

## **External Analysis of Strategic Market Management Based on Markov Property, Risk Modeling, Failure Rate and Chi-Square Distribution – A Statistical Approach**

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### **Abstract**

The paper highlights a novel approach of augmentation of features of an existing product in the light of Markov property. It is also evident that, the predicted quantified estimate of submarket growth lacks proper co-ordination between present estimate with past one and present estimate with future event and this has been justified using Abelian property. Mathematical interpretations of strategic market analysis based on risk modeling, failure rate and chi-square distribution have also been cited in this research work.

**Keywords:** Markov property, Abelian property, risk modeling, failure rate, chi-square distribution

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### **Introduction**

The stochastic change in market growth often fails to meet expectations. In this perspective, forecasting demand seems to be a difficult task in dynamic market. The market impact of a brand new product in terms of sales and profit has to be investigated within a time period of observation. In case of extreme strategic uncertainty towards prediction of market size, the predicted quantified estimate of submarket growth lacks proper co-ordination between present estimate with past one and present

estimate with future event. The augmentation or expansion of features of an existing product can be quantified and realized based on Markov property based state transition. External analysis can also be analyzed using failure rate and chi-square distribution. Mathematical justification should suffice the validity of application of statistical approaches in strategic market management.

### **Literature Review**

Strategic development or review (Aaker *et.al.*, 2015) deals with an analysis of the factors external to a business that affect strategy. A running business can be investigated on the basis of a priori events and statistical trend analysis ((Cobb *et.al.*, 1925), (Giri *et.al.*, 1999)). Marketing Myopia (Levitt, 1960) also indicates the essence of investigation of sales and profit in case of strategic uncertainty. Furthermore, profit and loss are two mutually exclusive events at any specific timing instant of the observation period. Therefore,  $R$ , the Bernoulli random variable (Olofsson, 2005) for the external analysis of business strategy in this situation, is applicable. Strategic uncertainties focus on specific unknown parameters that will affect the outcome of strategic decisions. In this claim we have proposed that the principle of hypothesis rule plays a pivotal role in strategic decisions. A statistical hypothesis (Samuelson, 1976) is an assertion about the distribution of one or more random variables which we want to verify on the basis of a sample. Fuzzy set theory was proposed in 1965 by Lotfi A. Zadeh. A fuzzy set (Zadeh, 1965) can be defined mathematically by assigning to each possible individual in the universe of discourse, a value representing its grade of membership in the fuzzy set. The null hypothesis of validity of an unknown event (gain or loss) for a biased individual is identical to alternate hypothesis of the same for a normal person. In this study (Alden *et.al.*, 1999), the authors examine the emergence of product appellation positioning customs in advertising that parallel the development of the wide-reaching marketplace. An auxiliary construct, global consumer culture positioning (GCCP), is proposed, operationalized, and tested. As per (Ataman *et.al.*, 2010), marketing managers spend billions of dollars on their marketing programs each year, but few studies systematically assess the long-term impact of these programs across many brands and categories. In addition, existing research focuses mainly on advertising and promotions, but not on product or distribution. This study attempts to consider both the data and the modeling requirements. This study (Atuahene-Gima *et. al.*, 2004) attempts to contribute to a better understanding of marketing strategy by examining the antecedents and outcomes at the project level. In this paper (Balducci *et.al.*, 2018) the rise of unstructured information (UD), propelled by novel technologies, is reshaping markets and therefore the management of promoting activities. Nevertheless these multiplied data stay principally untapped by several firms, suggesting the potential for additional analysis developments. Market analysts and marketing strategists (Dickson *et. al.*, 2001) emphasize understanding the fundamental dynamics of a market, but how deeply do they think about the interplay of these fundamentals and what frameworks do they use for such reasoning?

### **Findings and Discovered Facts**

Fact 1 - The augmentation or expansion of features of an existing product can be quantified and realized based on Markov property based state transition.

