# Remedial Instructional Programme for Teaching Addition of Fractions to Children with Mathematical Disability (CwMD) In Inclusive Schools 

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#### Abstract

The article explains the different types of error committed and difficulties exhibited by Children with Mathematical Disability (CwMD). The authors made an attempt to know different types of errors committed and difficulties exhibited by the participants while doing the addition of fractions. The study aims in developing the remedial instructional programme for CwMD in fractions related to (addition of fractions). The effectiveness of the programme has been studied and results indicated that the intervention provided was effective in improving the performance of participants from pre-test to post-test. The study has implications for teaching multiplication of fractions in inclusive schools.


Key words: Remedial Instructional Programme, Mathematical Disability, Inclusive Education.

## Introduction

Learning fractions is difficult for children in general and especially difficult for children with Mathematical Disability (CwMD). ). Fractions are well known to be difficult to learn. Fraction sense "refers to a person's general understanding of fractions and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling fractions and operations" (McIntosh et al., 1992, p. 3). However, children encounter fractions as the most complicated mathematical concepts in primary and even in their middle years in school. Moreover, fractions play a key role in mathematics, since they are involved in probabilistic, proportional and algebraic reasoning. Fractions are critical component of mathematics understanding and a gateway for too many sought after occupations. Fractions are an essential foundational skill for future mathematics success (NMAP, 2008). Children with mathematics difficulties (MD) lag behind in numerous aspects of fraction knowledge, including comparing and ordering fractions, estimating fraction on a number line, performing fraction arithmetic calculations, and solving word problems involving fractions (Bailey et al., 2015; Cawley, Parmer, Yan, \& Miller, 1996; Hecht \& Vagi, 2010; Mazzocco \& Devlin,

2008; Siegler \& Pyke, 2013). Fractions are well-known to constitute a stumbling block for primary school children (Behr et al., 1983; Moss and Case, 1999; Grégoire and Meert, 2005; Charalambous and Pitta-Pantazi, 2007). Understanding difficulties in learning fractions seems absolutely crucial as they can lead to mathematics anxiety, and affect opportunities for further engagement in Mathematics. The learning of fractions is traditionally a difficult topic for many students (Charalambous \& Pitta-Pantazi, 2007; Meert et al., 2010; Pitkethly \& Hunting, 1996) especially when dealing with quantities in numerator and denominator. Pitkethly and Hunting (1996) posited that students view these two quantities as two separate entities of whole numbers instead of part-whole conceptualizations.

The importance of fractions extends beyond the school years. Fractions are essential foundational skill for future mathematics success (NMAP, 2008). The importance of fractions makes it a major topic in elementary and middle school curricula. According to Common Core State Standard Initiative (CCSSI, 2010), students should develop understanding in fraction in Grade 3 and Grade 4 , they should gain competence in fraction and word problems from Grade 4 to Grade 6and they should be able to apply fraction to problem solving ratios and proportions of Grade 6 and Grade 7.

## Objectives

1. To analyze the type of errors committed by CwMD in mathematics studying in Grade -VI and VII while attempting items relating to Addition of Fractions w.r.t different criterion measures of Grade V, VI and VII.
2. To plan out the remedial instructional Programme in Addition of Fractions for CwMD studying in the Grade VI and VII w.r.t different criterion measures of Grade - V, VI and VII.

## Methodology

The methodology related to the participants, tools and techniques method of collection and analysis of data are discussed in this section.

## Participants

In order to achieve the objectives of the study the participants, CwMD were selected from seven Government and Private Aided schools with Kannada as Medium of Instruction from Mysore City by applying a set of Exclusionary and Inclusionary Criteria. A total of 21 participants with CwMD were considered as the sample for the study.

Table -1
Performance of the participants in the criterion measures pertaining to Fractions (Addition of fraction) of Grade- V, VI, and VII.

| Sl. <br> No | Grade | CRITERION MEASURE |  |  |  |
| ---: | :---: | :--- | :---: | :---: | :---: |
| 1 | V | Find the sum of the given fractions | PA | NM |  |
| 2 | VI | Addition of fractions (having same denominator) | -- | ---- | 100 |
| 3 | VII | Addition of fraction | 33.33 | --- | 66.67 |

