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Characterization of Semi-Ring by Rough Spherical Fuzzy ideals

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Abstract- In this paper we combine the rough set and spherical fuzzy set. We define spherical fuzzy ideal in semi-ring. Also we prove nonempty intersection of spherical fuzzy ideals is also spherical fuzzy ideal. The image and pre image of an spherical fuzzy ideal is also a spherical fuzzy ideal.Approximation of spherical fuzzy set on a crisp approximation space gives a rough spherical fuzzy set. Also we introduce algebraic properties of rough spherical fuzzy ideals in semi-ring. Some examples are established.

1. INTRODUCTION

Now a days the most interesting topic of researchers is fuzzy set theory, this was introduced by Zadeh[10] in 1965. This became a useful topic to study the problems of vagueness, uncertainty. After that, there are some extensions of fuzzy set. Such as interval valued fuzzy set, intutionistic fuzzy set, picture fuzzy set etc. Spherical fuzzy set were developed by Kahraaman and Gundogdu [3] as an extension of pythaogorean, neutrosophic, and picture fuzzy sets. A spherical fuzzy set must satisfy the following condition:

$$0 \leq \varepsilon_m^2(l) + \varepsilon_n^2(l) + \varepsilon_h^2(l) \leq 1, \text{ for all } l v \mathrm{U}$$

where U is the universe of discourse. Moreover $\varepsilon_m(l)$, $\varepsilon_n(l)$, and $\varepsilon_h(l)$, are the degree of membership, non-membership and hesitancy function of l to the fuzzy set ε respectively.

Rough set theory was developed by Pawlak[6] is another method to deal with vagueness and uncertainty. Equivalence classes in a set as the building blocks for the construction of lower and upper approximations of a set. Many authors[4, 5, 8, 9] apply algebraic properties to rough sets.

The summery of this manuscript is as follows: In section 2 we review some basic concepts related to this article. In section 3 we study the notion of spherical fuzzy ideals in semirings and some interesting properties of these ideals are discussed. Section 4 deals with homomorphism of spherical fuzzy ideals in semi-rings. In section 5 we discuss about the rough spherical fuzzy ideals in semi-rings

2. Preliminaries

This section deals with the basic concepts related to this article. For basic definitions let us see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Throughout this paper we have proved for spherical fuzzy left ideal. Proof for the spherical fuzzy right ideal are similar to spherical fuzzy left ideal. Let $\Re$ denote the semi-ring unless otherwise specified

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3.spherical fuzzy left Ideals(SFLI) of semi-ring

In this section we introduced the concept of SFLI in semi-ring $\Re$. To illustrate the concepts some examples are discussed. Some interesting properties of SFLI are proved.

Definition 3.1. [10] A SFset $\varepsilon$of the universe U is defined by

$$\varepsilon = \{l, \langle \varepsilon_m(l), \varepsilon_n\ (l), \varepsilon_h\ (l)\rangle\}$$

where $\varepsilon_m(l) : U \to [0, 1]$, $\varepsilon_n(l) : U \to [0, 1]$, $\varepsilon_h(l) : U \to [0, 1]$ and

$0 \leq \varepsilon_m^2(l) + \varepsilon n^2\ (l) + \varepsilon_h^2(l) \leq 1$ for every $l \in U$ for each l, the numbers

$\varepsilon_m(l)$, $\varepsilon_n(l)$ and $\varepsilon_h(l)$ are the degree of membership, non membership and hesitancy of l to ε, respectively.

Example 3.2. Let U = {v, b, l, k} be the universe. A SF set Ɛ of U is defined by,

ε_m (i) = {0.6, 0.3, 0.7, 0.4},

ε_n (i) = {0.3, 0.3, 0.6, 0.6} and

ε_h(i) = {0.5, 0.5, 0.2, 0.3} where i = v, b, l, k ∈ U

Definition 3.3. A SF set ε in $\Re$ is called a SFLI of $\Re$ if the following conditions are holds:

(1)$\varepsilon_m\ (i + j) \geq \varepsilon_m(i) \wedge \varepsilon_m(j)$

(2) $\varepsilon_n\ (i + j) \geq \varepsilon_n(i) \wedge \varepsilon_n(j)$

(3) $\varepsilon_h(i + j) \leq \varepsilon_h(i) \vee Ɛ_h(j)$

(4) $\varepsilon_m(ij) \geq \varepsilon_m(j)$(resp., $\varepsilon_m(ij) \geq \varepsilon_m(i)$)