**Justification –**

The augmentation or expansion of features of an existing product can be quantified based on the following relationship

$$Q_{s,t} = \frac{|t_p - t_a|}{t_a}$$

where  $t_p$  be the predicted estimation of viability of the augmented product at future timing instant  $t$

,  
 $t_a$  be the predicted estimation of viability of the existing product at timing instant  $t$  based on trend analysis

and  $Q_{s,t}$  be the measure of feature deviation.

After  $n$  successive stages of sensing predicted estimation of viability of the augmented product,

In generalized version,

$$w_{s_i > s_{i+1}} = \frac{f_i^* \times |t_{p_{i+1}} - t_{p_i}|}{t_{p_1} - t_a + t_{p_2} - t_a + \dots + t_{p_n} - t_a}$$

where  $f_i^*$  be the frequency of state transition over a specific period of observation

and  $w_{s_i > s_{i+1}}$  be the weight of state transition from  $i^{\text{th}}$  state to  $(i+1)^{\text{th}}$  state

Hence, it is justified to state that the augmentation or expansion of features of an existing product can be quantified and realized based on Markov property based state transition.

Fact 2 - In case of extreme strategic uncertainty , the realization of the quantified estimate of profit is governed by the property of Abelian group.

**Justification-**

In case of extreme strategic uncertainty towards prediction of market size, the predicted quantified estimate of submarket growth lacks proper co-ordination between present estimate with past one and present estimate with future event.

Let us assume  $T_E(\epsilon_t)$  be the time-stamp of observing the present quantified estimate of submarket growth ,

$T_E(\epsilon_{t-1})$  be the measure of lagging time with respect to quantified estimate of submarket growth

and  $T_E(\epsilon_{t+1})$  be the measure of leading time with respect to quantified estimate of submarket

growth. The entire time-stamps of realizing the past ,present and future quantified estimate of submarket growth can be represented by Abelian Group  $(G,+)$

Where  $T_E \epsilon_{t-1}, \epsilon_t, \epsilon_{t+} = \{0, \pm 1, \pm 2, \dots, \pm \infty\}$ ,

the identity element „0“ being  $T_E \epsilon_t$  .

The sets representing correlation of past and future quantified estimate of submarket growth with respect to present are

$$T_E \epsilon_{t-1}, \epsilon_t = \{ -1, 0 , -2, 0 , \dots , (-\infty, 0)$$

and

$$T_E \epsilon_t, \epsilon_{t+1} = \{ 1, 0 , 2, 0 , \dots , (\infty, 0)$$

In case of extreme strategic uncertainty, analyzing of event of timestamp  $T_E(\epsilon_t)$  leads to false belief. Hence, realization of the quantified estimate of submarket growth representing correlation of both present with past as well as present with future has to be distorted.

Fact 3 - The risk related to failure of the market to meet expectations can be realized based upon statistical modeling.

**Justification –**

The stochastic change in market growth often fails to meet expectations. In this perspective, forecasting demand seems to be a difficult task in dynamic market. The market impact of a brand new product in terms of sales and profit has to be investigated within a time period of observation.

Let  $T_1$  be the time stamp of new product launch in sub-market ,

$T_1 + \Delta t$  be the lower boundary of the time period of observation of the market impact of the product  $T_2$  be the upper boundary of the aforesaid period and  $\alpha$  be the bias market impact based on prior experiment of previously new products in the same market scenario.

The statistical modeling of the above case study in case of dynamic market , where there is no occurrence of a single instance such that observed impact meets the bias  $\alpha$  , is as follows :

Let  $P(T_1 + \Delta t)$  be the probability of occurrence of a single instance such that observed impact meets  $\alpha$  in timing instant  $(T_1 + \Delta t)$  and  $P(T_2)$  be that of a single instance such that observed impact meets  $\alpha$  in timing instant  $T_2$ .

