a fixed time to send or receive the messages. There is no guarantee that the message will be received on time or reach to its destination. Therefore, some components may become rational components in the system and they start their own paths.

In this paper, we incorporate game theory approach to maximize its Expected Utility. In this case, the utility will be to choose itself as a leader or to form a coalition of agents to elect a leader among

themselves. The implementation of this algorithm is done on unidirectional ring networks.

The rest of this paper is organized as follows. Section II briefly describes the background information. Section III presents the fundamentals of game theory and game models that are commonly used in blockchain. Section IV discusses applications of game theory for security issues in blockchain. Section V presents applications of game theory for the mining management in blockchain. Section VI discusses applications of game theory at top blockchain platforms. Section VII outlines challenges and future research directions. Section VIII summarizes and concludes the paper.

## 2. Background Information

### 2.1 Consensus

Designing and Implementation of a fault tolerant distributed system is considered as one of the complex endeavours in the last few years. The verification of these models is the next challenge that we face after implementation. Nowadays, several new techniques have been developed to identify and simplify these tasks. There are two techniques which are widely used for this purpose.

They are Atomic Broadcasting and Consensus. In distributed system, simple definition of consensus is agreement. It is a task to get all the processors in a network to agree on some specific protocol or condition based on the votes of each processors. It allows each processor to reach to a common conclusion and result, where each processor may or may not have similar inputs. Many problems that arise in distributed system can be solved through such as leader election.

### 2.2 Unidirectional Ring Network

In a unidirectional ring network as shown in Fig. 1, the data flows in only one direction either clockwise or anti-clockwise and such a network is called a half-duplex network. In a bidirectional ring network, each agent can communicate with its predecessor and successor as well. This allows each agent to detect faulty agents in its neighborhood by continuously matching the information sent by its neighbor agents in previous rounds. However, this is not the case in unidirectional ring networks. The message is passed in only single direction and that direction is decided beforehand. This limits the honest agent to detect incorrect IDs sent by adversarial agents. So, our main focus is on a synchronous unidirectional ring network.

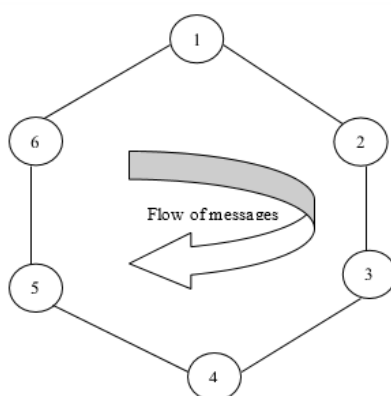


Fig. 1. Unidirectional Ring Network

### 2.3 Game Theory in Distributed System

Game theory approach has been a better option for solving the complicated problems in distributed system. In game theory concept, there are no good players or bad players, but rational players make moves in selfish manner by assigning payoffs (or utilities) to outcomes. With this type of play with rational players, game theory-predictions consists of several solution concepts considering the outcome of a game with rational players. In selfish type of play in game theory, the players make best responses to their beliefs. But, in the team, all the players are unknown what the others players' strategies are.

Let us assume a multi-party game model where there are  $n$  number of participants and none of them are honest. These participants or players are referred as either rational or adversarial. The rational players are those who act in their selfish interest to maximize their utility. Adversarial players are those who act arbitrarily (Lysyanskaya *et al.* 2006).

### 2.4 Assumptions

In our protocol, we assume that each processor knows about their IDs as well as the IDs of the processors present in the network. This assumption is necessary because getting to know the information about each and every processor during the execution of the protocol is trivial in electing a leader. Hence, we created a wake-up phase which runs before the actual protocol and facilitates each processor with the information of other processors. Once each processor knows about every other processor, our main protocol starts. Although, we should not deny the fact that knowing the IDs of other processors may have many other advantages over not knowing about it at all.

The first advantage is that each processor gets to know about the total number of processors present in the ring. This information should necessarily be available or else the protocol would never stop. The processors may not know when to stop sending messages to their neighbouring processors if there is no stopping condition such as when the last processor receives the final message, no new messages should be sent until the initiator starts sending again. The second advantage of this is that at the time of sending their IDs to its neighbours, processors will receive IDs from other processors as well. Hence, they use this information to verify that the IDs sent by other processors is the same id that they sent during wake-up phase.