(5) $\varepsilon_n(ij) \geq \varepsilon_n(j)$(resp., $\varepsilon_n(ij) \geq \varepsilon_n(i)$)

(6) $\varepsilon_h(ij) \leq \varepsilon_h(j)$(resp., $\varepsilon_h(ij) \leq \varepsilon_h(i)$) for all i, j ∈R Similarly, we define for the SFLI of $\mathfrak{R}$

Thoerem 3.4 Let ε be aSFLI of $\mathfrak{R}$. Then ε is a SFLI of $\mathfrak{R}$ if and only if any level subsets $\varepsilon_m^t = \{l \in \mathfrak{R}: \varepsilon_m \geq t, t \in [0,1]\}, \varepsilon_n^t = \{l \in \mathfrak{R}: \varepsilon_n \geq t, t \in [0,1]\}$ and $\varepsilon_h^t = \{l \in \mathfrak{R}: \varepsilon_m \leq t, t \in [0,1]\}$ are left ideals of $\mathfrak{R}$.

Proof. Assume that ε is SFLIof $\mathfrak{R}$. Then for some $l \in \mathfrak{R}$ any one of $\varepsilon_m, \varepsilon_m$ and $\varepsilon_m$are not zero. i.e., $\varepsilon_m^t, \varepsilon_m^t$ and ε_m^t are not zero. Suppose $l, g \in \varepsilon^t = (\varepsilon_m^t, \varepsilon_m^t, \varepsilon_m^t)$ and $s \in \mathfrak{R}$.

$$\varepsilon_m(l+g) \geq \varepsilon_m(l) \wedge \varepsilon_m(g) \geq t$$

$$\varepsilon_m(l+g) \geq \varepsilon_m(l) \wedge \varepsilon_m(g) \geq t$$

$$\varepsilon_m(l+g) \leq \varepsilon_m(l) \vee \varepsilon_m(g) \leq t$$

This implies $l + g \in \varepsilon_m^t, \varepsilon_m^t, \varepsilon_m^t$. i.e., $l + g \in \varepsilon^t$.

Also.,

$$\varepsilon_m(lg) \geq \varepsilon_m(g) \geq t$$

$$\varepsilon_m(lg) \geq \varepsilon_m(g) \geq t$$

$$\varepsilon_m(lg) \leq \vee\, \varepsilon_m(g) \leq t$$

Hence $\varepsilon^t$is a left ideal of $\mathfrak{R}$.

Conversely let us assume that ε^t is left ideal of $\mathfrak{R}$. Contrary let us assume that ε is not a SFLI of$\mathfrak{R}$. Then for $l, g \in \mathfrak{R}$ anyone of the following is true.

$$\varepsilon_m(l+g) < \varepsilon_m(l) \wedge \varepsilon_m(g) < t$$

$$\varepsilon_m(l+g) < \varepsilon_m(l) \wedge \varepsilon_m(g) < t$$

$$\varepsilon_m(l+g) > \varepsilon_m(l) \vee \varepsilon_m(g) > t$$

Choose $e_1 = \frac{1}{2}[\varepsilon_m(l+g) + (\varepsilon_m(l) \wedge \varepsilon_m(g)\varepsilon_m(l) \wedge \varepsilon_m(g))]$.

Then $\varepsilon_m(l+g) < e_1 < \varepsilon_m(l) \wedge \varepsilon_m(g)$. Which implies $l, g \in \varepsilon_m^{e_1}$ but $l + g\ not\ in\ \varepsilon_m^{e_1}$. Which is a contradiction to our hypothesis. Similarly we prove for $\varepsilon_n(l+g)$ and $\varepsilon_h(l+g)$. Hence tha converse part.

Proposition 3.5: Intersection of nonempty collection of SFLI is also aSFLI.

Proof:

Assume that $\{\varepsilon^k : k \in I\}$be a family of a SFLI of $\Re$. Let $r, s \in \Re$.