Therefore, we can state that Hence,  $P(T_2) = P(T_1 + \Delta t) [1 - \epsilon x]$  where  $x = T_2 - (T_1 + \Delta t)$  and  $\epsilon$  is the constant of proportionality.

$$\lim_{x \rightarrow 0} \frac{P(T_2) - P(T_1 + \Delta t)}{x} = -\epsilon P(T_1 + \Delta t)$$

or,  $P(T_1 + \Delta t) = a e^{-\epsilon x}$ , where  $a$  is a constant of proportionality. Hence the risk related to failure of the market to meet expectations can be realized based upon statistical

Fact 4 - The investigation related to risk associated with super competitor late entry in a market can be governed by the basic principle of geometric distribution.

**Justification –**

Analysis of risks in high-growth markets is essential for proper understanding of the market. The conventional wisdom that the strategist should seek out growth areas often overlooks a set of associated risks. In this perspective, super competitor late entry is one of the major concerns. Even if the market impact of the product is well accepted still a similar product of superior quality and cheaper rate may lead to a significant threat. The role of the business strategist is to perform continuous independent trials and observe the timing instants of success. In this context, success indicates that the product possesses best market impact in spite of entry of similar products. The objective of the strategist is to search for optimum timing instant when the product is well accepted in the market.

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Let success probability be  $p$  in context to event  $S$  while  $m$  being the number of independent trails till  $S$  is incident. Hence  $m$  is discrete with range  $1,2,3,\dots$  and an event  $\{m=n\}$  indicates the validity of the sequence  $FF\dots FS$ .

Hence, by independence, the above fact has the probability  $p(1-p)^{n-1}$ . This signifies the validity of the basic principle of geometric distribution. The condition expectation is  $\alpha$  and can be computed as follows –

Let  $X$  is the probability that the product has the best market share,  $Y$  is the probability that the product has a superior competitor.

Therefore,  $\alpha = E[m|S]X + E[m|F]Y = 1 * p + (1 + \alpha)(1 - p) \dots\dots\dots (1)$

Hence, from Eq(1)  $\alpha = 1/p$ .

This reveals that the conditional expectation, that the product has the best market impact, can be derived from the success probability of the product.

Fact 5 – The investigation related to risk associated with price instability resulting from overcapacity, can be governed by the basic principle of geometric distribution.

**Justification -**

If the supply of a product is in bulk amount, then due to price pressure, the industry profitability will be short-lived. Accordingly the price of the product is decreased in order to attract customers and thereby clearing the stock. Hence, we can represent the aforesaid situation in the light of static mathematical model as follows –  $x = a - by$ , where  $x$  = supply,  $y$  = price,  $a$  and  $b$  are positive co-efficients.

Similarly, in order to attract customers, if demand increases then the business strategist can also lower the price. Hence,  $z = c - dy$ , where  $z$  = demand,  $a$  and  $b$  are positive co-efficients. In stable condition, demand equals to supply.

Hence,  $a - by = c - dy$  or,  $y = \frac{a-c}{b-d} \dots\dots\dots (2)$

The Eq(2) signifies that revised market price of the product in a situation when optimum risk factor due to price instability is taken care off.

Fact 6 – The investigation related to risk associated with resource constraints in high-growth markets, can be overcome using the Cobb-Douglas model.

**Justification –**

Manpower and resource are the two prime factors leading to optimum supply. The objective of the business strategist should be achievement of maximum supply for a given investment. Huge capital investment and related inadequate supply will lead to a serious threat in profitability. Correlating the above fact with the Cobb-Douglas model,

$S = \beta M^x R^y \dots\dots\dots (3)$

where  $S$  is the supply,  $\beta$  is a constant,  $M$  is manpower,  $R$  is resource, and exponents being  $x$  and  $y$ . If  $x$  and  $y$  are the elasticities of production with respect to manpower and resource investments.