We assume that the agents are rational. If we assume that each processor prefers to have a leader rather than having no leader at all and that they are indifferent of who becomes the leader then any leader election algorithm can be applied. But if there exist a single rational agent among the honest agents who deviates from the protocol to get the maximum benefit, then these protocols fail to perform well. If we view the scenario as a formal intuition where all the processors are honest, then we implicitly assume that in standard distributed settings all processors follow the designated algorithm.

But what happens when agents prefer to have different leaders? One of the cases may be when some agent wants himself to be elected as a leader as it would lower the cost of routing with other agents. Here, the standard settings of distributed protocol (which assumes that each agent has distinct id, and the leader is elected based on the highest or the lowest id present in the network) fails to work. Thus, our major focus being on obtaining a fair Nash Equilibrium, where each processor has an equal

probability of becoming a leader. In simple words, the probability that someone will be elected as a leader should be 1. But before digging into the solutions, we need to understand what Nash equilibrium means in an asynchronous setting.

## 2.5 Advantages

In a distributed system, leader election is an idea of giving some special power to one of the processors in a network which include assigning work, scaling bottlenecks, handling responsibilities of requests in a system. It is a powerful tool for reducing operations, reducing coordination, improving efficiency and throughput, simplifying architecture, etc. Leader election provides a centralized way of management where a single processor makes decisions suitable for the rest of the processors and the entire system itself. Advantages of leader election are as follows:

- A leader provides consistency to the clients as it has the power to control the changes that can be made to the state of the system.
- Designing a software for one specific processor is much easier than designing different protocols for each individual processor. The leader does not care about the conditions of system whether working or not at the time of decision making.
- One of the major advantages of leader election is to reduce partial failures and keeping logs and metrics for failure and recovery. If there is one centralized system that takes all the decisions, it makes the system easier for humans to think about.
- Efficiency of a centralized system is better since it can simply circulate the changes need to be made rather than building consensus about the changes need to be made.
- A single leader can improve performance or reduce cost by providing a single consistent cache of data which can be used every time.

## 2.6 Disadvantages

Disadvantages of leader election are as follows:

- Paradigms like partial deployment, incremental deployment, A-B testing, automatic rollback is hard to apply in leader election system.
- Adversarial attacks of processors may result in disobeying leader's decision and following their own protocols to maximize their optimal expected utility.
- A single leader is prone to single point failure. The entire system will suffer from unavailability if the leader itself is failed or fix a faulty processor.
- If the leader itself is not following the designer's protocol and doing wrong work and if there is no other node to check for it, then it can make decisions for the processors which will be in favor of the leader and the entire system will have a high blast radius.

## 2.7 Motivation

In this paper, our motive is to solve the consensus problem based on game theory approach. Here, leader election is used because of:

- **Data Consistency:** In a bank server system, multiple servers exist in a cloud, but only a single server should be responsible for a single customer.
- **Data Replication:** If a group of servers decide to replicate files among themselves, they need to elect a server that contains the primary copy of the file that will communicate with other client machines.
- **Error-Free Transition:** In comparison to government elections, sometimes leaders communicate with clients that have different kinds of information which may or may not be understood by other server nodes. So, leader transforms the information so that it is understood by all the nodes.

This paper does not focus on how leader election can be applied using game theory approach, but it focuses on how leader election in a unidirectional ring can be made fault tolerant so that it is efficient and that no other node deviates from the protocol. The paper proposes some theorems and proofs to use game theory as a tool to define how leader is elected and that why other nodes get to the conclusion of electing that particular node as a leader. Also, it states why no other node (not leader) deviates from its decision of choosing a leader rather than being one.