Then $(\bigcap_{k\in I} \varepsilon_m^k)(r+s) = inf_{k\in I}\varepsilon_m^k(r+s)$

$$\geq inf_{k\in I}(\varepsilon_m^k(r) \wedge \varepsilon_m^k(s))$$

$$= inf_{k\in I}\varepsilon_m^k(r) \wedge inf_{k\in I}\varepsilon_m^k(s)$$

$$= \left(\bigcap_{k\in I} \varepsilon_m^k\right)(r) \wedge \left(\bigcap_{k\in I} \varepsilon_m^k\right)(s)$$

And

$$\left(\bigcap_{k\in I} \varepsilon_n^k\right)(r+s) = inf_{k\in I}\varepsilon_n^k(r+s) \geq inf_{k\in I}(\varepsilon_n^k(r) \wedge \varepsilon_n^k(s))$$

$$= inf_{k\in I}\varepsilon_n^k(r) \wedge inf_{k\in I}\varepsilon_n^k(s)$$

$$= \left(\bigcap_{k\in I} \varepsilon_n^k\right)(r) \wedge \left(\bigcap_{k\in I} \varepsilon_n^k\right)(s)$$

Also $(\bigcap_{k\in I} \varepsilon_m^k)(r+s) = sup_{k\in I}\varepsilon_m^k(r+s)$

$$\leq sup_{k\in I}(\varepsilon_m^k(r) \vee \varepsilon_m^k(s))$$

$$= sup_{k\in I}\varepsilon_m^k(r) \vee sup_{k\in I}\varepsilon_m^k(s)$$

$$= \left(\bigcap_{k\in I} \varepsilon_m^k\right)(r) \vee \left(\bigcap_{k\in I} \varepsilon_m^k\right)(s)$$

Moreover$(\bigcap_{k\in I} \varepsilon_m^k)(rs) = inf_{k\in I}\varepsilon_m^k(rs) \geq inf_{k\in I}\varepsilon_m^k(s) = \bigcap_{k\in I} \varepsilon_m^k\ (s)$

Similarly we can prove for $(\bigcap_{k\in I} \varepsilon_m^k)(rs) \geq \bigcap_{k\in I} \varepsilon_m^k\ (s)$

And $(\bigcap_{k\in I} \varepsilon_m^k)(rs) \leq \bigcap_{k\in I} \varepsilon_m^k\ (s)$

Hence proved.

4. HOMOMORPHISM OF SPHERICAL FUZZZY LEFT IDEALS (SFLI) IN SEMI-RINGS

This section deals with the homomorphism of SFLI. Also we prove the image and pre-imageof SFLIis also a SFLI of $\mathfrak{R}$.

Definition 4.1. Let G and H be two nonempty sets and $\beta: G \to H$ be function

(1) If F is a SF set in H, then $\beta^{-1}(F)$ is the SF set in G defined by $\beta^{-1}(F) = \{(l, \beta^{-1}(F_m(l)), \beta^{-1}(F_n(l)), \beta^{-1}(F_h(l))) : l \in G\}$,

Where $\beta^{-1}(F_m(l)) = F_m(\beta(l))$,

$$\beta^{-1}(F_n(l)) = F_n(\beta(l))$$

$$\beta^{-1}(F_h(l)) = F_h(\beta(l))$$

(2) If P is SF set in G then $\beta(P)$ is a SF set in H defined by $\beta(P) = \{k, \beta(P_m(k)), \beta(P_n(k)), \beta(P_h(k)) : k \in H\}$

Where

$$\beta(P_m(k)) = \begin{cases} sup_{l \in \beta^{-1}(k)} P_m(l), if\ \beta^{-1}(k) \neq 0 \\ 0, otherwise \end{cases}$$

$$\beta(P_n(k)) = \begin{cases} sup_{l \in \beta^{-1}(k)} P_n(l), if\ \beta^{-1}(k) \neq 0 \\ 0, otherwise \end{cases}$$

$$\beta(P_h(k)) = \begin{cases} sup_{l \in \beta^{-1}(k)} P_h(l), if\ \beta^{-1}(k) \neq 0 \\ 0, otherwise \end{cases}$$

Theorem 4.2. Let G and H be two semi-rings and β be a homomorphism of G onto H. If P is a SFLI of H, then $\beta^{-1}(P)$ is a SFLI of G.

Proof:

Let $e, r \in G$. Then

$$\beta^{-1}(P_m)(e + r) = P_m(\beta(e + r))$$

$$= P_m(\beta(e) + \beta(r))$$

$$\geq P_m(\beta(e)) \wedge P_m(\beta(r))$$

$$= \beta^{-1}(P_m)(e) \wedge \beta^{-1}(P_m)(r)$$

$$\beta^{-1}(P_n)(e + r) = P_n(\beta(e + r))$$

$$= P_n(\beta(e) + \beta(r))$$

$$\geq P_n(\beta(e)) \wedge P_n(\beta(r))$$

$$= \beta^{-1}(P_n)(e) \wedge \beta^{-1}(P_n)(r)$$

And

$$\beta^{-1}(P_h)(e + r) = P_h(\beta(e + r))$$

$$= P_h(\beta(e) + \beta(r))$$

$$\leq P_h(\beta(e)) \vee P_n(\beta(r))$$

$$= \beta^{-1}(P_n)(e) \vee \beta^{-1}(P_n)(r)$$

Also

$$\beta^{-1}(P_m)(er) = P_m(\beta(er))$$

$$= P_m(\beta(e)\beta(r))$$

$$\geq P_m(\beta(r)$$

$=\beta^{-1}(P_m)(r)$

Similarly $\beta^{-1}(P_n)(er) \geq \beta^{-1}(P_n)(r)$ and $\beta^{-1}(P_h)(er) \leq \beta^{-1}(P_h)(r)$ is a $\mathbb{SF}\boldsymbol{\mathcal{LI}}$ of G.

Theorem 4.3:

Let G and H be two semi-rings and β be a homomorphism of G onto H. If F is a $\mathbb{SF}\boldsymbol{\mathcal{LI}}$ of G, then $\beta(F)$ is a $\mathbb{SF}\boldsymbol{\mathcal{LI}}$ of H.

Proof.

Let $e_2, r_2 \in H$. Then

$$\beta\big(F_m(e_2 + r_2)\big) = sup_{s\in\beta^{-1}(e_2+r_2)} F_m(e_2 + r_2)$$

$$\geq sup_{e_1\in\beta^{-1}(e_2), r_1\in\beta^{-1}(r_2)} F_m(e_2 + r_2)$$

$$\geq sup_{e_1\in\beta^{-1}(e_2), r_1\in\beta^{-1}(r_2)} F_m(e_2) \wedge F_m(r_2)$$

$$= (sup_{e_1\in\beta^{-1}(e_2))} F_m\big(e_1\big)) \wedge (sup_{r_1\in\beta^{-1}(r_2))} F_m(r_1\big))$$

$$= \beta(F_m(e_2)) \ \wedge \ \beta(F_m(r_2))$$

$$\beta\big(F_n(e_2 + r_2)\big) = sup_{s\in\beta^{-1}(e_2+r_2)} F_n(e_2 + r_2)$$

$$\geq sup_{e_1\in\beta^{-1}(e_2),r_1\in\beta^{-1}(r_2)}F_n(e_2+r_2)$$

$$\geq sup_{e_1\in\beta^{-1}(e_2),r_1\in\beta^{-1}(r_2)}F_n(e_2)\wedge F_n(r_2)$$

$$=(sup_{e_1\in\beta^{-1}(e_2))}F_n\big(e_1\big))\wedge(sup_{r_1\in\beta^{-1}(r_2))}F_n\big(r_1\big))$$

$$=\beta(F_n(e_2))\ \wedge\ \beta(F_n(r_2))$$

Similarly, we can prove for other case

$$\beta\big(F_h(e_2+r_2)\big)=inf_{s\in\beta^{-1}(e_2+r_2)}F_h(e_2+r_2)$$

$$\leq inf_{e_1\in\beta^{-1}(e_2),r_1\in\beta^{-1}(r_2)}F_h(e_2+r_2)$$

$$\leq inf_{e_1\in\beta^{-1}(e_2),r_1\in\beta^{-1}(r_2)}F_h(e_2)\vee F_h(r_2)$$

$$=(inf_{e_1\in\beta^{-1}(e_2))}F_h\big(e_1\big))\vee(sup_{r_1\in\beta^{-1}(r_2))}F_h\big(r_1\big))$$

$$=\beta(F_h(e_2))\ \vee\ \beta(F_h(r_2))$$

Moreover

$$\beta\big(F_m(e_2r_2)\big)=sup_{s\in\beta^{-1}(e_2+r_2)}F_m(e_2r_2)$$

$$\geq sup_{e_1\in\beta^{-1}(e_2),r_1\in\beta^{-1}(r_2)}F_m(e_2r_2)$$

$$\geq sup_{r_1\in\beta^{-1}(r_2))}F_m(r_2)$$

$$=\ \beta(F_m(r_2))$$

Also we can prove $\beta\big(F_n(e_2r_2)\big)\geq\beta\big(F_n(r_2)\big)$ and $\beta\big(F_h(e_2r_2)\big)\leq\beta\big(F_h(r_2)\big)$. Hence

$\beta(F)$ is a $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$of H.

5. Rough spherical fuzzy ($\mathbb{RSF}$)sets

The aim of this section is to explore the idea of $\mathbb{RSF}$ sets. Throughout this section let $\mho$ denotes the congruence relation on $\mathfrak{R}$.