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Table - 02
Errors noticed in while doing the task in Fractions related to Addition by CwMD

| $\begin{aligned} & \text { Sl. } \\ & \text { No } \end{aligned}$ | Errors |  | Example | Probable Reasons |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Conceptual error Procedural committed | Error | To find the sum of the given fraction. <br> Example $\begin{aligned} & 7 / 9+3 / 9 \\ & =10 / 18 \end{aligned}$ | Does not know when there is a common denominator it should be taken directly and by not adding it. <br> Does not know that when denominators are same common denominator should be considered. <br> Lack of Procedural Knowledge |
|  | Did not attempt |  | $\begin{aligned} & 3 / 10+6 / 10+7 / 10 \\ & 31 / 4+4 / 5+2 / 3 \end{aligned}$ | Does not know how to do the mathematical operation i.e. addition. <br> Confusion prevailed while adding the fractions when numerators are different and denominator are same. <br> Does not know to add the fractions when the denominators are different. |
|  | Conceptual committed Procedural committed | Error <br> Error | $\begin{align*} & 5 / 6+1 / 3+5 / 2+6 / 3 \\ & 5+1+5+6 \\ & ----------=17 \\ & 6+3+2+3=---- \end{align*}$ $14$ | 1. Adds the numerator <br> 2. Adds the denominator. <br> 3. Does not know to take LCM when the denominators are not same. <br> 4.Does not have the conceptual understanding <br> 5.Does not have the Procedural Knowledge. |

Analysis was done to identify the types of error committed and the difficulties experience by the participants and the probable reasons for the errors/difficulties.

## Planning and Preparing the Remedial Instructional Programme

Based on the errors committed and difficulties exhibited by CwMD, the general principles suggested by various researchers a remedial instructional programme to teach Addition of Fractions was developed.
Some of the general principles to learn Fractions are

1. Readiness skill for learning fractions to be emphasized.
2. Teaching the same concept in different ways or representations.
3. Make use of simple vocabulary while teaching,
4. To foster the cognitive development.
5. Teaching should have direct impact on child's perception in learning.
6. Multisensory approach to be used while teaching.

## Specific principles to teach Addition of Fractions

1. To compensate for short-term memory performance cues should be used to remember steps while doing Addition of Fractions.
2. The terms and the symbols of addition of Fractions to be used frequently for better retention and better performance.
3. Activities should be drawn such that a child finds interest in learning.

## Main features of the Remedial Programme.

Keeping the above principles in mind the remedial instructional programme to teach addition of fractions to CwMD, was developed. Some of the main features of the programme are

1. The programme is designed in such a way that it caters the needs of the majority of children who have problem in learning the concept of addition of Fractions.
2. Each lesson has specifically designed instructional objectives.
3. Activities are arranged in sequential order.
4. The present learning activities were linked to the previous activities.
5. Achieving the objective of the previous class is a pre-requisite skill to go to the next lesson.
6. Concepts were taught using the concrete materials. Slowly, it was shifted to semi-concrete and finally the abstract form of addition of Fractions with different denominators was used.
7. Lessons were short requiring 20 min covering a specific concept.

## SAMPLE LESSON

The addition of fraction teaches us to add two or more fractions with same denominators and to take LCM different denominators are considered in addition of fractions. The addition of fractions depends on two major conditions.
a) Same denominator.
b) Different denominator.

## Addition of Fraction:

General Objective: To enable the children to understand the concept and procedure adopted in addition of fraction by taking common denominator and different denominators.

## Specific Objective:

a) The pupil will be able to identify fractions having common denominator and fractions having different denominator.
b) The pupil will be able to recognize fractions with same denominators and fractions having

## Teaching-Learning aids used:

1. Mathematics Table used.
2. Charts related to addition of fractions with common denominator and different denominator used
3. Writing board and color chalk used different denominators.

## Addition of Fractions with same denominator

If the denominators of two or more fractions are same then we can directly add the numerator keeping the denominator common.

## Example 1: General form of Addition of Fraction when the denominator are same.

1 . $\mathrm{a} / \mathrm{b}+\mathrm{c} / \mathrm{b} \quad--\rightarrow$ (The denominators are same, consider once, it is common denominator, here $b$ is common denominator, consider it once)

$$
\begin{array}{cl}
=\begin{array}{c}
\mathrm{a}+\mathrm{c} \\
------ \\
\mathrm{b}
\end{array} & \rightarrow \text { (Add the numerator) } \\
\text { (Common denominator) }
\end{array}
$$

## Example 2:

Add the fractions 4/6 + 7/6
$4 / 6+7 / 6 \rightarrow$ [Look at the numerator, add them]
$=4+7 \quad \rightarrow \quad$ [Add the numerator]
---------
$6 \rightarrow \quad$ (Look, at the denominator they are same, take the common denom

## Follow the below steps to add the fractions with same denominator.

$>$ Add the numerator together, keeping the denominator common.
$>$ Writing the simplified fraction

## Example 3: Addition of fractions with different denominators.

$[(9 / 6)+(3 / 4)]$

## Method-01:

Step -01: Cross multiply the left numerator with the right denominator and right numerator with the left denominator).[Cross multiplication done representing through the arrows using color chalk].
Step-02: Multiply the denominators, they are different. (There is no common denominator). [Asked students whether the denominators considered are same or different]
Step -03: Take LCM of the denominator
Step-04: Finally add the numerator and the denominator.