When  $x = y = 1$ , then the percentage change in output equals the percentage change in any one of the two variables. From Eq(4.18), a linear relationship can be represented such that

$$C = zM + R \dots\dots\dots(4)$$

where C is the capital investment and co-efficient z is constant.

Hence  $C \propto R$  which signifies that if the resource constraints are overcome, then the capital investment will increase thereby reducing the risk factor associated.

Fact 7- The output of external analysis in the strategic market management is in setting to the classification and understanding that which is opportunity and which is threat can be characterized as a Bernoulli random variable.

**Justification –**

An opportunity is mainly a movement that indicates the major upward variation in sales and profit relationships whereas a threat indicates the substantial or major downward change. In the context of strategic ambiguity, study of the present sales and the profit pattern with respect to that of the past can be seen as a discrete random variable. Let X and Y be the variables represented for the opportunity and threat individually. Moreover, X and Y are two equally exclusive occurrences at any specific timing instant of the study period. Hence, R, is the Bernoulli random variable which is used for the external analysis of the business strategy in this condition, can be shown as –

$R = 1$  if X occurs, else  $= 0$  if Y occurs. Here, R is a statistical value of X or Y. Let  $p_o$  be the possibility of existence of the given opportunity and  $p_t$  be that of the threat.

Then the probability mass function (pmf) of R is presented as follows -

$$p(m) = p_o \quad \text{for } R=1$$

and 
$$p(m) = p_t = (1 - p_o) \quad \text{for } R=0.$$

Therefore the measured estimated value and the related variance which is equivalent to the forecast of opportunity in strategic uncertainty position is statistically characterized as –

$$E[R] = p_o$$

and 
$$\text{Var}[R] = p_o \cdot (1 - p_o) = p_o p_t$$

Likewise the calculated expected value and related variance which is in correspondence to the forecast of threat in strategic uncertainty state is statistically characterized as –

$$E[R] = p_t$$

and 
$$\text{Var}[R] = p_t \cdot (1 - p_t) = p_t p_o$$

Fact 8 - Study of statistical estimation of threat in the strategic market management can be ruled by the basis of the numerical feature of the failure rate function.

**Justification –**

In example of a significant downward change in area of sales and profit, the quantification of the threat estimation is measured by how the expected failure is at a timing instant t which is of a latest product which is now t time units old. Therefore it is mostly an uncertain rate of failure. Let  $t_x$  be the timing instant of the appearance of a new product in the market and  $t_y$  be the maximum projected

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life expectancy of the new product. Now the probability mass function  $p(t_y)$  is pretty low as it is doubtful that the product will be going to conclude exactly at the age of  $t_y$ . On the other hand, the failure rate  $f(t_y)$  is the conditional probability that the product is going to conclude as soon as after  $t_y$  time units and that is much greater. Therefore, the statistical feature of the failure rate function which is thinking  $T$  as the life expectancy which is going to take the lead as the following –

$$p(t_y) = P ( T = t_y ) ;$$

$$f(t_y) = P ( T = t_y | T \geq t_y )$$

The directly above mathematical calculations show the point that as the highest anticipated lifespan of the new product rises, which is corresponding to the probability mass function  $p(t_y)$ , will have a tendency to 0 while the failure rate  $f(t_y)$  will be likely towards 1.

Fact 9– In case of the existence of the event threat ,the assurance interval based statistical interference enables the improvement of possibility of the amount of opportunity in the future.

**Justification –**

The threat implies the degradation in profit. Critical estimation with error limits serves an idea of the intentions behind the existence of the event. In this perspective, we suggest the study of the business increase in specific time-intervals so that the real failure rate can be examined and identified correctly. It is desirable to maintain the confidence interval as short as possible to have the high precision in the gain estimation. This is a very important step in issue of external evaluation of business strategy. The primary purpose is to discover the unidentified parameters that is going to impact considerably towards the rate of downward change in sales and the profit.