Definition 5.1. Let $\mathfrak{R}$. be the universal set and $\mho$ be a congruence relation on $\mathfrak{R}$.. Let $\mathcal{P}$ be a $\mathbb{SF}$ set in $\mathfrak{R}$.. Then the lower and upper approximations of $\mathcal{P}$ is define

$\underline{\Upsilon}(\mathcal{P})=\{(l,\underline{\Upsilon}(\mathcal{P}_{\boldsymbol{m}})(l),\underline{\Upsilon}(\mathcal{P}_{\boldsymbol{n}})(l),\underline{\Upsilon}(\mathcal{P}_{\boldsymbol{h}})(l)): l\in\mathfrak{R}\}$

$\overline{\Upsilon}(\mathcal{P})=\{(l,\overline{\Upsilon}(\mathcal{P}_{\boldsymbol{m}})(l),\overline{\Upsilon}(\mathcal{P}_{\boldsymbol{n}})(l),\overline{\Upsilon}(\mathcal{P}_{\boldsymbol{h}})(l)): l\in\mathfrak{R}\}$

$$\underline{\Upsilon}(\mathcal{P}_m)(l) = \bigwedge_{s\in[l]_\Upsilon} \mathcal{P}_m(s), \underline{\Upsilon}(\mathcal{P}_n)(l) = \bigwedge_{s\in[l]_\Upsilon} \mathcal{P}_n(s), \underline{\Upsilon}(\mathcal{P}_h)(l) = \bigvee_{s\in[l]_\Upsilon} \mathcal{P}_h\ (s)$$

$$\overline{\Upsilon(}(\mathcal{P}_m)(l) = \bigvee_{s\in[l]_\Upsilon} \mathcal{P}_m(s), \overline{\Upsilon\ (}\mathcal{P}_n(l) = \bigvee_{s\in[l]_\Upsilon} \mathcal{P}_n(s), \overline{\Upsilon}(\mathcal{P}_h)(l) = \bigwedge_{s\in[l]_\Upsilon} \mathcal{P}_h\ (s)$$

Then$\Upsilon(\mathcal{P}) = (\underline{\Upsilon}(\mathcal{P}), \overline{\Upsilon}(\mathcal{P}))$ is called a $\mathbb{RSF}$ set of $\mathcal{P}$ with respect to the approximation space $(\Upsilon, \mathfrak{R})$.

Example 5.2. Let Z = {i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8,i_9, i_{10}} be the universe set and Υ be the congruence relation on Z. The equivalence classes of Z are defined by

Z/ Υ= {{i_1, i_3, i_9}, {i_2, i_7, i_{10}}, {i_4}, {i_5, i_8}, {i_6}}

Let ε be the $\mathbb{SF}$ set of Z defined by

ε_m (x) = {0.3, 0.8, 0.4, 0.4, 0.2, 0.3, 0.6, 0.2, 0.5, 0.4}

ε_n(x) = {0.8, 0.4, 0.5, 0.3, 0.6, 0.9, 0.1, 0.3, 0.8, 0.8}

ε_h(x) = {0.5, 0.3, 0.7, 0.4, 0.7, 0.1, 0.6, 0.7, 0.2, 0.4} for all x ∈ Z.

Then the lower approximation of ε is given by

$\underline{\Upsilon}(\varepsilon_m)(x)$= {0.3, 0.4, 0.3, 0.4, 0.2, 0.3, 0.4, 0.2, 0.3, 0.4}
$\underline{\Upsilon}(\varepsilon_n)(x)$ = {0.8, 0.1, 0.8, 0.3, 0.3, 0.9, 0.1, 0.3, 0.8, 0.1}
$\underline{\Upsilon}(\varepsilon_h)(x)$ = {0.5, 0.6, 0.5, 0.4, 0.7, 0.1, 0.6, 0.7, 0.5, 0.6}

Then the upper approximation of ε is given by

$\overline{\Upsilon(}(\varepsilon_m)(x)$ = {0.5, 0.8, 0.5, 0.4, 0.2, 0.3, 0.8, 0.2, 0.5, 0.8}

$\overline{\Upsilon(}(\varepsilon_n)(x)$ = {0.8, 0.1, 0.8, 0.3, 0.3, 0.9, 0.1, 0.3, 0.8, 0.1}

$\overline{\Upsilon(}(\varepsilon_h)(x)$ = {0.2, 0.3, 0.2, 0.4, 0.7, 0.1, 0.3, 0.7, 0.2, 0.3} for all x ∈ Z.