## 3. Related work

There is one study where it is proved that a random protocol can achieve resilience to coalition of linear size. This means that if the coalitions are linearly dependent to  $n$  then the protocol is resilient. This was found by Michael Saks which is given in (Saks and Michael 1989). In the later years, Boppana optimal resilience. i.e., resilience to size  $(\frac{1}{2} - \epsilon)$ . It is important to note that our approach can be and Narayanan (Boppana et al. 2000) provided a brief proof of this protocol that has approximate well suited to define the underlying principle of (Abraham et al. 2013) which shows that it is easy to design a leader election protocol when working under synchronous or completely connected network. For asynchronous settings, these protocols fail to perform well even if there is a single processor failure since we cannot identify whether the processor is really at fault or just responding really slowly. They also stated that in bidirectional ring network, each processor can send messages to its predecessor as well as its successor since the ring is bidirectional. This means that if there is a single fault in the network, the processors can confirm it by sending the message in the opposite direction from where the fault has initially occurred. The protocol that works for unidirectional network in synchronous settings will work same as in bidirectional network in synchronous settings. The only thing needed to change is the direction of the message transfer at the time of initiation. If there is no agreement among the processors on the orientation of message passing then the initiator processor can choose the direction arbitrarily and rest of the protocol will be executed as designed; all the other processors will continue to forward their messages in the same direction. If any of the processor registers its buffer with 'null', then the protocol will fail and no leader will be chosen. Since this approach reduces the adversarial attacks on the protocol, it will not be fair in terms of getting an elected leader. If the processor keeps on deviating from the

protocol, no leader will be chosen at all. That is why, we assume in our model that each processor follows a solution-driven approach.

### 3.1 Secret sharing and Multiparty Computation

**Secret Sharing method:** A method to divide a secret into parts, called shares that are distributed among a group of agents such that the secret can only be reconstructed when an authorized group of agents combine their shares. Combinations of shares of unauthorized agents do not reveal the secret.

**Multiparty computation method:** Insecure multi-party computation (MPC), a set of parties, each having a secret value, want to compute a common function over their inputs, without revealing any information about their inputs other than what is revealed by the output of the function. In (Abraham *et al.* 2006) they mathematically proved that if there is a coalition  $C$  of size  $k$  and all the members of the coalition deviate from the protocol then no member can do better in electing a leader among them. Few other things other than this was that the adversarial agents can attack and get a leader elected if they prefer a solution driven approach, which is exactly what we assumed in our model. In solution driven approach the processors tend to elect a fair leader rather than having no leader at all. Finally, Abraham *et al.* in (Abraham *et al.* 2006) discussed the mediator technique and simulated games with mediator by games without

mediator. Their main contribution lies on the improvement of the results by (Andrychowicz *et al.* 2014) where they used rationality to stick out from the traditional method of ‘good-guy’ ‘bad-guy’ model which is a standard distributed computing community model. (Andrychowicz *et al.* 2014) provided a model that had a result with 1-resilient equilibrium. (Abraham *et al.* 2013) improved this result to  $(n-1)$  resilience.

This means that if any multiparty computation is carried out by a trusted mediator, then it can also be done without a mediator with a high resilience output. Their result gives a deeper understanding of cheap talk where a model can be built to securely simulate resilient equilibrium without the use of a trusted third party mediator. The motivation to do so is because there is a possibility that we may not find a trusted third party all the time. In (Izmalkov *et al.* 2011) and (Lepinski *et al.* 2004) theoretically described two ways to actually get an equilibrium in a game which is converted from using trusted mediator to no trusted mediator at all. They relied on two very strong fundamentals known as ballot box and envelope. These fundamentals are undeveloped to be implemented in a model yet. (Vida *et al.* 2013) proposed a model to simulate Nash equilibrium with the help of a trusted mediator. He used the strategy called ‘punishing the player’ which threatens the players who deviate from the protocol by not electing any processor as a leader if they come to know someone is not following the protocol. Later (Heller and Yuval 2010) extended Ben Porath’s result to model this approach in a coalition. They show that if standard assumptions are made in cryptography, an equilibrium of a game can be simulated with a mediator such that players use punishment strategy.

### 3.2 BAR Model

**BAR Model (Byzantine, Altruistic, Rational):** Byzantine nodes may deviate arbitrarily from the suggested protocol for any reason. They may be broken (e.g., misconfigured, compromised, malfunctioning, or mis-programmed) or may just be optimizing for an unknown utility function that differs from the utility function used by rational nodes—for instance, ascribing a value to harm inflicted on the system or its users.