Assume the gain estimates of a brand-new product which is in the timing interval  $[ T_1, T_n ]$  be  $G_{T_1}, G_{T_2}, \dots, G_{T_n}$ .

Let  $\alpha$  and  $\beta$  be the projected unknown parameters which is associated to each  $G_{T_i}$ .

Hence ,  $T_1 \leq \alpha \leq T_k (a)$

and  $T_1 \leq \beta \leq T_k (b)$

where  $[ T_1, T_k ]$  be the trust interval (  $k \leq n$  ),

$a$  be the trust level for  $\alpha$

and  $b$  be the trust level for  $\beta$ .

Let us suppose more that  $P(\alpha)$  be the probability of the involvement of  $\alpha$  for the existence of the threat and  $P(\beta)$  be the probability of the involvement of  $\beta$  for same.

In this condition, any one of the subsequent instances will happen –

Case 1 -  $P(\alpha) > P(\beta)$

Where  $E$  is an error bound which is defined such that  $P ( \alpha'' - E \leq \alpha \leq \alpha'' + E ) = a$  where  $\alpha''$  is an estimator for the  $\alpha$  . Therefore the factor  $\alpha$  is now established and built upon this an estimation for realizing the prospect in the future can be done.

Case 2 -  $P(\beta) > P(\alpha)$

Where  $E$  is an error bound which is defined such that  $P ( \beta'' - E \leq \beta \leq \beta'' + E ) = b$  where  $\beta''$  is an estimator for the  $\beta$  .

Case 3 -  $P(\alpha) = P(\beta)$

$$P(\alpha'' - E \leq \alpha \leq \alpha'' + E) = P(\beta'' - E \leq \beta \leq \beta'' + E)$$

Therefore, the confidence level for the both of the unknown factors are identical. On the base of the aforementioned statistical design and cases, we can declare that the confidence interval which is based on the statistical interference enables the development of possibility of occurrence of the opportunity in the future.

Fact 10– The chi-square distribution is established on the trend analysis and it plays a very important role in the context of external analysis of strategic market management.

### **Justification –**

An external analysis contributes to the strategy not directly by examining the trend of the gain assessment. In any business situation, evaluation of the sales and profit is very important for evaluating the current condition with respect to the present market demand.

Figuratively we suggest a trivalent state in such a way that

-1 implies loss in the sales and profit of a business,

0 implies no change

+1 implies the gain.

Likewise, the aforementioned states are mutually full and extensive in nature.

Let the sales and profit status of the business is examined and it is on the basis of n past sampled estimates,  $X_i$  signifies the number of observations that fall over in category i for  $i = 1, 2, 3$  as trivalent state has been understood and  $p(i)$  be the probability of existence of each state. Therefore,

$$\sum_{i=0}^3 (X_i - n \cdot p(i))^2 / n \cdot p(i) \text{ signifies the simple chi-square distribution.}$$

### **Conclusions**

The augmentation or expansion of features of an existing product can be quantified and realized based on Markov property based state transition. In case of extreme strategic uncertainty, the realization of the quantified estimate of profit is governed by the property of Abelian group. The risk related to failure of the market to meet expectations can be realized based upon statistical modeling. The investigation related to risk associated with super competitor late entry in a market can be governed by the basic principle of geometric distribution. The investigation related to risk associated with price instability resulting from overcapacity, can be governed by the basic principle of geometric distribution. The investigation related to risk associated with resource constraints in high-growth markets, can be overcome using the Cobb-Douglas model. The output of external analysis in the strategic market management is in setting to the classification and understanding that which is opportunity and which is threat can be characterized as a Bernoulli random variable. - Study of statistical estimation of threat in the strategic market management can be ruled by the basis of the numerical feature of the failure rate function. In case of the existence of the event threat, the assurance interval based statistical interference enables the improvement of possibility of the amount of opportunity in the future. The chi-square distribution is established on the trend analysis and it plays a very important role in the context of external analysis of strategic market management.



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