Then $\Upsilon(\varepsilon)$ = ($\underline{\Upsilon}(\varepsilon)$, $\overline{\Upsilon(}\varepsilon)$)

Theorem 5.3. Let Υ be a congruence relation on $\mathfrak{R}$. If P and Q are any two $\mathbb{SF}$ set of $\mathfrak{R}$ then the following conditions are hold:

(1) $\underline{\Upsilon}\ (\mathcal{P}) \subseteq \mathcal{P} \subseteq \overline{\Upsilon(}\mathcal{P})$.

(2) $\underline{\Upsilon}\ (\underline{\Upsilon}\ (\mathcal{P})) = \underline{\Upsilon}\ (\mathcal{P})$

(3) $\overline{\Upsilon(}$ $(\overline{\Upsilon(}$ $(\mathcal{P})) = \overline{\Upsilon(}$ $(\mathcal{P})$

(4) $\overline{\Upsilon(}\underline{\Upsilon(}(\mathcal{P})) = \underline{\Upsilon}(\mathcal{P})$

(5) $\underline{\Upsilon}(\overline{\Upsilon(}P)) = \overline{\Upsilon(}\mathcal{P})$

(6)$\underline{\Upsilon}$ $(\mathcal{P}^c$ $) = \overline{(\Upsilon}(\mathcal{P}))^c$

(7) $\overline{\Upsilon(}\mathcal{P}^c) = (\underline{\Upsilon}(\mathcal{P}))^c$

(8) $\overline{\Upsilon(}(P \cap Q) \subseteq \overline{\Upsilon(}(P) \cap \overline{\Upsilon(}(Q)$

(9) $\underline{\Upsilon}(P \cup Q) \supseteq \underline{\Upsilon}(P) \cup \underline{\Upsilon}(Q)$

(10) P ⊆ Q implies$\underline{\Upsilon}(P) \subseteq \underline{\Upsilon}(Q)$

(11) P ⊆ Q implies $\overline{\Upsilon(}P) \subseteq \overline{\Upsilon(}Q)$

Proof. By Definition 5.1 proof is obvious. .

6.Rough spherical fuzzy left ideals($\mathbb{RSF}\boldsymbol{\mathcal{LJ}}$) in semi-rings

In this section we introduce the concept of approximations of spherical fuzzy ideals. i.e., lower and upper approximation of $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$). Also we prove the lower and upper approximations of $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$ is also a $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$). More over we prove the nonempty intersection of $\mathbb{RSF}\boldsymbol{\mathcal{LJ}}$) is also $\mathbb{RSF}\boldsymbol{\mathcal{LJ}}$of $\mathfrak{R}$

Definition 6.1. Let Ɛ be a $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$. Then $\underline{\Upsilon}$(Ɛ)(resp., $\overline{\Upsilon(}$Ɛ))) is also a $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$)of $\mathfrak{R}$ then it is known as ϒ-lower(resp., ϒ -upper) $\mathbb{RSF}\boldsymbol{\mathcal{LJ}}$)of $\mathfrak{R}$..

Definition 6.2. A $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$ is said to be $\mathbb{RSF}\boldsymbol{\mathcal{LJ}}$)of $\mathfrak{R}$ if it is both ϒ -lower and ϒ -upper $\mathbb{RSF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$.

Theorem 6.3. Let ϒbe a congruence relation on $\mathfrak{R}$. If Ɛis a $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$ then ϒ (Ɛ) is a $\mathbb{SF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$.

Proof. Since E is a SFLI of R. Let e, t ∈ R. Then consider

$$\underline{\Upsilon}(\varepsilon_m)(e+t) = \bigwedge_{s\in[e+t]_\Upsilon} \varepsilon_m(s),$$

$$= \bigwedge_{s\in[e]_\Upsilon+[t]_\Upsilon} \varepsilon_m(s),$$

$$= \bigwedge_{p+q\in[e]_\Upsilon+[t]_\Upsilon} \varepsilon_m(p+q),$$

$$= \bigwedge_{p\in[e]_\Upsilon, q\epsilon[t]_\Upsilon} \varepsilon_m(p) \wedge \varepsilon_{m(q)},$$

$$\geq \left(\bigwedge\nolimits_{p\in[e]_\Upsilon} \varepsilon_m(p),\right)\wedge\left(\bigwedge\nolimits_{q\in[t]_\Upsilon} \varepsilon_m(q),\right)$$

$$=\underline{\Upsilon}\,(\varepsilon_m)(\mathrm{e}) \wedge \underline{\Upsilon}\,(\varepsilon_m)(\mathrm{t})$$