Altruistic nodes follow the exact protocol. They represent “seed nodes” in real system. Rational nodes are self-interested and seek to maximize their benefit according to a known utility function. They deviate from their suggested protocol only if doing so, gives them better utility in participating in the system. (Aiyer et al. 2005) discussed BAR model and protocols that can tolerate Byzantine behaviours when the protocol is not followed, misconfigured or broken. Rational behaviour of an agent when selfish processor deviates from the designer’s specification to increase their expected utility outcome. Amidst this, they reduced the complexity of a general three-level architecture model under BAR to get optimal building services. They described the first ever backup service to condone Byzantine processors and the number of unbounded rational users.

(Chandra et al. 1996) introduced a fault detection algorithm which is based on the study of an actual RAIN system. RAIN stands for Reliable Array of Independent Nodes which is implemented to recovering faults using distributed checkpoint. Other than this they designed a fault tolerant database and distributed web services. For a system to be fault tolerant, the system must monitor each node independently and keep track on failures and neighbouring processors must take actions if their successor or predecessor are at fault. They specifically make use of two approaches to help detect failure and take necessary actions for recovery.

The first approach is group membership that provides coalition of processors which work in unison to detect their neighbour’s status (Lysyanskaya et al. 2006). The second approach is that this paper is based on i.e., leader election. Leader election protocol brings an inconsistency in global state. Tasks like consensus, load balancing, resource allocation, etc can be solved once a leader has been elected (Abraham et al. 2013). But even these approaches have some problems related to them. Both of them require some kind of agreement during the presence of failure. One should keep in mind the impossible result that (Fang and Chunsheng 2011) which showed that it is impossible to get consensus in an asynchronous network if there exist a single silent failure. (Chandra et al. 1996) discussed that we can remove weakly specified group members that are prone to system failure sooner than other healthy agents.

### 3.3 Similar Algorithms

(Bakshi et al. 2011) presented a leader election algorithm in asynchronous bidirectional ring based on probabilistic approach. The final election of leader is done through random identity selection, to detect identity clashes they used hop counters and round modulo 2. The validation and verification are done by building the model, final-state, so that the probability of elected leader should be 1. Their work is an extended study of the leader election algorithm in (Chan et al. 1979). They required to have unique identities for each of the processor and that it should be ordered. Their algorithm elects a leader with the largest identity available in the network. This may give rise to adversarial attack of the processors. Processors can deliberately choose a highest number and confirms it to be its ID.

(Bauk and Sung Uoon 2005) designed a leader election algorithm for unidirectional ring where the size of the ring was known and it was similar to the leader election model of (Abraham et al. 2006). (Fischer et al. 2006) presented a self-stabilizing leader election algorithm for unidirectional ring where the size of the ring was unknown which, at the end, returns the same leader to each agent.

After careful consideration of this problem, we conclude that it is helpful to use a mediator in electing

leader in network. But taking into accounts, a third-party mediator will increase the chances of data leak and reduced security. Basically, giving authority to an agent, who is not participating in the leader election, to elect leader among various agents is risky. Hence, there has been studies on whether a problem that can be solved with a third-party mediator can be converted to be solved using cheap-talk, without the concern of a mediator. The method of using a mediator has gained currency in the recent years since it can be used to solve many distributed computing problems. For example, byzantine agreement is achievable using a mediator even if half of the agents are corrupted. Each agent can tell the mediator their ID's and their preference to who should be the leader, and the mediator chooses the majority. We focus on this type of problem, i.e., to study the result of the protocol in (Abraham et al. 2013), and formulate an enhanced version of it.

#### 4. Problem Statement

In this paper, we focus on the problem of consensus by leader election in asynchronous settings. We say that a distributed system is asynchronous if there is no time limit to execute a step like message delivery, clock drift, etc. It has simple syntax; the portability of the applications based on this model is easier than assuming time constraints; and practically we can say that unexpected load over a specific processor is a speciality of system in asynchronous. The protocol breaks if even a single node fails to follow the protocol in unidirectional or bidirectional ring. However, there are some chances of recovery when protocol is applied in a completely connected network. The faulty agents can be reduced by applying the algorithm designed in the failed areas which can detect failures consistently in distributed system. The processors should be able to differentiate between a fault-free population and faulty one.