1. Add the given two fractions $[(9 / 6)+(3 / 4)]$.
= ---------------------
(6x4)

6

4

| 6,4 |
| :--- |
| 1,4 |
| 1,1 |

LC M 6x4=
24
24
$=54$
24

| $6 \times 1=6$ | $4 \times 1=4$ |
| :--- | :--- |
| $6 \times 2=12$ | $4 \times 2=8$ |
| $6 \times 3=18$ | $4 \times 3=12$ |
| $6 \times 4=24$ | $4 \times 4=16$ |
| $6 \times 5=30$ | $4 \times 5=20$ |
| $6 \times 6=36$ | $4 \times 6=24$ |
| $6 \times 7=42$ | $4 \times 7=28$ |
| $6 \times 8=48$ | $4 \times 8=32$ |
| $6 \times 9=54$ | $4 \times 9=36$ |
| $6 \times 10=60$ | $4 \times 10=40$ |

## II

6
[(9/6)
$+$
Method:
(3/4)]

| 6,4 |
| :--- |
| 1,4 |
| 1,1 |

4

Step 1: Consider each of the fractions separately and multiply with the L C M
a) 9
$-\quad$ x $24 \rightarrow$ (The denominator and the L C M has to be divided)
6
$6 \times 4=24$

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$$
9 \times 4=36 \rightarrow(1)
$$

Step 2: Consider the other fraction and multiply the numerator with the LCM or divide the denominator with LCM.

| $6 \times 1=6$ | $4 \times 1=4$ |
| :--- | :--- |
| $6 \times 2=12$ | $4 \times 2=8$ |
| $6 \times 3=18$ | $4 \times 3=12$ |
| $6 \times 4=24$ | $4 \times 4=16$ |
| $6 \times 5=30$ | $4 \times 5=20$ |
| $6 \times 6=36$ | $4 \times 6=24$ |
| $6 \times 7=42$ | $4 \times 7=28$ |
| $6 \times 8=48$ | $4 \times 8=32$ |
| $6 \times 9=54$ | $4 \times 9=36$ |
| $6 \times 10=60$ | $4 \times 10=40$ |
|  |  |

b) --- x $24--\rightarrow$ ( The denominator and the L C M has to be divided) 4
$3 \times 6=18 \rightarrow$ (2)

Step 3: Add the product of both the fractions With the denominator
4)24(6

24
$4) 24(6$
24
------
0

| $36+18$ |
| :---: |
| ------- |
| 24 |$=$| 54 |
| :---: |
| ---- |
| 24 |

Table - 3
Performance of the participants in the criterion measures pertaining to Fractions (Addition of fraction) of Grade- V, VI, and VII in Pre-Test and Post-Test.

| Sl. <br> No | Grade |  | Pre-Test |  |  | Post-Test |  |  |
| :---: | :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | CRITERION MEASURE | M | PA | NM | M | PA | NM |
| 1 | V | Find the sum of the given <br> fractions | 40.47 | 40.13 | 19.4 | 90.47 | 9.53 | -- |
| 2 | VI | Addition of fractions (having <br> same denominator) | -- | ---- | 100 | 47.61 | 47.63 | 4.76 |
| 3 | VII | Addition of fraction | 33.33 | --- | 66.67 | 100 | -- | -- |

## Table - 4

Percentage of mastery, Partial-achievers and Non-mastery in the criterion measures pertaining to Addition of Fractions in Pre-Test and Post-Test.


Here, M $\rightarrow$ Refers to Mastery (Mastery to be attained at 80\%)
PA $\rightarrow$ Partial Achievers (PA to be attained at $75 \%$ )
$\mathrm{NM} \rightarrow$ Non-Mastery (NM to be attained when does not attain)



## Major Findings

1. The research findings show an alarming $7 \%$ incidence of CwMD in upper primary school.
2. A systematic procedure was adopted to identify children with Mathematical Disability in upper primary school.
3. Errors related to addition of fractions helps in understanding the difficulties experienced by CwMD.
4. A suitable remedial instructional programme was developed to teach addition of fractions to CwMD.
5. Teachers should be trained in methods to teach CwMD and other children who face difficulties in learning mathematics.
6. Addition of fraction having same denominator was found to be most difficult in the pre-test and mastery was seen at $47.61 \%$ in the post-test and partial achievers at $47.63 \%$.
7. Addition of fraction was seen to have $100 \%$ mastery in the post-test from $66.66 \%$ of non mastery in the pre-test of Grade VII.

## Implications of the study

1. Since the programme developed to teach Addition of fraction to CwMD , this programme can be used for any children who are having difficulty in understanding the addition of fractions due to various other reasons in upper primary schools.
2. As factions are found to be difficult for most of the normal children also. So, the remedial strategies suggested here can be made use to teach n the regular classrooms so that it will be helpful to the normal children.
3. Children with Mathematical disability (CwMD), can overcome their problem if the specific deficits are identified and faulty strategies adopted to do the operations are rectified.

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