Consequently for non membership function

$$\underline{\Upsilon}(\varepsilon_n)(e+t) = \bigwedge_{s\in[e+t]_\Upsilon} \varepsilon_n(s),$$

$$= \bigwedge_{s\in[e]_\Upsilon+[t]_\Upsilon} \varepsilon_n(s),$$

$$= \bigwedge_{p+q\in[e]_\Upsilon+[t]_\Upsilon} \varepsilon_n(p+q),$$

$$= \bigwedge_{p\in[e]_\Upsilon, q\epsilon[t]_\Upsilon} \varepsilon_n(p) \wedge \varepsilon_{m(q)},$$

$$\geq \left(\bigwedge\nolimits_{p\in[e]_\Upsilon} \varepsilon_n(p),\right)\wedge\left(\bigwedge\nolimits_{q\in[t]_\Upsilon} \varepsilon_n(q),\right)$$

$$=\underline{\Upsilon}\,(\varepsilon_n)(\mathrm{e}) \wedge \underline{\Upsilon}\,(\varepsilon_n)(\mathrm{t})$$

Similarly for hesitancy function,

$$\underline{\Upsilon}(\varepsilon_h)(e+t) = \bigvee_{s\in[e+t]_\Upsilon} \varepsilon_h\,(s)$$

$$= \bigvee_{s\in[e]_\Upsilon+[\mathrm{t}]_\Upsilon} \varepsilon_h\,(s)$$

$$= \bigvee_{p+q\in[e]_\Upsilon+[\mathrm{t}]_\Upsilon} \varepsilon_h(p+q)$$

$= \vee_{p\epsilon[e]_\Upsilon, q\epsilon[t]_\Upsilon} \varepsilon_h(p) \vee \varepsilon_h(q)$

$\leq (\vee_{p\epsilon[e]_\Upsilon} \varepsilon_h(p)) \vee (\vee_{q\epsilon[t]_\Upsilon} \varepsilon_h(q))$

$$=\underline{\Upsilon}(\varepsilon_h)(e) \vee \underline{\Upsilon}(\varepsilon_h)(t)$$

Moreover

$$\underline{\Upsilon}(\varepsilon_m)(et) = \bigwedge_{s\in[e+t]_\Upsilon} \varepsilon_m(s),$$

$$= \bigwedge_{s\in[e]_\Upsilon[t]_\Upsilon} \varepsilon_m(s),$$

$$= \bigwedge_{pq\in[e]_\Upsilon[t]_\Upsilon} \varepsilon_m(pq),$$

$$= \bigwedge_{p\in[e]_\Upsilon, q\epsilon[t]_\Upsilon} \varepsilon_m(pq),$$

$$\geq \left(\bigwedge_{q\in[e]_\Upsilon} \varepsilon_m(q),\right)$$

$$=\underline{\Upsilon}(\varepsilon_m)(t)$$

Similarly we prove for nonmembership and hesitancy function $\underline{\Upsilon}(\varepsilon_n)$(et) ≥ $\underline{\Upsilon}(\varepsilon_n)$(t) and

$\underline{\Upsilon}(\varepsilon_h)$()(et) ≤ $\underline{\Upsilon}(\varepsilon_h)$(t). Hence $\underline{\Upsilon}(\varepsilon)$ is $\mathrm{SF}\boldsymbol{\mathcal{LJ}}$)of $\mathfrak{R}$.

Theorem 6.4. Let Υ be a congruence relation on $\mathfrak{R}$. If ε is a $\mathrm{SF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$ then$\Upsilon(\varepsilon)$ is a $\mathrm{SF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$

Proof. Proof similar to Theorem 6.3 .

By combining Theorem 6.3 and Theorem 6.4 we get a $\mathrm{SF}\boldsymbol{\mathcal{LJ}}$) of is a $\mathrm{RSF}\boldsymbol{\mathcal{LJ}}$)of $\mathfrak{R}$.

Theorem 6.5. Intersection of nonempty family of $\mathrm{RSF}\boldsymbol{\mathcal{LJ}}$) of $\mathfrak{R}$ is $\mathrm{RSF}\boldsymbol{\mathcal{LJ}}$).