#### 5. Acceptance-Based-a-Lead <sup>uni</sup> A Robust File Protocol

In this section, we will be discussing about the enhanced approach of leader election which is an extended version of A-Lead<sup>uni</sup> by (Abraham et al. 2013). This protocol is expected to be resilient to  $O(\sqrt{n})$

In this section, we will be discussing about the enhanced approach of leader election which is an extended dishonest processors. We extend the A-Lead<sup>uni</sup> protocol and add an "acceptance" stage which keeps all the processors in synchrony. We apply "dashing of data" technique where the adversarial agents wait to receive all the messages from other agents before sending its own message. In this way, the adversarial processors minimize the number of messages being traversed along the ring. The major drawback that we observed in A-Lead<sup>uni</sup> is that during the execution of the protocol, all the processors should necessarily be in synchrony or else the honest processors can detect the fault and abort the protocol. We know that in A-Lead<sup>uni</sup>, the processors keep sending the secret messages till all the processors has received the IDs of every other processor in the ring. The processors do this by keeping the number of times they have sent and received IDs from their respective predecessor processor. This is simulated using "circular trip" starting and ending from and to the originator. In each circular trip, every processor receives a secret message from its predecessor and transfer the secret message (also known as the ID of previous processor) that it received in the preceding trip.



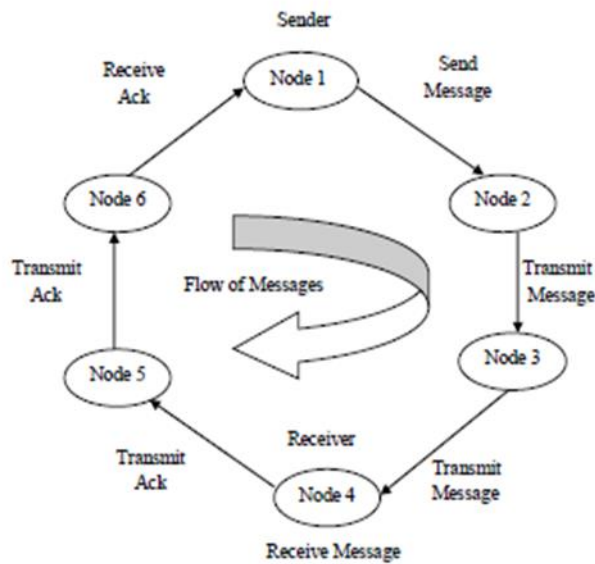
To understand it simply, let us assume  $Transmit_i$  to be the total number of secret messages there are no adversaries in the ring then we have,  $|Transmit_i - Transmit_j| \leq l$ . The equation already sent by a processor  $i$  (be it honest or adversarial processor). If we consider a case where confirms that all the processors are in synchrony. Since, we are focused on solving the leader election problem in asynchronous ring network, our attack, invasion of adversarial processor, makes use of the disadvantage of asynchronous settings in distributed system and de-synchronize non-adversarial processors. The crucial remark that can be made here that enabled the attack to occur is the difference between  $|Transmit_i - Transmit_j|$  is considerably substantial as compared to longest genuine portion,  $H_1$ . This is the major reason how an adversarial processor  $A_i$  comes to know about all the secret messages before choosing its own message. The “acceptance” stage works in the following manner:

Every processor needs to choose a uniformly distributed “acceptance value”,  $a_i \in X$  where  $X = n^2$ . The decision for how many number of circular trips are required will be based on the total number of processors participate for the leader election present in the ring.

Each processor will be given the opportunity to validate the acceptance value of other processors. Only one processor will be allowed to become the acceptor in a particular circular trip. That is, in the  $i^{th}$  trip,  $i^{th}$  processor will become the acceptor and sends its acceptance value,  $a_i$ . All the rest of the processors will receive and send this acceptance value immediately to their neighbouring processor with no delay in the ring.

No other processor can save this acceptance value or send its own value in the ring unless its his turn to do so. When all the processors have transferred the value, the value will be checked by the trip’s validator that whether it is the same acceptance value that he sent and committed to in the beginning. The acceptance value is not same as the secret message that every processors send. We distinguished both the values for different purposes. The secret message is used to calculate the final result, that is, which processor should be elected as a leader and acceptance value is used to validate whether or not is there any processor trying to manipulate the incoming messages. In other words, we use acceptance value to keep all the processors in synchrony. These secret messages are the random values that each processor,  $i$  selects to send. The final result will be calculate  $\sum_{i=1}^n m_i \pmod n$ .

The vital distinguished design behind the enhanced protocol is to force all the processors to be more synchronized than before. Since, there are some drawbacks in synchronization settings like small information can move faster, so the method used to prove previous resilience fails. Therefore, in order to cope up with this problem, the new protocol makes use of random function that generates random values for each processor. Hence, adversarial processors receive lots of information before getting biased to calculate the output. Therefore, we prove that there is a need to send an enormous amount of messages by an adversarial processor before knowing about the secret messages of other processors.



**Fig. 2.** Secret Sharing Protocol

If we look back on the A-Lead<sup>uni</sup> protocol, it was based on Shamir’s secret sharing sub-protocol. Our protocol is based on the similar paradigm which deals with sharing secret messages along the ring and wait to receive an acknowledgement from the receiver.

**Algorithm 1** Acceptance-based-A-LEAD<sup>uni</sup>

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*Phase 1 – For originator processor 1*

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- 1: Initially select an acceptance value  $a_{1,t}$  and a secret message  $m_i$ ,  $t = m_i$ , where  $t$  is the trip number.
  - 2: Send the secret message in the ring and wait for the secret message  $m_{1,nt+1}(modn)$
  - 3: **if** it is the first trip, i.e.  $t = 1$ ,
    - then** Send its acceptance value in the ring and wait to receive it at the end of the trip to validate the synchrony of processors.
  - 4: **if** it is not the first trip
    - then** forward immediately the incoming acceptance value to the successive processor. Finally, forward the remaining secret messages  $m_{1,nt+1}(modn)$
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*Phase 2 – For standard processor i*

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- 5: Choose a secret message  $m_i$ ,  $i = m_i$  and an acceptance value  $a_{i,t} = a_i$
- 6: Since it is a standard processor, it will receive the secret message  $m_{1,nt+1}(modn)$  and then it will send the message stored in the buffer  $m_{1,nt+1}(modn)$
- 7: **if** The value of trip number,  $t$  is equal to  $i$  i.e., the  $i^{th}$  processor

**then** send the acceptance value  $a_{i,t}$  and wait to receive it at the end of the trip to validate the synchrony of processors.

8: **if** They are not equal

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**then** forward immediately the incoming acceptance value to the successive processor.

At first, the originator processor 1 chooses a random secret message  $m_1$  and an acceptance value  $a_1$ . Using Shamir's secret sharing protocol send the secret message and acceptance value  $\{m_1\}$  and  $\{a_i\}$  to every processor in the ring. For a processor  $i$  it receives  $m_{i,1}, \dots, m_{i,n}, a_{i,1}, \dots, a_{i,n}$ . These values are value with the one that it sent at the beginning of the trip. If  $a_{i,j} \neq a_i$  the processor aborts the protocol received alternatively and not during the same trip. Each processor then verifies the incoming acceptance by giving output = *NULL*.

In our model, we assume that the processors are consecutively placed in an increasing order of their IDs. However, this arrangement can be changed according to the need. Our protocol works even when the processors are located randomly in the ring. We prove that Sync-A-Lead<sup>uni</sup> is a reliable protocol by showing that adversaries cannot transfer large amount of information among themselves which is the crucial step for them to know the secret messages of every processor and bias the outcome. We apply a random function  $R$  instead of the summation function to calculate the final result/output. To stop the adversaries from misusing the acceptance stage for their own benefit, we apply the random function to both secret message and acceptance value stage so that even if the processors calculate the partial sum of the secret messages during the execution of the protocol, they will not be able to bias the result of random function at the end that determines a fair leader for the asynchronous ring.