Proof. Assume that $\{\Upsilon(\kappa^j)\ I\}$ *be a family of RSFLI of* $\mathfrak{R}$. *Then* $(\underline{\Upsilon}(\kappa^j)$ *and* $\overline{\Upsilon}(\kappa^j)$ *are* $\mathrm{SF}\boldsymbol{\mathcal{LJ}}$)of $\mathfrak{R}$

$$Also\ \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa^j)(e) = \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa^j)(e)\ and\ \overline{\Upsilon}\left(\bigcap_{j\epsilon I} \kappa^j\ (e)\right) = -\left(\bigcap_{j\epsilon I} \overline{\Upsilon}(\kappa^j)(e)\right)$$

First we prove for lower approximation.

For this let us take e, t $\in\Re$

$\underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j(e+t) = \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_m^j(e+t))$

$=\inf_{j\epsilon I} \underline{\Upsilon}(\kappa_m^j(e+t))$

$\geq \inf_{j\epsilon I} (\underline{\Upsilon}(\kappa_m^j(e)) \wedge \underline{\Upsilon}(\kappa_m^j(t)))$

$\geq \inf_{j\epsilon I j\epsilon I} (\underline{\Upsilon}(\kappa_m^j(e)) \wedge (\inf(\underline{\Upsilon}(\kappa_m^j(t)))$

$= \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_m^j(e) \wedge \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_m^j(t)$

$= \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j)(e) \wedge \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j)(t)$

Let us prove for non membership function

$\underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j(e+t) = \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_n^j(e+t))$

$=\inf_{j\epsilon I} \underline{\Upsilon}(\kappa_n^j(e+t))$

$\geq \inf_{j\epsilon I} (\underline{\Upsilon}(\kappa_n^j(e)) \wedge \underline{\Upsilon}(\kappa_n^j(t)))$

$\geq \inf_{j\epsilon I} (\underline{\Upsilon}(\kappa_n^j(e)) \wedge (\inf_{j\epsilon I}(\underline{\Upsilon}(\kappa_n^j(t)))$

$= \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_n^j(e) \wedge \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_n^j(t)$

$= \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j)(e) \wedge \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j)(t)$

Next let us prove for hesitancy function

$\underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_h^j(e+t) = \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_h^j(e+t))$

$=\inf_{j\epsilon I} \underline{\Upsilon}(\kappa_h^j(e+t))$

$\geq \inf_{j\epsilon I} (\underline{\Upsilon}(\kappa_h^j(e)) \vee \underline{\Upsilon}(\kappa_h^j(t)))$

$\geq \inf_{j\epsilon I} (\underline{\Upsilon}(\kappa_h^j(e)) \vee (\inf_{j\epsilon I}(\underline{\Upsilon}(\kappa_h^j(t)))$

$= \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_h^j(e) \vee \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_h^j(t)$

$= \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_h^j\ )(e) \vee \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_h^j)(t)$

Also

$\underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j(et) = \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_m^j(et))$

$= \inf \underline{\Upsilon}(\kappa_m^j(et)$

jϵI

$\geq \inf\ (\underline{\Upsilon}(\kappa_m^j(et))$

jϵI

$= \bigcap_{j\epsilon I} \underline{\Upsilon}(\kappa_m^j(t))$

$= \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j\ )(t)$

Similarly ,

$\underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j(et)) \geq \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j(t)$ and $\underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_h^j(et) \leq \underline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_h^j(t)$

In the same way we can prove

$\overline{\Upsilon}\left(\bigcap_{j\epsilon I} \kappa_m^j\right)(e+t) \geq \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j)(e) \wedge \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j)(t)$,

$\overline{\Upsilon}\left(\bigcap_{j\epsilon I} \kappa_n^j\right)(e+t) \geq \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j)(e) \wedge \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j)(t)$,

$\overline{\Upsilon}\left(\bigcap_{j\epsilon I} \kappa_h^j\right)(e+t) \leq \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j)(e) \vee \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j)(t)$.

Moreover

$\overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j(et)) \geq \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_m^j(t)$

$\overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j(et)) \geq \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_n^j(t)$

$\overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_h^j(et)) \leq \overline{\Upsilon}(\bigcap_{j\epsilon I} \kappa_h^j(t)$.

Hence intersection of family $\mathbb{RSF}\mathcal{LI}$)is also a$\mathbb{RSF}\mathcal{LI}$).

Conclusion:

In this paper we have studied spherical fuzzy ideals in semi-rings. Also we combined rough set and spherical fuzzy set. Moreover apply algebraic properties to rough spherical fuzzy sets in semi-rings. Our aim is to extend these results to study some other algebraic structures such as near ring, semihyperrings, semigroups, ordered semigroups etc.

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