## 6. Results

Game Theory and Distributed System problems go hand in hand. Both strive in determining the best possible outcome for a player/processor in a game/network. We studied the drawbacks of A-Lead<sup>uni</sup> protocol and proposed an enhanced version of it resilient to  $O(\sqrt{n})$  adversaries and design a new protocol after some modifications. Our protocol performed well even in the presence of invasion of adversarial processors. Although there were few assumptions in our model, they helped us in overcoming the drawbacks of A-Lead<sup>uni</sup> and design a new protocol after some modifications.

We can say that our protocol used game theoretic approach as a tool to define how leaders are elected and also how each processor comes down to electing only a single leader among various candidates. Lastly, we can say that with the help of our model, we managed to keep all the processors in synchrony and that they follow a solution based approach where they will choose to have any leader rather than having no leader at all.

## 7. Conclusion and Future Scope

Our protocol performed well even in the presence of invasion of adversarial processors. Although there were few assumptions in our model, they helped us in overcoming the drawbacks of A-Lead<sup>uni</sup> and design a new protocol after some modifications. We can say that our protocol used game theoretic

approach as a tool to define how leaders are elected and also how each processor comes down to electing only a single leader among various candidates. Lastly, we can say that with the help of our model, we managed to keep all the processors in synchrony and that they follow a solution based approach where they will choose to have any leader rather than having no leader at all.

There are many assumptions on which our model is based upon. In this section, we discuss about the various assumptions and behavior of processors based on these assumptions that helped us designing the improved version of the protocol in (Abraham et al. 2006). Below are some of the questions that arises when we make those assumptions:

- In (Abraham et al. 2006), processors do not know the size or IDs of other processors in the ring. So they are transferred during the wake-up phase and then the processors decide the direction for message passing. We assumed that we don't need a wake-up phase because our invasion of adversaries do not affect the wake-up phase nor it tries to deviate from the designated protocol. Hence, we assume that processors already know about the size and IDs of each processor. Our invasion protocol can be extended to execute honestly, the wake-up phase of the protocol in (Abraham et al. 2006). However, we cannot say that adversaries will never abuse this phase. Hence, the question still remains open for discussion.
- For rational agents, we defined that the output of the given protocol will be  $[n] \cup [NULL]$ . But the problem still revolves around the fact when IDs are unknown. Simply put, it is still difficult to find a resilient FLE protocol when the IDs are unknown. To understand this, let's consider an example. Let the IDs are from huge sample space  $\Sigma$ . Now, the expected utility of a processor,  $i$  will be,  $EU: \Sigma \cup [NULL] \rightarrow [0,1]$ . This function will be considered rational when  $EU_i[NULL]=0$ . Consider the protocol in (Abraham et al. 2006) to be  $P$  having the  $EU_P[\Sigma]$ . Although, an adversarial agent may or may not obey the protocol and abuse to have  $EU_A[\Sigma] = \frac{k}{n}$ . This means that in a unidirectional ring, for  $k$  adversaries with  $k > 1$ , there does not exist a  $k$  resilient protocol.
- In order to find a solution to this problem, what we can do is define an  $\epsilon$ - $k$ -unbiased protocol for all the IDs. Let the deviation from every dishonest processor be  $\forall x \in \Sigma : Pr(output = x) \leq \epsilon + \frac{1}{n}$ . Hypothetically, we proved that A-Lead<sup>uni</sup> and Acceptance based-A-Lead<sup>uni</sup> are  $\epsilon - k - unbiased$  for  $k$  and  $\epsilon$ .
- Among various challenges that we faced to design an invasion of adversarial processors, we tackle a particular problem which is an adversarial coalition can bluff the honest set into believing that there exist an Originator processor in every set. Considering the protocol discussed in (Abraham et al. 2006) and (Abraham et al. 2013), the processors tend to know the IDs of every other processor during the wake-up phase by exchanging and selecting the lowest ID among them to be the Originator. No honest set can have two Originator processors as each processor receives a set of IDs which is used to identify if all the processors are honest and that no ID is repeated more than once. However, an adversarial coalition can abuse this phase by hiding the highest ID among honest processors  $h \in H_i$  and change it to 0 before forwarding it to the next set  $H_j, j \neq i$ . The adversary can use this highest ID to set it for itself.

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